The International Pure Public Goods*

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要約

世界に2国（第1国と第2国）が存在するが、第2国は何かの理由で国際純粋公共財を提供できないとする。そのとき、一般には、第2国は第1国へトランスファーを与えるか、またはすでに持っている第1国への偏値の少なくとも1部を放棄することが有利となる。それにもかかわらず、第1国によって供給される国際純粋公共財は、（想像上の）世界政府が供給する量はもちろんのこと、両国がともに独立に供給することができた場合の量に及ばない。

Summary

Suppose that there are two countries, Country 1 and Country 2 in the world but that Country 2 cannot provide the international pure public goods for some reason. Then, in general, it is advantageous to country 2 to provide the grant to Country 1 or abandon at least a part of that initial claim against Country 1 (if any). Nevertheless the amount of the international pure public goods provided by Country 1 is less than not only that provided by the (imaginary) world government but also that which could be provided by both independent countries.

The main purpose of this paper is to translate Prof. Hoshikawa’s interesting paper (Hoshikawa (2004)) into a simple mathematical model.1)

Keywords: international pure public goods, transfer, a claim against a foreign country, the world government.

* This paper is dedicated to Prof. Yoshitaka Hattori and the late Ms. Kayoko who was his lovely wife. The original version of this paper was read as a comment on Mr. Hoshikawa’s report at “Seidoron-Kenkyukai” of Osaka University of Economics. I am grateful to the participants for their helpful discussion.

1) Almost all recent papers (including one in this volume) written by Prof. Hoshikawa are stimulating and instructive for me. This paper was also written under influence of his paper (Hoshikawa (2004)). The differences between us are as follows.

(i) Hoshikawa is concerned with the virtual transfer or reduction in a claim against the U. S. through the unexpected change in foreign exchange rates that actually occurred between Japan that is prohibited from supplying the IPPG directly and the U. S., but I almost ignore such financial phenomena and focus on the real aspect that underlies them.

(ii) Hoshikawa seems to consider that the almost sufficient supply of the IPPG could be realized.
[1] The model

Consider the representative individual who lives in Country 1. He cannot emigrate to Country 2. But he can be free to borrow or lend at the common real rate of interest\(^3\). He can also buy any private goods at the same price from any country. In this sense both the parity of interest rate and the parity of purchasing power hold in the foreign exchange market.

The individual maximizes the following utility with respect to \(C_1\) (the consumption), given \(G\) (the international pure public goods, IPPG\(^3\)). That is, maximize

\[
U_1 = \int_0^\infty e^{-\rho t} (\theta \ln C_1(t) + (1-\theta) \ln G(t)) \, dt
\]

s.t.

\[
(1) \quad \dot{D}_1(t) = r(t) D_1(t) + C_1(t) + T_1(t) - Y_1(t),
\]

where \(G, D_1, T_1, Y_1, \rho\) and \(r\) denote the amount of IPPG, the debt, the lump-sum tax, the income, the time preference and the real rate of interest, respectively and \(1 > \theta > 0\).

This leads to the Hamiltonian

\[
H_1 = e^{-\rho t} (\theta \ln C_1(t) + (1-\theta) \ln G(t)) + \lambda_1(t) e^{-\rho t} (r(t) D_1(t) + C_1(t) + T_1(t) - Y_1(t))
\]

Hence

\[
\frac{\theta}{C_1(t)} \lambda_1(t) = 0
\]

and

\[
\dot{\lambda}_1(t) - (\rho - r(t)) \lambda_1(t) = 0.
\]

Therefore

\[
(2) \quad \frac{\dot{C}_1(t)}{C_1(t)} = r(t) - \rho
\]

The transversality condition (the non-ponzi condition) is

\[
(3) \quad \lambda_1(t) e^{-\rho t} D_1(t) = \frac{\theta D_1(t)}{C_1(t) e^{\rho t}} \to 0 \quad (t \to \infty)
\]

\(^3\) by movement of the foreign exchange rate at the cost of its stability. But I believe that, apart from the volatility of exchange rates, transfer or reduction in a claim cannot achieve even the level of IPPG that would be provided independently by each country.

2) I assume the perfect foresight at the initial time point.

3) According to usual textbooks the term of the pure public goods is defined as one that represents the goods with the properties both of non-excludeability and of non-rivalry. I will follow this usual definition regardless of the original definition in Samuelson's seminar paper (Pichhandt (2001)).
Similarly about Country 2 it holds that

\begin{equation}
\dot{D}_2(t) = r(t)D_2(t) + C_2(t) + T_2(t) - Y_2(t),
\end{equation}

\begin{equation}
\frac{\dot{C}_2(t)}{C_2(t)} = r(t) - \rho
\end{equation}

and that

\begin{equation}
\lambda_2(t)e^{-\rho t}D_2(t) = \frac{\theta D_2(t)}{C_2(t)e^{\rho t}} \to 0 \quad (t \to \infty)
\end{equation}

Now in the world economy as a whole it holds that

\begin{equation}
G(t) = G_1(t) + G_2(t)
\end{equation}

\begin{equation}
2Y = C_1(t) + C_2(t) + G(t)
\end{equation}

where \(G_i\) denotes the supply of the IPPG by Country \(i (i=1, 2)\) and \(Y = Y_1 = Y_2\) denotes one country's GDP (or NDP) which is assumed to be equal between two countries and to be constant over time. Moreover we assume that the population is equal to unity in each country and constant over time. Thus

\begin{equation}
D(t) = D_1(t) = -D_2(t)
\end{equation}

For simplicity we assume below that \(G_1\) and \(G_2\) are constant over time.

Then from (2) and (8)

\begin{equation}
C_1(t)(r(t) - \rho) + C_2(t)(r(t) - \rho) = 0
\end{equation}

Then it holds that \(r(t) = \rho\). Then, from (2) and (5) \(C_1(t)\) and \(C_2(t)\) are constant over time. Thus from (1) the following differential equation holds.

\begin{equation}
\dot{D}(t) = \rho D(t) + C_1 + G_1 - Y
\end{equation}

This equation has the following solution.

\[D(t) = \left( D(0) - \frac{Y - C_1 - G_1}{\rho} \right)e^{\rho t} + D(0),\]

where \(D(0)\) denotes the initial level of debt to country 2. However, the transversality condition (3) requires that

\[D(0) - \frac{Y - C_1 - G_1}{\rho} = 0.\]

This implies that the optimal level of private consumption is

\begin{equation}
C_1 = Y - G_1 - \rho D(0),
\end{equation}

which is assumed to be positive. Similarly, noticing (9), we obtain for country 2.

\begin{equation}
C_2 = Y - G_2 + \rho D(0).
\end{equation}

The world economy is in the steady state from the initial time.
[ 2 ] The world government

Suppose that the world government is constructed. Consider the following game.

**Stage 1**: The world government determines the supply of IPPG so as to maximize welfare of all the people in the world.

**Stage 2**: The individual in both countries determines the private consumption.

The world government would maximize

\[ W = \theta \ln C + (1 - \theta) \ln G \]

subject to

\[ 2C + G = 2Y. \]

This results in

\[ C = \theta Y \]

\[ G = 2(1 - \theta) Y. \]

(15) represents the ideal amount of IPPG.\(^4\)

[ 3 ] The case that both countries determine the supply of IPPG independently\(^5\)

Suppose the following two-stage game.

**Stage 1**: The governments in both Country 1 and Country 2 determine the supply of the IPPG independently so as to maximize welfare of their own countries.

**Stage 2**: The individual in both countries determines the private consumption.

Then the government of Country 1 maximizes

\[ W_1 = \theta \ln C_1 + (1 + \theta) \ln G \]

subject to

\[ Y = C_1 + G_1 + \rho D(0) \]

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4) In our simple model the world government need not ask one country to transfer to another country. But in more complicate models it would need to do so in order to maximize the "social welfare function". The most important complication is the difference in the production function between the countries. See Sando (2002), which emphasizes the distinction between the case that the transfer from the rich country to the poor one (or vice versa) is freely allowed and the case that it is prohibited.

5) Kindle (1966) emphasizes the importance of "leadership" or "regime" among countries in supplying the IPPG. But I assume that such substitutes for the world government do not exist since it is imaginary or difficult to establish. Then, although the supply of the IPPG is evidently lower than otherwise, my purpose is to show that its supply is even much lower when a part of countries is prohibited from supplying the IPPG directly.
and

\[(17) \quad G = G_1 + G_2,\]

Assume that the government of Country 1 takes \(G_2\) as given when it determines \(G_1\) (Cournot-type decision). Notice that we assumed in (12) that \(Y > \rho D(0)\). Then the result is as follows:

\[(18) \quad G_1 + \theta G_2 = (1 - \theta)(Y - \rho D(0)).\]

Similarly for Country 2

\[(19) \quad G_2 + \theta G_1 = (1 - \theta)(Y + \rho D(0)).\]

Therefore from (18) and (19) we obtain that

\[(20) \quad G = G_1 + G_2 = \frac{2(1 - \theta)}{1 + \theta} Y\]

\[(21) \quad C_i = \frac{2 \theta}{1 + \theta} Y\]

Notice that \(G\) and \(C_i\) \((i = 1, 2)\) is independent of \(D(0)\) (Warr (1983)). However, from (18) we must assume (and will assume below) that

\[(22) \quad Y > \rho D(0)\]

Thus expectedly it is impossible to achieve the ideal state in the preceding section in the real world. In other words, by comparing (15) with (20) we see that too little amount of the IPPG is supplied if the world government is not constructed. Evidently the individual’s welfare is lower.

[4] The case that Country 2 is prohibited from supplying international public goods

[4.1] The game with neither transfer nor abandonment of a claim

Consider that Country 2 is prohibited from supplying IPPG for some reason. Then Country 1 maximizes with respect to \(G\)

\[W_i = \theta \ln C_i + (1 - \theta) \ln G\]

subject to

\[(23) \quad Y = C_i + G + \rho D(0).\]

This leads to the following solutions.

\[(24) \quad G = (1 - \theta)(Y - \rho D(0))\]

\[C_i = \theta(Y - \rho D(0))\]

Naturally \(G\) in (24) is much less than \(G\) in (22).

By the way Country 2’s consumption is
\[ C_2 = Y + \rho D(0). \]

It seems that Country 2 enjoys her free-rider's position but the level of Country 2' welfare may be raised by transfer to Country 1 and/or abandonment of a claim against Country 1, as will be shown in the following subsections.


For simplicity assume in this subsection that Country 1 owes no debt to Country 2 at the initial time point.

Now suppose that Country 2 can transfer some amount to Country 1 and consider the following three-stage game.

Stage 1: Country 2 determines the amount of the transfer \( R \) to Country 1.

Stage 2: Country 1 determines \( G \).

Stage 3: The individual in both countries determines the private consumption.

As usual, we solve the game backwards. Firstly Government 1 maximizes, given \( R \),

\[ W_1 = \theta \ln C_1 + (1 - \theta) \ln G \]

subject to

\[ Y + R = C_1 + G. \]

Then we obtain

\begin{align*}
(25) & \quad C_1 = \theta (Y + R) \\
(26) & \quad G = (1 - \theta)(Y + R). 
\end{align*}

Secondly Country 2 maximizes

\[ W_2 = \theta \ln C_2 + (1 - \theta) \ln G \]

subject to (26) and

\[ Y - R = C_2, \]

in which \( R \) is equal to \( T_2 \). This leads to

\begin{align*}
(27) & \quad C_2 = 2\theta Y \\
(28) & \quad R = Y(1 - 2\theta). 
\end{align*}

By substituting (28) into (26) we obtain

\begin{align*}
(29) & \quad C_1 = 2\theta(1 - \theta)Y \\
(30) & \quad G = 2(1 - \theta)^2 Y \\
(31) & \quad G = (1 - \theta)Y. 
\end{align*}

Notice, however, that if \( \theta > 1/2 \), the right hand side of (28) is negative. Then since country 1 is not enforced to transfer to country 2, \( R \) vanishes. Therefore from (26)

\[ G = (1 - \theta)Y. \]

Naturally if Country 2 can voluntarily transfer to Country 1, the former's welfare is not
smaller and in some case larger than otherwise. Comparing, however, the case in this subsection with the case in the preceding section in which the IPPG is independently provided by both countries, we can prove that $G$ in (30) or (31) is less than $G$ in (20). That is, the situation that one country is permitted to supply the IPPG only indirectly is unfavorable for the provision of the IPPG than the situation that both countries supply the IPPG independently. This proposition also holds in two cases examined in [4.3] and [4.4].

[4.3] Country 2’ abandonment of her claim against Country 1

In the game in the preceding subsection let us replace the “transfer” with the “reduction in Country 2’s claim against Country 1”. Then the larger $D(0)$ which is abandoned, the higher $G$ which is achieved (see the upper part of Fig. 1) since from (26)

$$G = (1 - \theta) (Y - \rho D^*)$$

where asterisk (*) denotes the claim that still remains after reduction in $D(0)$. Notice
that even if $D^*$ vanishes, $(1-\theta)Y$ is still lower than $G$ in (20) achieved in the case that two countries could supply the IPPG independently. At any rate the supply of IPPG is the largest one, if Country 2 gives up all the financial asset. However, can this “give up” be realized?

For the moment we assume that $\theta > 1/2$. In the stage 1 of the game, welfare of Country 2 is maximized at

$$\rho D^* = (2\theta - 1)Y,$$

which can be realized if and only if

$$\rho D(0) \leq (2\theta - 1)Y.$$

Then we obtain

$$G = 2(1-\theta)^2 Y.$$

As is shown in Fig. 1, although $W_1$ is larger the smaller $D(0)$ is, $W_2$ is maximized at $D^*$ in (33). Therefore if (34) does not hold, Country 2 has no incentive to reduce $D(0)$.

So far we assumed that $\theta > 1/2$. On the other hand if $\theta \leq 1/2$, all the $D(0)$ will be abandoned. Then and only then $G$ is given by $(1-\theta)Y$.

**[4.4] The mixed policy**

Of course Country 2 can adopt the mixed policy that includes both transfer to Country 1 and reduction of the claim against Country 1. By referring to two preceding subsections this case is illustrated in Fig. 2 (1) and Fig. 2 (2). Fig. 2 (1) illustrates the case that $\theta < 1/2$. In this case as long as $D(0) > D^*$, we can choose any point of the bold line. For example, if $\rho D(0) = 2\theta Y$, as is illustrated, Country 2 can choose a point $(0, (1-2\theta)Y)$ or a point $(\rho D(0), Y)$ or any point $(\rho D^*, R)$ on the bold line between the above two points. Then we obtain

$$G = 2(1-\theta)^2 Y.$$  

Fig. 2 (2) illustrates the case that $\theta > 1/2$. In this case, if $\rho D(0) \geq (2\theta - 1)Y$, Country 2 can choose any point on the sloped bold line between $(\rho D(0) + (1-2\theta)Y, \rho D(0))$ and $(0, \rho D(0) + (1-2\theta)Y)$. Then we obtain

$$G = 2(1-\theta)^2 Y.$$  

However, if $\rho D(0) < (2\theta - 1)Y$, Country 2 cannot but choose the point $(\rho D(0), 0)$. Then we obtain

$$G = (1-\theta)(Y - \rho D(0)),$$

which is less than $G = 2(1-\theta)^2 Y$, as is easily proved.
[5] The conclusion

Although I recognize that it is very dangerous to apply such an abstract theory as developed above to the real world, I dare to try to do so. At present Japan is almost prohibited by the Constitution to have the military force. So most of the defense against terrorism and intrusion into the democratic territory cannot but be entrusted to the U. S. This situation seems to be saved if Japan grants some amount to the U. S. and/or abandons some amount of an accumulated claim against the U. S.. Certainly it will be effective to some extent.

But an amount of the IPPG obtained by these means is even lower than that provided when each of two countries supply the IPPG independently so as to maximize its own welfare, although the latter is still lower than that provided by the world government if it could be established.

Apart from dreaming of the world government there are two solutions to this problem for Japan.6) The first is to strengthen the alliance with (the dependence on) the U. S. and grant too much amount or abandon too much claim at the cost of its own welfare. The second is to amend the Constitution and have the independent military force.

Finally in this conclusion I implicitly assume the existence of terrorists. Although there are many papers which tried to explain the occurrence of terrorism from the economic point of view,7) I believe that it cannot be explained simply by the purely economic conditions. This is one of the most important tasks that remain in my life.

REFERENCES
Pickard, Michael (2001), “Fifty Years after Samuelson’s ‘the Pure Theory of Public Expenditure’ ”., Presentation at the 52th International Atlantic Economic Conference.

6) I ignore the third option. It is that Japan becomes a "paid rider", which means that Japan, the secondary target of terrorists, gives the sanctuary to terrorists' groups (Lee (1988)).
7) A survey is given by Blomberg and others (2004).