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シンポジウム：複雑性・合理性・定常性をめぐって

## Professor Shiozawa's Case against General Equilibrium : A Comment

Jean Cartelier

Professor Shiozawa's positions favouring stationarity as a central element of a research program alternative to General Equilibrium Theory (GET) are both interesting and stimulating. They are founded on the very lucid idea that Classical economics is not a special case of GET. Classical economics, in the sense of Ricardo and Torrens, rests on von Neuman's model, which is irreducible to Arrow-Debreu's one. Some of the critiques Professor Shiozawa directs against GET are perfectly relevant and his general orientation deserves to be taken seriously.

But it would not be very useful to extensively insist on the points in agreement. It is more interesting to mention differences in interpretations and intuitions, mainly if they allow to raise some still pending questions. To say it too quickly, Professor Shiozawa's critique of GET, however radical it may be, leaves aside a very fundamental question about equilibrium : *why is it that standard GET cannot tell us anything about market outcomes out of equilibrium and what are the assumptions which are necessary in order to remove this drawback ?* Professor Shiozawa would answer by pointing the preference of economists for equilibrium at the expense of stationarity. It is a sensible explanation. Neoclassical economists are at a loss with stationarity<sup>1)</sup>. He reminds us also that von Neumann's model is an alternative to Arrow-Debreu's one. But, important as they are, these considerations do not concern the specific question of a market economy which encompasses both decentralization and coordination of individuals. This point is the one around which the following comment is organized.

Professor Shizawa is certainly right when he recalls that equilibrium in GET is defined by two conditions :

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1) see Franco Donzelli (1989), *The Concept of Equilibrium in Neoclassical Economic Theory. An Inquiry into the Evolution of General Competitive Analysis from Walras to the Neo-Walrasian Research Programme.* Ph. D. Dissertation, University of Cambridge.

1. individual agents are on their preferred positions, parameters being given (either prices or other individuals' actions)
2. individual actions are all feasible (mutually compatible)

But he should have remind us as well that equilibrium conditions are the only possibility of determining prices and allocations. Equilibrium conditions are nothing but the economic counterpart of the mathematics of building and solving systems of equations. Excess-demand functions result from the aggregation of functions derived from individual maximization of utility under budgetary constraints at parametric prices (condition 1). Excess-demand functions must be all simultaneously non-positive (the so-called equality between supply and demand) as a necessary condition of mutual compatibility of desired transactions (condition 2). A system of equations is thus obtained with certain mathematical properties, coming from technical (continuity) and economic (convexity) assumptions. To solve this system of equations is nothing but to determine prices. In this view, to demonstrate the existence of general equilibrium prices for a competitive economy may appear as a splendid achievement.

But, at the same time, this quick reminiscence of GET clearly shows that the mere idea of the existence of prices and allocations out of equilibrium is pure non-sense. Equilibrium magnitudes and solutions of equations are the same thing. It would be therefore self-contradictory to search for 'solutions which are not solutions' or 'prices not being equilibrium prices'.

The constitutive inability of dealing with situations out of equilibrium makes GET almost irrelevant for the study of market economies. All economist agree on the idea that a market economy is a decentralized one. Common sense would admit that individual actions, in such a context, may have unintended results (even if it is believed that market forces strongly stabilize the whole : market forces make sense only out of equilibrium!). It is not sufficient to say that *tâtonnement* processes are non-stable or that they take a too long time : they simply have nothing to do with the notion of a market economy. The Walrasian *tâtonnement* process may at most be interpreted as a centralized negotiation taking place before the market, which is a notion more appropriate to democratic planned economies than to market ones.

What are the features of GET which are responsible for its poor adequation to the alleged purpose of giving an account of a market economy? The answer to this question is not so difficult to find out : assumptions about individuals' characteristics (endowments and preferences defined on a commodity-space) and behaviour (maximization of utility or profit functions) are simply not sufficient to get a relevant description of such an economy, *i. e.* to model the working of the market. They are not even sufficient to built a system of excess-demand functions

determining rates of exchange. It is necessary to assume also that prices are parametric, which means that no individual agent can manipulate them. Note that introducing parametric prices amounts to add a social element to individual data. Prices exist but not through the will of individual agents. They come from elsewhere. Moreover, they are supposed to be coherent and the same for every agent. Such an assumption, which is most often considered as synonymous of perfect competition, put under the rug two difficult problems : uniqueness (why not several prices for the same commodity?) and consistence of prices (why not  $n(n-1)/2$  different rates of exchange?). This 'social' element which is added to the first one (the commodity-space) allows one to determine nothing but equilibrium prices (under the usual assumptions). A model founded on this set of assumptions does not allow to understand a market coordination between individuals except in the very special case of equilibrium.

If this diagnosis is correct, the remedy does not simply consist in passing to a dynamic analysis. Non-*tâtonnement* models have the same drawback. This is clear as soon as it is recognized that they differ from Walrasian *tâtonnement* models mainly by the adjunction of an arbitrary rationing scheme. At each point of the trajectory, there is an equilibrium in the sense that prices and allocations are such that they satisfy an arbitrarily given rationing scheme.

It is not even sufficient to introduce additional assumptions concerning the exchange technology (which is badly lacking in Arrow-Debreu model) in order to cure traditional GET models from their inability to give an account of a market economy. Search-theoretic models exhibit some important results, namely a positive price for money and/or its acceptance at equilibrium<sup>2)</sup>. But, they share with standard GET the equilibrium method of determining prices and allocations and therefore the same basic drawback : non-equilibrium situations are out of the picture.

A solution must be searched for in a more fundamental direction. The point is : what is necessary and sufficient to add to GET's fundamental assumptions in order to get a relevant description of a market economy?

At this point, two lines of reasoning are possible. The first one consists in making rationality assumptions responsible of all the evils of GET. This line has been followed by most of economists dissatisfied with GET. Bounded rationality is claimed to be a good candidate for replacing 'parametric rationality', satisfying for maximizing, uncertainty for risk. The second one is to acknowledge the need for an additional assumption concerning a market mechanism.

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2) K. Iwai (1988), 'The evolution of money—A Search-Theoretic Foundation of Monetary Economics', CARESS working paper #88-3, University of Pennsylvania A. Trejos and R. Wright (1995), 'Search, Bargaining, Money and Prices', *Journal of Political Economy*, 103, 118-141.

Professor Shiozawa prefers the first type of critique. To the traditional reasons of suspecting rationality assumptions, he adds an interesting argument founded on the impossibility of solving maximization programs when complexity arises (pp. 90-91). This argument is very smart but not so much convincing. As a matter of fact, it applies well in the case of centrally planned economies where it is clear that computations must be effective. Computation here is not a metaphor but a necessary procedure. This is far less clear for a market economy. Nobody sensible would maintain that individuals, in the real world, embark upon long and complex computations. Walras, for instance, was very conscious of that. Maximization was for him an 'as if' assumption which does not imply that individuals effectively run computations of any sort. It is only a useful way of stating that individuals do not behave at random. A billiard player does not solve any system of equations. Nevertheless, it is possible to determine what are the mathematical conditions he has to comply with in order to succeed. Nobody would maintain that a good billiard player must be mathematically trained! Moreover at the high level of abstraction of the GET, which corresponds to an equally high level of abstraction of the question to be solved, maximization is not a that bad assumption. Satisfying would not be less arbitrary and less mechanical.

What is unacceptable with GET is less assumptions concerning the behaviour of agents than the lack of concern with the problem of coordination between individual actions. It is why a second line of reasoning seems more fruitful which put emphasis on the way market outcomes result from individual decisions.

A very interesting case is given by some strategic market games models<sup>3)</sup>. These models are characterized by a market mechanism defined independently of (logically prior to) any behavioural assumption. To take the most simple case, a market mechanism indicates :

- (1) what are the markets : say,  $n-1$  markets,  $i=1, 2, \dots, n-1$
- (2) how people meet in the markets: say, individuals desiring to sell bring quantities  $q_i$  of commodity  $i$  in the market for  $i$  and individuals desiring to buy put a quantity of commodity  $n$  (money)  $m_i$  in the market for  $i$
- (3) what are the market outcomes : say, market price for commodity  $i$  is given by the ratio of the sum over all the agents of money put in the market to the aggregate quantity of commodity brought in the market, market allocation of any individual in commodity  $i$  is given by the sum of his initial endowment minus the quantity he has brought in the

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3) For an overview, see Shubik, Martin (1990), 'A Game Theoretic Approach to the Theory of Money and Financial Institutions' in *Handbook of Monetary Economics*, ed. by B. M. Friedman and F. H. Hahn, vol 1, Elsevier Science Publishers.

market plus the ratio of the quantity of money he has put in the market for  $i$  divided by the market price of commodity  $i$ .

The example above<sup>4)</sup> shows that a mechanism determining market prices and individual market allocations as functions of individual actions can be defined without mentioning maximization, satisfying or whatever individual behaviour. As a consequence, market prices and allocations are obtained independently of any reference to equilibrium.

Assuming a market mechanism breaks the link between equilibrium and price and allocation determination, typical of GET. Given any individual actions (positive vectors in quantities), it is possible to determine market prices while it is impossible to know if the economy is or not in equilibrium.

Equilibrium is defined by reference to individual behaviour assumptions which are logically posterior to mechanism assumptions. The most usual notion is that of Nash equilibrium which implies that no one has an interest in deviating from a situation where each agent plays his best response to actions of others.

It is worth noting that the assumption of a market mechanism was part of preclassical economics. The rule above can be traced back to Cantillon: 'Les prix se fixent [sur les marches] par la proportion des denrees qu'on y expose en vente et de l'argent qu'on y offre pour les acheter'<sup>5)</sup>. It can be found again in Keynes' *Treatise on Money* (see the fundamental equations of chapter 10). More generally, it should be remarked that such a rule requires that money and not only commodities should be given at the very starting point of the reasoning.

Although it is not possible to develop this point here, the very question about the role and place of equilibrium is closely related to that of money. In order to define a market mechanism, which is the condition to disentangle price determination and equilibrium, it is necessary to put money into the model as a fundamental ingredient. This amounts to acknowledge that money is not a special commodity but rather a social mechanism. It appears as a set of rules shaping in a decisive way the working of a market economy<sup>6)</sup>.

Recognizing that economic theory should be founded on an explicit market mechanism is necessary but does not wholly answer to the questions raised by Professor Shiozawa. As developments above make it clear, a market mechanism leaves aside the question of individual behaviour and, as a result, that of equilibrium.

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4) The example is drawn from Shapley L. and Shubik M. (1973), 'Trade using one commodity as a means of payment', *Journal of Political Economy*, 85, 937-968.

5) Cantillon R. (1952), *Essai sur la nature du commerce en général*, INED, Paris, p. 7.

6) Cartelier Jean (1995) *La monnaie*, Flammarion, Paris.

At this point Professor Shiozawa puts forward an interesting proposition. His main argument is that unless people are not too far from their usual environment, economic system is stable and works because people know how to behave. It is worth trying to rely this proposition to what precedes.

First of all, it can be noticed that Professor Shiozawa's remark is in accordance with a thesis advocated by Kreps about the relevance of Nash equilibrium. Kreps maintains that Nash equilibrium is an interesting concept only in games where people have an 'obvious way to play'. This is also reminiscent of Keynes's notion of convention (*General Theory's* chapter 12).

Professor Shiozawa prefers to relate his point to stationary states. 'A stationary state is a running state of an economy where everything is repeated *quite* regularly' (p. 95, author's emphasis). He sees in Marx's scheme of reproduction and in Sraffa's *Production of commodities* an example of such stationary processes. But this does not go without problems. Defining a stationary state by a 'voisinage' of the stationary solution of a dynamic system is interesting but raises the usual objections to dynamic analysis, i.e. its relative lack of robustness. Criteria for Lyapunov stability are not easy to exhibit and rarely support a clear-cut economic interpretation.

A suggestion, for relating professor Shiozawa's point to the question of market coordination, would be to refer less to the value of some variables than to the permanence of the 'rules of the game'. To take the example of Cantillon's rule backed by the rules of a system of payment, solvency of agents is a fundamental consideration. Disequilibrium appears under a monetary individual aspect, that is positive and negative monetary balances. Obviously, there are limits that negative balances should not overpass. Beyond a certain amount of indebtedness (relative to income or wealth) agents' ability to pay back is suspected and there is a very high risk of going into bankruptcy with possible risk system effects. To put it shortly, along any market mechanism comes a *constraint set* encompassing all the states of an economy which are admissible in the sense that they do not put the economy into danger. Such a constraint set is a first useful theoretical reference in the institutional approach briefly sketched in this comment.

This is not the end of the story. The system of payment associated to any market mechanism does not work but under a regulation. In modern payment systems, credit money is a common means of payment. Despite the ultra-liberal arguments of Free Banking partisans, it can be shown that credit or private money is not sufficient. A lender of last resort is a necessary element of the system. That means that, besides *state variables*, a monetary economy contains also *control variables* which must be taken into consideration.

Manipulation of control variables is then part of the story and enters the model. It

contributes to modify dynamic trajectories in a very complex manner. Too complex indeed to be studied by usual methods of dynamics. Without embarking upon a technical development, far beyond the author's ability, it is worth mentioning here the mathematical theory of viability<sup>7)</sup>.

In this context, the problem is to determine a subset of this constraint set having the following property : from any point of this subset starts at less one trajectory which thanks to controls manipulation remains in the constraint set. The largest among these subsets (if they exist) is called the *viability kernel*.

To come back to Professor Shiozawa's argument, it appears that viability considerations allow to give it an interesting *rationale*. For instance, it is possible to define *viability niches* which are viability sets characterized by a complete inertia in the controls. More generally, while the economy evolves into viability sets, in which its reproduction (in a loose sense) is not put at stakes agents behave routinely and inertia is the main force at work in the economy. As soon as the frontiers of the viability set is reached something has to change either in the behaviour of agents or in the regulation rules followed by the authorities.

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7) A complete view of viability theory as applied to economics is given in Aubin Jean-Pierre (1997), *Dynamic Economic Theory, A Viability Approach*, Springer.