Temperature in a subducting plate when subduction velocity changes

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(with 10 figures)

Abstract

Based on a simple subduction model similar to McKenzie (1972), temperature in a plate of constant thickness subducting into an isothermal mantle was solved analytically in a general form, so that transient features of the seismic segment of the subducting plate as a function of time were discussed for three cases that (1) the subducting motion stopped, (2) new subduction began after a short period of the pause and (3) the subduction velocity changed stepwisely. In any cases, the plate assimilates thermally with the mantle and becomes aseismic after a constant time since it started to subduct in spite of difference in history of its movement.

I. Introduction

Turbidite sediments of Pliocene age were found in the DSDP cores of a site 297 drilled at the out side of the Nankai trough, and this finding has given an idea that subducting motion of the Philippine Sea plate had stopped at the Nankai trough during a short period around 3–5 Ma (e.g. Ingle et al., 1975). A simple imagination gives an idea that the seismic zone would shorten after the subducting motion stopped, and that if new subducting motion began the zone would become long until a steady state would be achieved. However, in order to discuss further, we need to know more how temperature in the plate changes with time during and after the pause. Many authors have discussed temperature in a subducting plate (e.g. McKenzie, 1969, 1970, 1972, Hasebe et al., 1970, Toksöz et al., 1971, Griggs, 1972), but most of them concerned with a steady state subduction and there has been very few discussions on transient features of temperature after the subducting motion stops or begins again.

As the first step to solve the problem, the present study will discuss mathematically how temperature changes after the subducting motion stops or begins again, and will give some ideas on transient features of a subducting plate when the subduction velocity changes stepwisely with time.

The general equation for temperature within a moving material is

\[ \rho C_p \left( \frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T \right) = \kappa \nabla^2 T + \mathbf{H}, \]

where \( \mathbf{v} \) is the velocity of the material, \( \rho \) is its density, \( C_p \) is its specific heat, \( \kappa \) is its heat
generation, and $k$ is its conductivity of heat. If the $x$ axis is taken to be parallel to the dip of a subducting plate, the $z$ axis is to be normal to the plate and the origin of coordinates is chosen to be on the lower boundary (See Fig. 1). (1.1) becomes

$$\rho C_p \left( \frac{\partial T}{\partial t} + v \frac{\partial T}{\partial x} \right) = k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2} \right) ,$$

where $v$ gives the $x$ component of velocity and $H$ is ignored because of the small radioactivity in the plate.

For the convenience of mathematical treatise, we will assume in this preliminary study that the first term of the right hand of (1.2) is greatly smaller than the second term. This assumption means to neglect the heat conduction parallel to the $x$ axis. Because the temperature gradient across the plate is greater than the gradient along the plate in most cases, this assumption may be reasonable. Then (1.2) becomes

$$\rho C_p \left( \frac{\partial T}{\partial t} + v \frac{\partial T}{\partial x} \right) = k \frac{\partial^2 T}{\partial z^2} .$$

If $\partial T/\partial t = 0$, (1.3) gives steady state equation, which McKENZIE (1972) solved analytically to give temperature in the plate subducting into an isothermal mantle. In the present study, we will examine features of his solution and solve (1.3) to obtain...
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(A) the temperature is isothermal in the mantle outside of the subducting plate,
(B) the subducting plate has initially an uniform temperature gradient,
(C) the thickness of the plate is constant, and
(D) heat conducts only in the direction normal to the surface of the plate.

The assumption (A) does not seem realistic because the temperature increases both outside and inside of the plate through heating due to the adiabatic compression, radioactivity and phase transition. However, as McKENZIE (1970, 1972) suggested, if we assume that the increase of temperature with depth is almost adiabatic enough to neglect the effect of radioactivity and phase transition, the assumption (A) is replaced by the assumption (A') the potential temperature is isothermal in the upper mantle. Then, we will be able to discuss features of the potential temperature in a subducting plate with a slight modification. According to McKENZIE (1967), the temperature gradient in the oceanic plate becomes uniform after a considerably long time since it was created at a spreading center. The assumption (B) suggests that an oceanic plate of a very old age subducts. The plate must bend under the trench, but the effect is ignored in such a manner as a material OA moves to OB quickly (See Fig. 1).

II. Steady-state subduction

The equation governing temperature \( T_s(x, z) \) in a plate subducting into an isothermal mantle at a velocity \( v \) is

\[
\rho C_p v \frac{\partial T_s}{\partial x} = \kappa \frac{\partial^2 T_s}{\partial z^2}.
\]

McKENZIE (1972) has given a solution of (II.1) under the boundary conditions (A) and (B):

\[
\begin{align*}
T_s(x, 0) &= T_0 \quad \text{and} \\
T_s(x, l) &= T_0, \\
T_s(0, z) &= T_0(1-z/l),
\end{align*}
\]

where \( l \) is the thickness of the plate and \( T_0 \) is the temperature in the isothermal mantle.

The solution is

\[
T_s(x, z) = T_0[1 + \sum_{n=1}^{\infty} C_n \exp(-n^2\pi^2z/Rl) \sin(n\pi z/l)],
\]

where \( C_n = 2(-1)^n/n\pi \) (\( n = 1, 2, \ldots \)) and \( R = \rho C_p v l / \kappa \). Fig. 3(a) gives contours of isotherms \( T_s/T_0 = 0.2, 0.4, 0.6 \) and 0.8 for the case of \( l = 80 \) km and \( v = 5 \) cm/yr.

As the partial derivative with \( z \) becomes null at the leading edge of an isotherm, the maximum \( x \)-coordinate \( x_a \) of an isotherm \( T_s \) can be determined by solving equations for two unknown values \( (x_a, z) \):
If we assume that the subducting plate assimilates thermally with the mantle and, therefore, loses the potential to generate earthquakes (or becomes aseismic) where the minimum temperature across the plate becomes higher than a critical temperature $T_a$, the solution $x_a$ of (II.5) gives the length of the seismic segment of the subducting plate.

The time $t_a$ defined by $t_a = x_a/v$ gives the time requiring for the plate to assimilate with the surrounding mantle since it started to subduct, or the time constant of a subducting plate for thermal assimilation. Then, (II.5) becomes

$$T_0[1 + \sum_{n=1}^{\infty} C_n \exp(-n^2 \pi^2 x_a/Rl) \sin(n\pi z/l)] = T_a,$$

$$\sum_{n=1}^{\infty} nC_n \exp(-n^2 \pi^2 x_a/Rl) \cos(n\pi z/l) = 0,$$

where $a = \rho C_p l^2/k$. As (II.6) does not include a parameter $v$, $t_a$ is independent of subduction velocity.

If we introduce $t'_a$ and $z'$ defined by $t'_a = t_a/l^2$ and $z' = z/l$, (II.6) becomes

$$T_0[1 + \sum_{n=1}^{\infty} C_n \exp(-n^2 \pi^2 t'_a/b) \sin(n\pi z'/l)] = T_a,$$

$$\sum_{n=1}^{\infty} nC_n \exp(-n^2 \pi^2 t'_a/b) \cos(n\pi z') = 0,$$

where $b = \rho C_p l^2/k$. As (II.7) does not include a parameter $l$, the solution $t'_a$ is independent of the thickness $l$.

Summarizing the above discussions, the length $x_a$ of the seismic segment is proportional to the product of velocity and square of thickness, and the time constant $t_a$ for assimilation is proportional to square of thickness:

$$t_a = l^2/c,$$

$$x_a = vt^2/c = vt_a,$$

where $c = 1/t'_a$. The above relations will be used in Discussion to understand the behavior of the seismic segment when the subduction velocity changes.

### III. Temperature after subducting motion stopped

If the subducting motion with a velocity $v$ stopped at $t=0$ and the plate remains without motion at the same place at $t>0$ (See Fig. 2), the equation governing temperature $T_c(x, z, t)$ in the plate is given by neglecting the term of $v \partial T/\partial x$ in (I.3):

$$\rho C_p \frac{\partial T_c}{\partial t} = \kappa \frac{\partial^2 T_c}{\partial z^2}.$$

(III.1)
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Fig. 2. Situation when subducting motion stops. (1) A plate subducted at a velocity \( v(t<0) \) and (2) it remains at the same place in the mantle after subducting motion stopped \((t>0)\).

The temperature at \( t=0 \) is given by (II.4):

\[
T_r(x, z, 0) = T_0[1 + \sum_{n=1}^{\infty} C_n \exp\left(-n^2\pi^2x/R\right) \sin(n\pi z/l)].
\]  

From the assumption (A), the temperatures at \( z=0 \) and \( l \) are

\[
T_r(x, 0, t) = T_0 \quad \text{and} \quad T_r(x, l, t) = T_0.
\]

Because the temperature gradient across the plate must be greater than that along

\[
l=80.0 \text{ km} \quad v=5.0 \text{ cm/yr} \quad v*=0.0 \text{ cm/yr} \quad t_p=0.0 \text{ m.y.}
\]

Fig. 3. Temperatures in the subducting plate after the subducting motion stopped. \( l=80 \text{ km}, v=5 \text{ cm/yr} \) and other physical constants are as same as in McKENZIE (1969). Practical calculations are made using Equation (IV. 5) under the conditions that \( l=80 \text{ km}, v=5 \text{ cm/yr}, v^*=0 \text{ cm/yr} \) and \( t_p=0 \text{ m.y.} \). Contours are isotherms for \( T_r/T_0=0.2, 0.4, 0.6 \) and 0.8 at (a) \( t=0 \text{ m.y (steady state)} \), (b) \( t=5 \text{ m.y.} \), (c) \( t=10 \text{ m.y.} \) and (d) \( t=15 \text{ m.y.} \).
the dip of the plate just after the subducting motion stopped as expressed by (III.2), it is reasonable to consider that (III.1) which neglects the heat conduction parallel to the dip of the plate is valid for a relatively short period. Because (III.1) does not include any terms related to \( x \), \( x \) in a function \( T_c(x, z, t) \) should be treated as a constant parameter but not a variable. So, we can not assign the boundary condition at \( x=0 \), indicating that this model is free from the assumption (B).

As shown in Appendix A, the solution of (III.1) satisfying (III.2) and (III.3) is

\[
T_c(x, z, t) = T_0[1+\sum_{n=1}^{\infty} C_n \exp(-n^2\pi^2(x+vt)/Rl) \sin(n\pi z/l)]. \tag{III.4}
\]

Fig. 3 compares the temperatures \( T_c(x, z, t) \) at \( t=0, 5, 10 \) and 15 my after the subducting motion stopped. As the right hand of (II.4) becomes the right hand of (III.4) if \( x \) is exchanged with \( x+vt \),

\[
T_c(x, z, t) = T_c(x+vt, z). \tag{III.5}
\]

Equation (III.5) indicates that temperature at a point \((x, z)\) after the subducting motion stopped is equal to one at a point \((x+vt, z)\) in the steady state subduction and, therefore, that the isotherm moves upward at a velocity \( v \) (See Fig. 3).

### IV. Temperature after new subducting motion began

In this section, we will discuss temperature in a plate which began to subduct again after a short period of a pause in the subducting motion as follows (See Fig. 4):

1. \( t<0 \)
2. \(-t_p < t < 0\)
3. \( 0 < t \)

Fig. 4. Situation before and after a pause in subducting motion. (1) A plate PB subducted at a velocity \( v \) \((t<-t_p)\). (2) The subducting motion stopped when a plate PA reached at the trench and the plate PB remained at the same place in the mantle \((-t_p < t < 0)\). (3) After the plate PA began to subduct again, it subducts at a velocity \( v^* \) together with the plate PB \((t>0)\).

- (1) a plate PB had subducted at a constant velocity \( v \),
- (2) the subducting motion stopped at \( t=-t_p \), when a plate PA reached at the trench and
- (3) the present subducting motion began at \( t=0 \) at the same place and the plate PA subducts at a velocity \( v^* \) together with the plate PB.
The thermal equation governing temperature $T_r(x, z, t)$ in the subducting plate is

$$\rho C_p \left( \frac{\partial T_r}{\partial t} + v^* \frac{\partial T_r}{\partial x} \right) = \kappa \frac{\partial^2 T_r}{\partial z^2} \quad \text{(IV.1)}$$

As the subducting motion had stopped during a period $t_p$, the temperature at $t=0$ is given by (III.4)

$$T_r(x, z, 0) = T_0[1+\sum_{n=1}^{\infty} C_n \exp(-n^2\pi^2(x+vt_p)/Rl) \sin(n\pi z/l)] \quad \text{(IV.2)}$$

From the assumptions (A) and (B), the temperatures at $z=0$ and $l$, and at $x=0$ are

$$T_r(x, 0, t) = T_0 \quad \text{and} \quad T_r(x, l, t) = T_0,$$

$$T_r(0, z, t) = T_0(1-z/l). \quad \text{(IV.3)}$$

$$T_r(0, z, t) = T_0(1-z/l). \quad \text{(IV.4)}$$

As shown in Appendix B, the solution of (IV.1) satisfying (IV.3) and (IV.4) is

$$\begin{align*}
\alpha &= 80.0 \text{ km} \\
v &= 5.0 \text{ cm/yr} \\
v^* &= 5.0 \text{ cm/yr} \\
t_p &= 7.5 \text{ my}
\end{align*}$$

Fig. 5. Temperatures in the subducting plate after a pause in the subducting motion. Calculations are made using (IV. 5) under the conditions that $l=80.0 \text{ km}$, $v=v^*=5.0 \text{ cm/yr}$ and $t_p=7.5 \text{ my}$. Physical constants are as same as in McKenzie (1969). Contours are isotherms for $T_r/T_0=0.2$, 0.4, 0.6 and 0.8 at (a) $t=0.0 \text{ my}$, (b) $t=7.5 \text{ my}$, (c) $t=15.0 \text{ my}$ and (d) $t=22.5 \text{ my}$. A dotted line gives the boundary between the plates PA and PB (See Fig. 4).
\[ T_r(x, z, t) = \begin{cases} T_0[1 + \sum_{n=1}^\infty C_n \exp\left(-\frac{n^2 \pi^2 x}{R^* l}\right) \sin\left(n \pi z/l\right)], & x \leq v^* t, \\ T_0[1 + \sum_{n=1}^\infty C_n \exp\left(-\frac{n^2 \pi^2 (x+(v-v^*)t+vtp)}{R l}\right) \sin\left(n \pi z/l\right)], & x > v^* t, \end{cases} \]  

where \( R^* = \rho C_p v^* l / \kappa \). Fig. 5 compares temperatures \( T_r(x, z, t) \) after the subducting motion began for the case of \( v^* = v \). After the subducting motion began, the boundary between the old plate PB and the new plate PA advances at a velocity \( v^* \). Therefore, the solution for \( x \leq v^* t \) gives temperature in the plate PA, while the solution for \( x > v^* t \) gives one in the plate PB.

As the temperature varies abruptly across the boundary between the plate PA and PB, heat must conduct across the boundary in the real situation. However, I think that \((IV.5)\) holds fundamental features of temperature in the subducting plate, because the effect of heat conduction across the boundary is restricted within a small distance as long as we discuss the transient temperature during a relatively short period.

If \( T_s(x, z) \) and \( T_s^*(x, z) \) are steady state temperatures in the plates subducting at velocities \( v \) and \( v^* \), respectively, \((IV.5)\) becomes

\[ T_r(x, z, t) = \begin{cases} T_s^*(x, z), & x \leq v^* t, \\ T_s(x+(v-v^*)t+vtp, z), & x > v^* t. \end{cases} \]  

\text{(IV.6)}

If \( t_p = 0 \), \( v = v^* \) and therefore \( R = R^* \), \((IV.6)\) becomes

\[ T_r(x, z, t) = T_s(x, z). \]

This is reasonable because the condition implies that the plate continues to subduct at a velocity \( v \).

If \( v^* = 0 \), \((IV.6)\) becomes

\[ T_r(x, z, t) = T_s(x+v(t+t_p), z) = T_s(x, z, t+t_p). \]

This is also reasonable because the condition implies that the subducting motion has continued to stop since \( t = -t_p \).

If \( v = v^* \), \((IV.6)\) becomes

\[ T_r(x, z, t) = \begin{cases} T_s(x, z), & x \leq vt, \\ T_s(x+vt, z), & x > vt. \end{cases} \]

As Fig. 5 shows, the boundary between the plates PA and PB advances at a velocity \( v \), while the isotherms in the plate PB does not move.

If \( t_p = 0 \), \((IV.6)\) becomes
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\[ T_s(x, z, t) = \begin{cases} T_s^*(x, z), & x \leq v^* t, \\ T_s(x + (v - v^*) t, z), & x > v^* t. \end{cases} \]

This gives temperature in the plate after the subduction velocity changed from \( v \) to \( v^* \) stepwisely at \( t = 0 \) (See Fig. 6). Fig. 7 compares temperatures \( T_s(x, z, t) \) after the subduction velocity changed from 5.0 cm/yr to 2.5 cm/yr. As Fig. 7 shows, the isotherm in the plate PB advances at a velocity \( (v^*-v) \), while the boundary between the plates PA and PB advances at a velocity \( v^* \).

Fig. 6. Situation before and after the change in subduction velocity. (1) A plate PB subducted at a velocity \( v \) \((t<0)\). (2) Subduction velocity changed from \( v \) to \( v^* \) when a plate PA reached at the trench \((t=0)\). (3) The plate PA subducts at a velocity \( v^* \) together with a plate PB \((t<0)\).

\( l = 80.0 \text{ km} \), \( v = 5.0 \text{ cm/yr} \), \( v^* = 2.5 \text{ cm/yr} \), \( t_p = 0.0 \text{ my} \).

(a) \( t = 0.0 \text{ my} \) (steady state) 
(b) \( t = 7.5 \text{ my} \) 
(c) \( t = 15.0 \text{ my} \) 
(d) \( t = 22.5 \text{ my} \).

Fig. 7. Temperatures in the subducting plate after the change in subduction velocity. Calculations are made using (IV. 5) under the conditions that \( l = 80 \text{ km} \), \( v = 5.0 \text{ cm/yr} \), \( v^* = 2.5 \text{ cm/yr} \) and \( t_p = 0.0 \text{ my} \). Physical constants are as same as in McKenzie (1969). Contours are isotherms for \( T_s/T_0 = 0.2, 0.4, 0.6 \) and 0.8 at (a) \( t = 0.0 \text{ my} \) (steady state), (b) \( t = 7.5 \text{ my} \), (c) \( t = 15.0 \text{ my} \) and (d) \( t = 22.5 \text{ my} \).
Thus, equations (IV.5) and (IV.6) represent temperature in the plate of which subduction velocity changes, in a relatively general form.

V. Discussion

As discussed in Section II, the length \( x_a \) of the seismic segment of the plate that subducts steadily at a velocity \( v \) is equal to \( vt_a \). The time constant \( t_a \) of a plate for thermal assimilation does not depend on the subduction velocity, but is proportional to square of thickness of the plate. We consider here \( t_a \) as a constant parameter because of the assumption (C) that the thickness is constant. Therefore, (IV.6) indicates that the subducting plate is seismic in the segment where the distance \( x \) from the trench satisfies the following condition (a) or (b)

(a) \( x \leq v^* t \) and \( x \leq v^* t_a \), \hspace{1cm} (V.1)
(b) \( x > v^* t \) and \( x + (v - v^*) t + v t_p \leq v t_a \), or \( v^* t < x \leq v (t_a - t_p) + (v^* - v) t \). \hspace{1cm} (V.2)

If \( v^* = 0 \) and \( t_p = 0 \) or the subducting motion has stopped since \( t = 0 \) (See Fig. 8 (a)), (V.1) and (V.2) become

![Diagram](image)

Fig. 8. Velocity (a), length of seismic segment (b) and movement of material (c) as a function of time for the case that subducting motion stops at \( t = 0 \). Thick, broken and dotted lines in (b) correspond to those in (a). A material which subducts at \( t < 0 \) moves from \( B_1 \) through \( B_2 \) to \( B_3 \). A thin line with spikes show the boundary where the subducting material becomes aseismic.
This suggests that the seismic segment shortens at a velocity \( v \) and that the seismic segment disappears after a constant time \( t_a \) (See Fig. 8(b)). Next, consider the movement of the material which starts to subduct at \( t = -t_s \) (See Fig. 8(c)). After the material moves from \( B_1 (0, -t_s) \) to \( B_2 (vt_s, 0) \), it remains at the same place and finally becomes aseismic at \( B_3 \). A simple calculation gives the coordinates of \( B_3 (vt_s, t_a - t_s) \). Because the material which starts to subduct at \( t = -t_s \) becomes aseismic at \( t = t_a - t_s \), the time constant for assimilation is \( t_a \), being equal to one in the case of the steady subduction.

The conditions (V.1) and (V.2) give, in a general form, the length of the seismic segment of the plate after a pause in the subducting motion as a function of time (See Fig. 9). Because the boundary between the remaining old plate PB and the new plate PA advances at a velocity \( v^* \), (V.1) represents features in the plate PA. From (V.1), we obtain \( x \leq v^* t \) for \( t \leq t_a \) and \( x \leq v^* t_a \) for \( t_a < t \). The former indicates that the leading edge of the plate PA moves at a velocity \( v^* \) and the latter indicates that the length of the seismic segment becomes equal to that in the steady state subduction with a velocity \( v^* \) after a time \( t_a \). On the other hand, (V.2) represents features of the plate PB. The first term in the right hand gives the length of the seismic segment when the subducting
motion has stopped during a period \( t_p \) and the second term shows that the length grows at a velocity \((v^*-v)\). The intersection time \(t_a-t_p\) between the equations \( x=v^*t \) and \( x=v(t_a-t_p)+(v^*-v)t \) gives the time when the seismic segment of the plate PB has disappeared, and the critical time is independent of the velocities \(v\) and \(v^*\). The lowest part of the seismic segment consists of the plate PB for \( t \leq t_a-t_p \) and of the plate PA for \( t > t_a-t_p \).

Considering the above discussions, the length \(x_a\) of the seismic segment changes with time as follows (See Fig. 9):

\[
x_a = \begin{cases} 
vt_a, & t \leq -t_p, \\
(v(t_a-t_p)-t), & -t_p < t \leq 0, \\
v(t_a-t_p)+(v^*-v)t, & 0 < t \leq t_a-t_p, \\
v^*t, & t_a-t_p < t \leq t_a, \\
v^*t_a, & t_a < t.
\end{cases}
\]

Next, consider the movement of the material which starts to subduct at \( t=-t_s \) \((t_s>t_p)\). After the material moves from \( B_1 \) \((0,-t_s)\) to \( B_2 \) \((v(t_s-t_p),-t_p)\), it remains at the same place during a period from \( B_2 \) to \( B_3 \) \((v(t_s-t_p),0)\). At \( t=0\), it begins to move again at a velocity \(v^*\) for \( B_4\), where it becomes aseismic. A simple calculation gives the coordinates of \( B_4 \) \((v^*(t_a-t_s)+v(t_s-t_p), t_a-t_s)\). Because the material, which starts to subduct at \( t=-t_s \), becomes aseismic at \( t=t_a-t_s \), the time constant of the plate PB for assimilation is \( t_a \). As the plate PA moves from \( A_1 \) to \( A_2 \), the time constant of the plate PA is naturally \( t_a \).

Above discussion includes the special case that the subduction velocity changed from \(v\) to \(v^*\) stepwisely at \( t=0\) (See Fig. 10 (a)). Then, as \( t_s=0 \), (V.1) and (V.2) become

\[
x \leq v^*t \quad \text{and} \quad x \leq v^*t_a, \\
v^*t < x \leq vt_a + (v^*-v)t.
\]

Equations (V.3) and (V.4) give the length \(x_a\) of the seismic segment as a function of time (See Fig. 10):

\[
x_a = \begin{cases} 
vt_a, & t \leq 0, \\
vt_a + (v^*-v)t, & 0 < t \leq t_a, \\
v^*t_a, & t_a < t.
\end{cases}
\]

The length of the seismic segment shifts from \(vt_a\) to \(v^*t_a\) through the transient stage while the segment grows at a velocity \((v^*-v)\). Next, consider the movement of the material which starts to subduct at \( t=-t_s \). The material moves from \( B_1 \) \((0,-t_s)\) to \( B_2 \) \((vt_s,0)\) at a velocity \(v\) and next moves at a velocity \(v^*\) from \( B_2 \) to \( B_3 \) \((v^*(t_a-t_s)+vt_s, t_a-t_s)\), where the material becomes aseismic. Thus, the time constant for assimilation is \( t_a \).
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VI. Summary and Acknowledgement

Based on a simple subduction model similar to McKENZIE (1972)'s one, the thermal conduction equation was solved in order to understand the transient features of temperature in the plate subducting into an isothermal mantle after a period of the pause in the subducting motion. If the previous subduction with a velocity  \( v \) stopped at \( t = -t_p \) and the present subduction with a velocity \( v^* \) began at \( t = 0 \), the temperature \( T_r(x, z, t) \) is expressed as

\[
T_r(x, z, t) = \begin{cases} 
T_s^*(x, z), & x \leq v^* t, \\
T_s(x + (v - v^*) t + v t_p, z), & x > v^* t, 
\end{cases}
\]  

(VI.1)

where \( T_s(x, z) \) and \( T_s^*(x, z) \) give steady state temperature distributions in the plate which subducts at velocities \( v \) and \( v^* \), respectively. Equation (VI.1) represents in a general form of the transient temperature in the plate when the subduction velocity changes stepwisely.

If we consider that the plate has assimilated thermally with the mantle and becomes aseismic where the minimum temperature across the plate becomes higher than a critical temperature \( T_a \), the length of the seismic segment of the plate varies with time as follows:
(1) After the subducting motion stopped, the seismic segment shortens at the same velocity as one of the previous subduction.

(2) After new subducting motion began before the previous seismic segment would disappear, the seismic segment, which consists of the old plate, becomes long at a velocity \((v^* - v)\) until the new plate overtakes it. After that, the seismic segment, which consists of the new plate, becomes long at a velocity \(v\) until it becomes as long as one in the steady state.

(3) After the subduction velocity changed from \(v\) to \(v^*\) stepwisely, the seismic segment grows at a velocity \((v^* - v)\) until it becomes as long as one in the steady state subduction with a velocity \(v^*\).

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References


Appendix A

Similar to MCKENZIE (1969), if we introduce nondimensional variables \(x' = x/l\), \(z' = z/l\), \(t' = t/\tau\) and \(T'_c = T_c/T_0\), where \(\tau = \rho C_p l^2/\kappa = Rl/\nu\) and \(R = \rho C_p zl/\kappa\), (III.1) becomes

\[
\frac{\partial T'_c}{\partial t'} = \frac{\partial^2 T'_c}{\partial z'^2},
\]  

(A.1)

(III.2) becomes

\[
T'_c(x', z', 0) = 1 + \sum_{n=1}^{\infty} C_n \exp \left( -n^2 \pi^2 x'/R \right) \sin (n\pi z'),
\]  

(A.2)

and (III.3) becomes
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\[ T'_e(x', 0, t') = 1 \quad \text{and} \quad T'_e(x', 1, t') = 1. \] (A.3)

The solution of (A.1) satisfying (A.3) is expresses in a form

\[ T'_e(x', z', t') = 1 + \sum_{n=1}^{\infty} P_n \exp(-n^2\pi^2t') \sin(n\pi z'). \]

The constants \( P_n \) (\( n=1, 2, \cdots \)) are determined from (A.2)

\[ 1 + \sum_{n=1}^{\infty} P_n \sin(n\pi z') = 1 + \sum_{n=1}^{\infty} C_n \exp\left(-n^2\pi^2x'/(2R)\right) \sin(n\pi z'). \]

\[ : \quad P_n = C_n \exp\left(-n^2\pi^2x'/(2R)\right). \]

Therefore, the solution of (A.1) becomes

\[ T'_e(x', z', t') = 1 + \sum_{n=1}^{\infty} C_n \exp\left(-n^2\pi^2(x'/R + t')\right) \sin(n\pi z'). \]

Appendix B

If we introduce non-dimensional variables \( \chi' = x'/l, \ z' = z/l, \ t' = t/\tau \) and \( T'_* = T_* / T_0 \), where \( \tau = \rho C_p l^2/\kappa \) and \( R_* = \rho C_v l^2/\kappa \), (IV.1) becomes

\[ \frac{\partial T'_*}{\partial t'} + R_* \frac{\partial T'_*}{\partial x'} = \frac{\partial^2 T'_*}{\partial x'^2}, \] (B.1)

and (IV.2) becomes

\[ T'_*(x', z', 0) = 1 + \sum_{n=1}^{\infty} C_n \exp\left(-n^2\pi^2(x'/R + t'_0)\right) \sin(n\pi z'), \] (B.2)

where \( t'_0 = t_0/\tau \), and (IV.3) and (IV.4) become

\[ T'_*(x', 0, t') = 1 \quad \text{and} \quad T'_*(x', 1, t') = 1, \] (B.3)

\[ T'_*(0, z', t') = 1 - z' \]

\[ = 1 + \sum_{n=1}^{\infty} C_n \sin(n\pi z'). \] (B.4)

The solution of (B.1) satisfying (B.3) is

\[ T'_*(x', z', t') = 1 + \sum_{n=1}^{\infty} F_n(x', t') \sin(n\pi z'), \] (B.5)

if functions \( F_n(x', t') \) (\( n=1, 2, \cdots \)) satisfy

\[ \frac{\partial F_n}{\partial t'} + R_* \frac{\partial F_n}{\partial x'} = -n^2\pi^2 F_n. \] (B.6)
Then, (B.2) becomes
\[ F_n(x', 0) = C_n \exp\left(-n^2\pi^2(x'/R+tp')\right), \]  
and (B.4) becomes
\[ F_n(0, t') = C_n. \]  
If \( F_n(x', t') \) \((n=1, 2, \cdots)\) are expressed in a form
\[ F_n(x', t') = G_n(x')H_n(x'-R^*t'), \]  
functions \( G_n(x') \) \((n=1, 2, \cdots)\) are determined by
\[ \frac{dG_n}{dx} = -\frac{n^2\pi^2}{R^*}, \]  
and functions \( H_n(\xi) \) \((n=1, 2, \cdots)\) are determined by (B.7) and (B.8):
\[ G_n(x')H_n(x') = C_n \exp\left(-n^2\pi^2(x'/R+tp')\right), \]  
\[ G_n(0)H_n(-R^*t') = C_n. \]  
Thus, \( F_n(x', t') \) \((n=1, 2, \cdots)\) are expressed by
\[ F_n(x', t') = C_n \exp\left(-n^2\pi^2x'/R^*\right)H_n(x'-R^*t'), \]  
where
\[ H_n(\xi) = \begin{cases} 1, & \xi \leq 0, \\ \exp\left(-n^2\pi^2(\xi/R-\xi/R^*+tp')\right), & \xi > 0. \end{cases} \]  
Using (B.5), (B.13) and (B.14), the solution of (B.1) becomes
\[ T'_e(x', z', t') = 1 + \sum_{n=1}^{\infty} C_n \exp\left(-n^2\pi^2x'/R^*\right)H_n(x'-R^*t') \sin(n\pi z'), \]  
or
\[ T'_e(x', z', t') = \begin{cases} 1 + \sum_{n=1}^{\infty} C_n \exp\left(-n^2\pi^2x'/R^*\right) \sin(n\pi z'), & x' \leq R^*t', \\ 1 + \sum_{n=1}^{\infty} C_n \exp\left[-n^2\pi^2(x'/R+(1-R^*/R)t'+tp')\right] \sin(n\pi z'), & x' > R^*t'. \end{cases} \]