CORRECTION TO

"ON $J^G$-HOMOMORPHISMS"

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The proof of Theorem 4.1 in [3] is incorrect, for the relation $(ig) \circ \gamma = \gamma \circ (if)$ made on p. 587 does not hold. The mistake was pointed out also by Professor Victor Snaith in Math. Rev. (1981). The following theorem for $G = \mathbb{Z}/2$ proves immediately a modified form of Theorem 4.1 (Corollary) via the canonical transformation $\sigma$ from $KR(-)$ to $KO_G(-)$. Because $\sigma$ commutes with the $p$-th Adams operations for $p$ odd (This is proved by a similar discussion as the usual realification homomorphism). This theorem seems to follow from [2], Theorem 11.4.1 and the result on p. 295. But the latter is not necessary for a proof of our case. So we give here a direct simple proof of it by using only Proposition 11.3.7 of [2].

We use the notations of [3] and [1]. Let $X$ be a $\Sigma^{1,1}$-suspension of a finite pointed $G$-complex, and let $p$ be an odd prime. By $\psi^p$ we denote the $p$-th Adams operation in $KO_G$- or $KR$- theory.

**Theorem.** Let $J_G$ denote the equivariant stable real $J$-homomorphism defined by the usual construction. Then, for any $x \in \widetilde{KO}^{0,1}_G(X)$

$$J_G(1-\psi^p)(x) = 0$$

as an element of $\pi^{0,0}_{\mathbb{Z}}(X)_{(2)}$. ($L_{(2)}$ is the localization of a group $L$ at the prime 2.)

**Corollary.** For any $x \in \widetilde{KR}^{-1}(X)$

$$J_K(1-\psi^p)(x) = 0$$

as an element of $\pi^{0,0}_s(X)_{(2)}$.

1. Observe the composites

$$\widetilde{KO}^{0,1}_G(X) \xrightarrow{\psi} \widetilde{KO}^{-1}(X) \xrightarrow{\delta} \widetilde{KO}^{0,1}_G(X) \text{ and } \pi^{0,0}_s(X) \xrightarrow{\psi} \pi^{0,0}_s(X) \xrightarrow{\delta} \pi^{0,0}_s(X)$$

then by the definitions of $\psi$ and $\delta$ we have

$$\delta \psi = 1 - \rho \text{ times} \quad (\text{cf. [1], Lemma 12.6}).$$
Let \( f: X \to SO(2n) \) be a pointed G-map. Here \( SO(2n) \) denotes the rotation group of degree \( 2n \) with the involution \( A \mapsto \tau A \tau \) where \( \tau \) is the involution of \( \mathbb{R}^n \), and the unit matrix as basepoint. Define a G-map \( \tilde{f}: X \times \mathbb{R}^n \to X \times \mathbb{R}^n \) by \( \tilde{f}(x, u) = (x, f(x)u) \) for \( x \in X, u \in \mathbb{R}^n \). We denote by \( E_f \) a real G-vector bundle over \( \Sigma^{0,1}X \) obtained from the clutching of trivial bundles \( E_1 = C^+X \times \mathbb{R}^n \) and \( E_2 = C^-X \times \mathbb{R}^n \) by \( \tilde{f}: E_1 |_x \to E_2 |_x \), where \( C^\pm X = \{ t \in \Sigma^{0,1}X | \pm t \geq 0 \} \) respectively. As is well known any element of \( KO_G(\Sigma^{0,1}X) \) is the stable equivalence class \( \{ E_f \} \) of such a G-vector bundle \( E_f \).

Since \( p \) is odd we can set

\[
\psi^p(\{ E_f \}) = \{ E_g \}
\]

where \( g \) is a pointed G-map from \( X \) to \( SO(2n) \).

By \( \tilde{F} \) we denote the fibrewise one-point compactification of a real G-vector bundle \( F \). According to [2], Proposition 11.3.7 we may have

(2) There exists a fibrewise G-map \( \tilde{\gamma}: \tilde{E}_f \to \tilde{E}_g \) with fibre-degree \( p^i \) for some \( i \geq 0 \), which preserves the basepoints of fibres.

2. Let \( \gamma \) be the restriction of \( \tilde{\gamma} \) to the fibre on the basepoint of \( \Sigma^{0,1}X \), which is a pointed G-map of \( \Sigma^{0,1} \) into itself. Write \( s + tp \in A(G) = Z[\rho]/(1 - \rho^2) \) for the element determined by \( \gamma \) (see [1], §§1 and 2). Then

\[
s - t = p^i.
\]

Moreover let \( \tilde{\gamma}_k \) be the restrictions of \( \tilde{\gamma} \) to \( \tilde{E}_k \) for \( k = 1, 2 \). Then, as a fibrewise G-map

\[
\tilde{\gamma}_k \circ g = 1 \times \gamma
\]

for \( k = 1, 2 \) evidently. Considering \( \gamma \) to be the constant G-map from \( X \) to \( \Omega^n \Sigma^n \) given by \( X \mapsto \{ \gamma \} \), and also \( f \) and \( g \) to be the G-maps from \( X \) to \( \Omega^n \Sigma^n \) respectively through the natural inclusion \( SO(2n) \subset \Omega^n \Sigma^n \), we therefore have

\[
\gamma \circ f \equiv g \circ \gamma: X \to \Omega^n \Sigma^n
\]

and hence

\[
f \circ \gamma \equiv g \circ \gamma
\]

as a G-map from \( X \) to \( \Omega^{2n} \Sigma^{2n} \). Add \( -\gamma \) to the both sides of this relation, then it follows from the definition of \( J_\rho \) that \( (s + tp)J_\rho(1 - \psi^p)(\{ E_f \}) = 0 \). That is, we obtain

(3) \((2t + p^i) + t(p - 1))J_\rho(1 - \psi^p)(x) = 0 \) for any \( x \in \mathcal{K}O_\rho^{-1}(X) \).

Applying \( \psi \) to this equality we have

(4) \( J_\rho(1 - \psi^p)(\psi(x))(\Omega) = 0 \)
where $J_e$ is the stable real $J$-homomorphism. (Here we write $z \in L$ viewed as an element of $L_{(2)}$.)

By (3) and (1)

$$
(2t+p)t)J_e(1-\psi^e) (x) = t\delta \psi J_e(1-\psi^e) (x)
$$

Thus, by (4)

$$
J_e(1-\psi^e) (x)_{(2)} = 0
$$

since $2t+p$ is odd. This completes the proof of Theorem.

References


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