Elasto-Plastic and Finite Element Analysis using Beam-Column Element for Concrete Filled Steel Tubes Subjected to Torsion

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Synopsis

Presented in this paper is mechanical properties and shear stress-shear strain curve for concrete, which is filled into a steel tube subjected to torsion. The concrete-filled steel tube is idealized by the assembly of beam-column elements consisting of the steel tubes and encased concrete for an analytical program on the basis of elasto-plastic and finite displacement theory, now being developed by the authors. The shear stress-shear strain curve of the encased concrete is assumed to vary on the Kiousis’ hardening parameter. The validity of the suggested stress-strain relationship for the encased concrete is verified through the comparison of the analytical results with the experimental results.

KEYWORDS: Stress-strain curve, Concrete-filled steel tube, Kiousis’ hardening parameter, Torsion

1. Introduction

Concrete filled steel members are widely used as structural member for structural rationality and improvement in the ductility of steel members. Concrete filled steel tube takes advantages of the improvements on the ultimate strength and ductility because the local buckling of the outer steel plates toward the inside of the cross section can be prevented. Moreover, the ultimate strength is larger than a cumulative strength of the steel and concrete parts due to the confined effect of the encased concrete.

Many researches on such concrete filled steel tubes and the confined effect have been conducted and these results are completed to the reference 1), for instance. However, most researches focus on concrete filled tubes subjected to compression and bending and few researches are objecting torsion.

Then, in this research, the shear stress-shear strain relationship of the encased concrete in concrete filled steel tubes subjected to torsion is suggested by referring to the experimental results2,3). Results from elasto-plastic finite element analysis of concrete filled steel tubes subjected to torsion are verified through the comparison with the experiment results.

2. Test Specimens and Materials

2.1. Test specimens

2 types of test specimens of square cross-section are prepared and a steel hollow specimen with the dimensions

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of $163 \times 123 \times 4.5\, \text{mm}$ is set as a basic specimen of the two. The other 1 type specimen is a concrete specimen of the same dimensions of the inside concrete of the basic specimen. 2 types of test specimens of circular cross-section are also prepared; one is a steel hollow specimen with the dimensions of $\phi 139.8 \times 4.5\, \text{mm}$ as a basic specimen, another is a concrete filled version of the basic specimen.

2.2. Material properties

Tension test to obtain mechanical properties of the steel plate used in the experiment are conducted by using the coupon test specimens of JI55 type for the square specimens and of JI512 type for the circular specimens. The tension test result is shown in Table1. Since the yielding point did not appear clearly, the yielding stress is calculated by referring to the 0.2% offset value.

Using two types of column specimens, $\phi 100 \times 200\, \text{mm}$ and $\phi 200 \times 100\, \text{mm}$, the material testing of concrete used is performed on the day when the loading test using each specimen is carried out. Table2 is summaries the test results.

3. Ultimate Strength of Concrete Filled Steel Tubes and Stress-strain Relationship of Encased Concrete

3.1. Calculation of ultimate strength of concrete filled steel tubes

The fully plastic torsional moments, $T_{psc}$ of a concrete filled steel tube can be defined by the following equation according to the cumulative strength method.

$$T_{psc} = T_{ps} + T_{pc}$$

Where $T_{ps}$ is the fully plastic torsional moments of steel tube and is calculated by the following equation.

$$T_{ps} = 2A \tau_{sy} t$$

Where $A$ is the cross sectional area surrounded by the center lines of the outer steel plates, $\tau_{sy}$ is the shear yielding stress of the steel plate and $t$ is the thickness of flange and web plates.

$T_{pc}$ is the fully plastic torsional moment of the encased concrete and is given for the square cross-section by the equation (3) and for circular cross-section the equation (4).

$$T_{pc} = \frac{1}{6} a_c^2 (3b_c - a_c) \tau_{cc}$$

$$T_{pc} = \frac{1}{12} \pi D_c^3 \tau_{cc}$$

Where, $a_c$ and $b_c$ are the width and height of the encased concrete, respectively, $D_c$ is the diameter of the encased concrete and $\tau_{cc}$ is the shear strength of the encased concrete. $\tau_{cc}$ is defined by the following equation

$$\tau_{cc} = 0.5 \sigma_{el}$$

3.2. Stress-strain relationship of encased concrete

The yielding function by Drucker-Prager is employed as the yielding function of the encased concrete.

$$\sigma_1 + \sqrt{J_2} - k - \kappa = 0$$
Where \( \alpha \) and \( k \) are material coefficients, \( I_1 \) is the first invariant of stress tensor, \( J_2 \) is the second invariant of deviatoric stress tensor and \( \kappa \) is Kiousis’s hardening parameter derived from the following equation:

\[
\kappa = \frac{H}{PA} \left[ (PA + 1 - Ax)e^{\alpha x} - (PA + 1) \right]
\]

(7)

Where \( x \) is the equivalent plastic strain, \( P, H \) and \( A \) are constant values, which are settled according to the material properties of concrete.

The shear stress-shear strain relationship of the concrete and Kiousis hardening parameter-equivalent plastic strain relationship can be illustrated Fig. 1.

In considering that only the torsion is subjected to the concrete filled steel tube, the equation (6) can be changed into the following equation.

\[
\tau_c = \tau_{cy} + \kappa
\]

(8)

Where \( \tau_{cy} (= \tau_{co}) \) is the shear yielding stress of the encased concrete, \( \tau_{ct} (= 1.5 \sigma_{ct}) \) is the shear strength, defined by the concrete tension strength3).

The shear strain \( \nu_{ce} \), equivalent to shear strength \( \tau_{ce} \) of the encased concrete, is assumed by the following equation:

\[
\nu_{ce} = \left[ 1 + 4.7(K - 1) \right] \nu_{c0}
\]

(9)

\[
\nu_{c0} = \frac{\tau_{ct}}{C_c}
\]

(10)

Where \( K = \tau_{cd} / \tau_{cy} \), \( G_c \) is the shear modulus in the elastic region.

### 3.3. Stress-strain relationship of steel and residual stress

The stress-strain relationship of steel materials is modeled into a tri-linear relationship based on the tension test result of the steel materials. The residual stress distribution of the square cross-section is assumed as shown in Fig. 2.

<table>
<thead>
<tr>
<th>Cross-section</th>
<th>Thickness of plate (mm)</th>
<th>Young’s modulus ( E_c )(N/mm²)</th>
<th>Poisson’s ratio ( \mu_p )</th>
<th>Yielding stress ( \sigma_{yf} )(N/mm²)</th>
<th>Tension strength ( \sigma_{ct} )(N/mm²)</th>
<th>Percentage of elongation (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Square</td>
<td>4.5</td>
<td>( 1.99 \times 10^5 )</td>
<td>0.294</td>
<td>274.0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Circular</td>
<td>4.5</td>
<td>( 2.15 \times 10^5 )</td>
<td>0.273</td>
<td>352.8</td>
<td>424.3</td>
<td>39.0</td>
</tr>
</tbody>
</table>

Table 1 Material properties of steel plates used

<table>
<thead>
<tr>
<th>Cross-section</th>
<th>Young’s modulus ( E_c )(N/mm²)</th>
<th>Poisson’s ratio ( \mu_c )</th>
<th>Compression strength ( \sigma_{cf} )(N/mm²)</th>
<th>Tension strength ( \sigma_{cd} )(N/mm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Square</td>
<td>( 2.05 \times 10^4 )</td>
<td>0.190</td>
<td>20.0</td>
<td>2.3</td>
</tr>
<tr>
<td>Circular</td>
<td>( 3.35 \times 10^4 )</td>
<td>0.211</td>
<td>63.4</td>
<td>3.1</td>
</tr>
</tbody>
</table>

Table 2 Material properties of concrete used

<table>
<thead>
<tr>
<th>Cross-section</th>
<th>Tested value ((P_u)(kN \cdot m))</th>
<th>Calculated value ((P_p)(kN \cdot m))</th>
<th>( P_u / P_p )(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Square</td>
<td>36.0</td>
<td>35.0</td>
<td>102.9</td>
</tr>
<tr>
<td>Circular</td>
<td>46.2</td>
<td>45.5</td>
<td>101.7</td>
</tr>
</tbody>
</table>

Table 3 Comparison of the ultimate strengths

61
Fig.1 Stress-strain relationship of encased concrete

Fig.2 Distribution of residual stress in square cross-section of test specimen

Fig.3 T-θ relationships for square cross-section

Fig.4 T-θ relationships for circular cross-section

4. Comparison with Experiment and Analytical Results

The tested and calculated values of the ultimate strengths are summarized in Table 3. The torsional moment $T$-twisted rate $\theta$ relationships of the test specimens with the square cross-section are plotted in Fig. 3. Y-axis is a non-dimensioned applied torsional moment divided by the fully plastic torsional moment. According to this figure, the obtained curves of torsional moment $T$-twisted rate $\theta$ show good agreements in the linear portion, and the portion which the stiffness changes suddenly. Focusing on the ultimate strength, the calculated value, $T_{pcs}$ tends to be smaller than the tested value. The ultimate strength $T_{ucs}$ is 103% of $T_{pcs}$. The stiffness of the concrete filled specimens is larger than that of steel hollow tube.

The torsional moment $T$-twisted rate $\theta$ relationships of the circular cross-section are plotted in Fig. 4. According to Fig. 4, both the steel tube and the concrete filled steel tube show the same tendency in the elastic region. It is considered that the filled concrete and the outer steel tube behave almost separately due to slide at the contacted part. Based on this result, the elastic shear modulus of concrete $G_c$ is changed into a modified one like $G_{cm} = k G_c$ ($k$ is the coefficient of modification) in the analysis, that is the coefficient $k=1.0$ is replaced to $k=0.5$ and 0.2. As shown in Fig. 4, tested and the analyzed curve shows greatly good agreement when the modified coefficient $k=0.2$ is used, and the calculated ultimate strength $T_{uCS}$ is 102% of $T_{pcs}$.

5. Conclusions

1) The ultimate strength and torsional moment-twisted rate relationship of the concrete filled steel tube subjected to torsion can be simulated.
2) A modified coefficient $k=0.2$ for the elastic shear modules of encased concrete of the concrete filled steel tubes with circular cross-section is suggested. Then, numerical analysis using the modified coefficient can simulate the test result with high accuracy.

References


