1. Introduction

Let $G$ be a finite group and $k$ be a field of characteristic $p>0$. Let $\Theta$ be a connected component of the stable Auslander-Reiten quiver $\Gamma_{s}(kG)$ of the group algebra $kG$ and set $V(\Theta)=\{\text{vx}(M)|M \text{ is an indecomposable } kG\text{-module in } \Theta\}$, where $\text{vx}(M)$ denotes the vertex of $M$. As we shall see in Proposition 3.2 below, if $Q$ is a minimal element in $V(\Theta)$, then $Q \leq_{c} H$ for all $H \in V(\Theta)$. In particular we see that $Q$ is uniquely determined up to conjugation in $G$.

Let $N=N_{G}(Q)$ and let $f$ be the Green correspondence with respect to $(G, Q, N)$. Choose an indecomposable $kG$-module $M_{o}$ in $\Theta$ with $Q$ its vertex. Let $\Delta$ be the connected component of $\Gamma_{s}(kN)$ containing $fM_{o}$. The purpose of this paper is to show that there is a subquiver $\Lambda$ of $\Delta$ and a graph isomorphism $\psi: \Lambda \rightarrow \Theta$ such that $\psi^{-1}$ behaves like the Green correspondence $f$ as a bijective map between modules in $\Lambda$ and those in $\Theta$. In particular $\Theta$ is isomorphic with a subquiver of $\Delta$. Also it will be shown that if $H\in V(\Theta)$, then $H \leq_{c} N_{G}(Q)$.

The notation is almost standard. All the modules considered here are finite dimensional over $k$. We write $W \mid W'$ for $kG$-modules $W$ and $W'$, if $W$ is isomorphic to a direct summand of $W'$. For an indecomposable non-projective $kG$-module $M$, we write $\mathcal{A}(M)$ to denote the Auslander-Reiten sequence terminating at $M$. A sequence $M_{0}-M_{1}-\cdots-M_{t}$ of indecomposable $kG$-modules $M_{i}$ ($0\leq i \leq t$) is said to be a walk if there exists either an irreducible map from $M_{i}$ to $M_{i+1}$ or an irreducible map from $M_{i+1}$ to $M_{i}$ for $0\leq i \leq t-1$. Concerning some basic facts and terminologies used here, we refer to [1], [5], [6] and [8].

The author would like to thank Dr. T. Okuyama for his helpful advice.
The trace map \( t^G_H : (W, W')^H \rightarrow (W, W')^G \) is defined by \( t^G_H(\phi) = \sum_{i=1}^n \phi \cdot g_i \), for \( \phi \in (W, W')^H \). For a set \( \mathcal{S} \) of subgroups of \( G \), write \( (W, W')^\mathcal{S} = \sum_{V \in \mathcal{S}} \text{Im}(t^G_H) \) and \( (W, W')^\mathcal{S,G} = (W, W')^G/(W, W')^\mathcal{S} \). A \( kG \)-homomorphism \( \phi \) is said to be \( \mathcal{S} \)-projective, if \( \phi \in (W, W')^\mathcal{S} \). A \( kG \)-module \( W \) is said to be \( \mathcal{S} \)-projective, if \( W \mid \sum_{V \in \mathcal{S}} \oplus (W \downarrow V) \downarrow G \).

For a set \( \mathcal{S} \) of subgroups of \( G \), we set \( \mathcal{S} = \mathcal{S} \cap eH = \{ V \in \mathcal{S} \mid V \in \mathcal{S}, g \in G \} \).

**Lemma 2.1** ([8], Theorem 2.3). With the notation above, let \( \phi \in (W, W')^G \).

1. \( \phi \) is \( \mathcal{S} \)-projective if and only if \( \phi \) factors through a \( \mathcal{S} \)-projective module.
2. If \( W \) or \( W' \) is \( \mathcal{S} \)-projective, then \( \phi \) is \( \mathcal{S} \)-projective.

**Lemma 2.2.**

1. ([8], Cor. 5.4) For a \( kG \)-module \( A \) and a \( kH \)-module \( B \), the following \( k \)-isomorphisms hold:
   
   \[
   (A \downarrow H, B)^{S,H} = (A, B^G)^{S,G},
   
   (B, A \downarrow H)^{S,H} = (B^G, A)^{S,G}.
   
   2. In particular, for \( kH \)-modules \( A \) and \( B \), the following \( k \)-isomorphism holds:

   \[
   ((A^G) \downarrow H, B)^{S,H} = (A, (B^G) \downarrow H)^{S,H}.
   
   The next two results are also well-known.

**Lemma 2.3** ([1], Prop. 2.17.10). Let \( M \) be an indecomposable non-projective \( kG \)-module and \( H \) be a subgroup of \( G \). Then the Auslander-Reiten sequence \( \mathcal{A}(M) \) splits on restriction to \( H \) if and only if \( H \) does not contain \( \text{vx}(M) \).

**Lemma 2.4** ([4], Lemma 1.5 and [7], Theorem 7.5). Let \( H \) be a subgroup of \( G \). Let \( M \) and \( L \) be indecomposable non-projective modules for \( G \) and \( H \) respectively. Assume that \( L \) is a direct summand of \( (L^G) \downarrow H \) with multiplicity one, and that \( M \) is a direct summand of \( L^G \) such that \( L \mid M \downarrow H \). Then \( \mathcal{A}(L)^G = \mathcal{A}(M) \oplus E \), where \( E \) is a split sequence.

Finally we note:

**Lemma 2.5.** Let \( P \) be a non-trivial \( p \)-subgroup of \( G \). Let \( M \) and \( L \) be indecomposable non-projective modules for \( G \) and \( N_G(P) \) respectively. Assume that \( \mathcal{A}(L)^G = \mathcal{A}(M) \oplus E \), where \( E \) is a split sequence and that \( P \leq \text{vx}(L) \). If \( M \) is not a direct summand of the middle term of \( \mathcal{A}(L)^G \), then \( \mathcal{A}(M) \downarrow_{N_G(P)} = \mathcal{A}(L) \oplus E' \), where \( E' \) is a \( P \)-split sequence.
Proof. Using the same argument as in the proof of [3], (2.3) Lemma (a), we have $\mathcal{A}(M) \downarrow_{N_G(P)} = \mathcal{A}(L) \oplus \mathcal{C}'$, where $\mathcal{C}'$ is some exact sequence. Therefore we have only to show that $\mathcal{C}'$ is a $P$-split sequence. Let $( , )$ denote the inner product on the Green ring $a(kG)$ induced by $\dim_k \Hom_{kG}( , )$ [2]. For an exact sequence of $kG$-modules $\iota: 0 \to A \to B \to C \to 0$, put $\mathcal{A}(\iota) = B - A - C$. By [2], Theorem 3.4, it is sufficient to show that $(\mathcal{A}(\mathcal{C}') \downarrow_P, W) = 0$ for any $kP$-module $W$. Using the Frobenius reciprocity, we have

\[
(\mathcal{A}(\mathcal{C}') \downarrow_P, W) = (\mathcal{A}(\mathcal{A}(M)) \downarrow_P, W) - (\mathcal{A}(\mathcal{A}(L)) \downarrow_P, W) = (\mathcal{A}(\mathcal{A}(M)), W^\mathcal{A}) - (\mathcal{A}(\mathcal{A}(L)), W^\mathcal{A}) = (\mathcal{A}(\mathcal{A}(L)), (W^\mathcal{A}) \downarrow_N) - (\mathcal{A}(\mathcal{A}(L)), (W^\mathcal{A}) \downarrow_N),
\]

where $N = N_G(P)$. By the Mackey decomposition, $(W^\mathcal{A}) \downarrow_N = W^\mathcal{A} \oplus W'$, where $W'$ is $\{P \cap N | g \in G \backslash N\}$-projective. Since $L$ is not $\{P \cap N | g \in G \backslash N\}$-projective, we have $(\mathcal{A}(\mathcal{A}(L)), W') = 0$. Consequently we get $(\mathcal{A}(\mathcal{C}') \downarrow_P, W) = 0$ as desired.

3. Minimal element in $V(\Theta)$

Let $\Xi$ be a subgraph of the stable Auslander-Reiten quiver $\Gamma_s(kG)$ and set $V(\Xi) = \{v_x(M) | M \in \Xi\}$. Note that every element in $V(\Xi)$ is a non-trivial $p$-subgroup of $G$ since every $M$ is non-projective. The following Lemma 3.1 is essential in our argument.

**Lemma 3.1.** Let $\Xi$ be a subgraph of $\Gamma_s(kG)$. Assume that $\Xi$ is connected. Take any $Q \in V(\Xi)$ with the smallest order among those $p$-subgroups in $V(\Xi)$. Then for any indecomposable module $M \in \Xi$, $M \downarrow_Q$ has an indecomposable direct summand whose vertex is $Q$.

**Proof.** Let $M_0 \in \Xi$ be such that $Q = v_x(M_0)$. As $\Xi$ is connected, there is a walk $M_0 - M_1 - \cdots - M_t = M$, so that $M_i$ is a direct summand of the middle term of the Auslander-Reiten sequence $\mathcal{A}(M_{i+1})$ or $\mathcal{A}(\Omega^{-2}M_{i+1})$. We proceed by induction on the “distance” $t$. Suppose that $M_{t-1} \downarrow_Q$ has an indecomposable direct summand whose vertex is $Q$. We may assume that $v_x(M_t) \equiv_c Q$, since otherwise $v_x(M_t) =_c Q$ and $Q$-source of $M_t$ is a direct summand of $M_t \downarrow_Q$. By Lemma 2.3, $\mathcal{A}(M_t) \downarrow_Q$ and $\mathcal{A}(\Omega^{-2}M_t) \downarrow_Q$ split. Since $M_{t-1}$ is a direct summand of the middle term of $\mathcal{A}(M_t)$ or $\mathcal{A}(\Omega^{-2}M_t)$, $M_t \downarrow_Q$ has an indecomposable direct summand whose vertex is $Q$.

Lemma 3.1 implies that the minimal elements with respect to the partial order $\leq_c$ are those that have the smallest order. Thus the following holds.

**Proposition 3.2.** Let $\Theta$ be a connected component of $\Gamma_s(kG)$. Let $Q$ be
an element of $V(\Theta)$ which is minimal with respect to the partial order $\leq_G$. Then for any $H \in V(\Theta)$, we have $Q \leq_G H$. In particular $Q$ is uniquely determined up to conjugation in $G$.

4. Module correspondence

Now returning to the situation of the introduction, let $Q$ be a minimal element in $V(\Theta)$ throughout this section. Let $\Lambda$ be the subquiver of $\Delta$ consisting of those $kN$-modules $L$ in $\Delta$ such that there exists a walk $fM_0=L_0-L_1-\cdots-L_t=L$ with $Q \leq_G \text{vx}(L_i)$ ($i=0,1,\ldots,t$).

First of all we note

**Lemma 4.1.** Let $L$ be an indecomposable $kN$-module in $\Lambda$. Then $Q \leq \text{vx}(L)$.

**Proof.** This follows immediately from Lemma 3.1.

Let \( \mathfrak{X} \) be the set of all \( p \)-subgroups of $N$ of order smaller than $|Q|$. Also let \( \mathfrak{Y} = \{ N \cap Q^g | g \in G \setminus N \} \).

**Lemma 4.2.** Let $W$ be an indecomposable $kG$-module in $\Theta$. Then there exists a $kN$-module $T$ satisfying the following two conditions:

(i) \( (T \downderarrow q)^\uparrow N = T \oplus T' \), where $T'$ is \( \mathfrak{Y} \)-projective.

(ii) \( (W \downderarrow q, T)^\mathfrak{X},N \neq 0 \).

**Proof.** By Lemma 3.1, $W \downderarrow q$ has an indecomposable direct summand $S$ whose vertex is $Q$. Let $T'=S^\uparrow N$. We show that $T$ satisfies the above two conditions. By the Mackey decomposition we have $T \downderarrow q = \sum_{g \in Q^g \setminus N \in G} \oplus (S \otimes g)$ and so every indecomposable direct summand of $T$ has $Q$ as a vertex. Hence by the Green correspondence $(T \downderarrow q)^\uparrow N \cong T \oplus T'$, where $T'$ is $\mathfrak{Y}$-projective. Let us show the condition (ii). Letting $X = \mathfrak{X} \cap N Q$, we have by Lemma 2.2 (1)

\[
(W \downderarrow q, T)^\mathfrak{X},N = (W \downderarrow q, S^\uparrow N)^\mathfrak{X},N
\]

\[
\cong (W \downderarrow q, S)^\mathfrak{X},Q \oplus (S, S)^\mathfrak{X},Q \neq 0
\]

and the assertion follows.

**Lemma 4.3.** Let $T$ be a $kN$-module satisfying the condition (i) of Lemma 4.2. Let $L$ be an indecomposable $kN$-module in $\Lambda$. Then the following $k$-isomorphisms hold:

\[
((L \downderarrow q)^\uparrow N, T)^\mathfrak{X},N \cong (L, (T \downderarrow q)^\uparrow N)^\mathfrak{X},N \cong (L, T)^\mathfrak{X},N.
\]

**Proof.** The first $k$-isomorphism holds by Lemma 2.2 (2).

Let $(T \downderarrow q)^\uparrow N = T \oplus (\Sigma_i \oplus X_i)$, where $X_i$ is an indecomposable $\mathfrak{Y}$-projective $kN$-module. It is enough to show that $(L, X_i)^\mathfrak{X},N = 0$ for all $X_i$. So we have to show that any $\alpha \in (L, X_i)^N$ is $\mathfrak{X}$-projective. Since $X_i$ is $Q=(Q^\mathfrak{X} \cap N)$-projective
for some $g \in G \setminus \mathcal{N}$, there exists $\beta \in (L \downarrow \mathfrak{g}, X_{\mathfrak{g}}) \mathfrak{g}$ such that $\alpha = \iota_{\mathfrak{g}}^\mathfrak{g}(\beta)$. Now, there exists a walk $fM_0 = L_0 - L_1 - \cdots - L_t = L$ such that $Q \leq \text{vex}(L_i) (i = 0, 1, \ldots, t)$ by Lemma 4.1. As $Q$ is not conjugate to $Q$ in $\mathcal{N}$, $\mathcal{A}(L_0) \downarrow \mathfrak{g}$ splits $(i = 0, 1, \ldots, t)$ by Lemma 2.3. Since $L_0 \downarrow \mathfrak{g}$ is $\mathfrak{x}$-projective and $L_1$ is a direct summand of the middle term of $\mathcal{A}(L_0)$, it follows that $L_1 \downarrow \mathfrak{g}$ is also $\mathfrak{x}$-projective. Using this argument repeatedly, we conclude that $L \downarrow \mathfrak{g}$ is $\mathfrak{x}$-projective. Therefore $\beta$ is $\mathfrak{x}$-projective by Lemma 2.1 and hence $\alpha$ is $\mathfrak{x}$-projective.

**Lemma 4.4.** Let $L$ be an indecomposable $k\mathfrak{N}$-module in $\Lambda$. Then $L \uparrow \mathfrak{g}$ has a unique indecomposable direct summand $M$ whose vertex contains $Q$, and we have

1. $L$ is a direct summand of $M \downarrow \mathcal{N}$, and
2. $M$ lies in $\Theta$.

Moreover letting $T$ be a $k\mathfrak{N}$-module satisfying the conditions in Lemma 4.2 for $M$, we have:

$$((L \uparrow \mathfrak{g}) \downarrow \mathcal{N}, T)_{\mathfrak{x}, \mathcal{N}} = (M \downarrow \mathcal{N}, T)_{\mathfrak{x}, \mathcal{N}} = (L, T)_{\mathfrak{x}, \mathcal{N}} \neq 0.$$ 

In particular, $L$ is a direct summand of $(L \uparrow \mathfrak{g}) \downarrow \mathcal{N}$ with multiplicity one.

Proof. Since $L \mid (L \uparrow \mathfrak{g}) \downarrow \mathcal{N}$, $L \uparrow \mathfrak{g}$ has an indecomposable direct summand $M$ such that $L \mid M \downarrow \mathcal{N}$. Therefore the vertex of $M$ contains $Q$ and $L \uparrow \mathfrak{g}$ has at least one indecomposable direct summand whose vertex contains $Q$.

Let $fM_0 = L_0 - L_1 - \cdots - L_t = L$ be a walk. We prove the assertion by induction on the $t$.

If $t = 0$, i.e., $L = fM_0$, then the assertion follows since $f$ is the Green correspondence.

Suppose the assertion holds for $L_{t-1}$. We shall derive a contradiction assuming that $L \uparrow \mathfrak{g}$ has two indecomposable direct summands $M$ and $W$ whose vertices contain $Q$. Let $L \uparrow \mathfrak{g} = M \oplus W \oplus W'$. We may assume that $L \mid M \downarrow \mathcal{N}$. By Lemma 2.4 $\mathcal{A}(L_{t-1}) \uparrow \mathfrak{g} = \mathcal{A}(M_{t-1}) \oplus \mathcal{E}$, where $M_{t-1}$ is the unique indecomposable direct summand of $L_{t-1} \uparrow \mathfrak{g}$ whose vertex contains $Q$ and $\mathcal{E}$ is a split sequence. Note that the middle term of $\mathcal{E}$ does not have an indecomposable direct summand whose vertex contains $Q$, since $M_{t-1}$ (resp. $\Omega^2 M_{t-1}$) is a unique indecomposable direct summand of $L_{t-1} \uparrow \mathfrak{g}$ (resp. $(\Omega^2 L_{t-1}) \uparrow \mathfrak{g}$) whose vertex contains $Q$. Let $Y$ (resp. $Y'$) be the middle term of $\mathcal{A}(M_{t-1})$ (resp. $\mathcal{A}(\Omega^2 M_{t-1})$). Since $L$ is a direct summand of the middle term of $\mathcal{A}(L_{t-1})$ or $\mathcal{A}(\Omega^2 L_{t-1})$, it follows that $M \oplus W | Y$ or $M \oplus W | Y'$. In particular both $M$ and $W$ lie in $\Theta$.

Let $T$ and $U$ be $k\mathfrak{N}$-modules satisfying the conditions (i) and (ii) for $M$ and $W$ respectively in Lemma 4.2 and put $T' = T \oplus U$. Then
\[(L^\uparrow \wp) \downarrow_N, \quad T')^\wp,N\]
\[= (M \downarrow_N, \quad T')^\wp,N \oplus (W\downarrow_N, \quad T')^\wp,N \oplus (W'^\downarrow_N, \quad T')^\wp,N\]
\[= (L, \quad T')^\wp,N \oplus (Z, \quad T')^\wp,N \oplus (W\downarrow_N, \quad T')^\wp,N \oplus (W'^\downarrow_N, \quad T')^\wp,N,\]

where \(M \downarrow_N = L \oplus Z\). But by Lemma 4.3, \(((L^\uparrow \wp) \downarrow_N, \quad T')^\wp,N \simeq (L, \quad T')^\wp,N\). This implies that \((W\downarrow_N, \quad U')^\wp,N \subset (W\downarrow_N, \quad T')^\wp,N = 0\), which is a desired contradiction. Thus \(L^\uparrow \wp\) has a unique indecomposable direct summand \(M\) whose vertex contains \(Q\), and the statements (1) and (2) hold. Moreover we obtain that

\(((L^\uparrow \wp) \downarrow_N, \quad T')^\wp,N \simeq (M \downarrow_N, \quad T')^\wp,N \simeq (L, \quad T')^\wp,N \neq 0,\]

since \(M \mid L^\uparrow \wp\) and \(L \mid M \downarrow_N\). Hence \(L\) is a direct summand of \((L^\uparrow \wp) \downarrow_N\) with multiplicity one; for otherwise

\[(L, \quad T')^\wp,N \oplus (L, \quad T')^\wp,N \subset ((L^\uparrow \wp) \downarrow_N, \quad T')^\wp,N \simeq (L, \quad T')^\wp,N \neq 0,\]

a contradiction.

For an indecomposable \(kN\)-module \(L\) in \(\Lambda\), let \(\wp\) be a unique indecomposable direct summand of \(L^\uparrow \wp\) whose vertex contains \(Q\).

**Lemma 4.5.** Let \(L\) and \(L'\) be indecomposable \(kN\)-modules in \(\Lambda\). Then \(\wp L \simeq \wp L'\) if and only if \(L \simeq L'\).

Proof. If \(L \simeq L'\), then \(\wp L \simeq \wp L'\) clearly. To show the converse, assume by way of contradiction that \(\wp L \simeq \wp L'\) but \(L \neq L'\). Since \(L \mid \wp L \downarrow_N\) and \(L' \mid \wp L' \downarrow_N\), we have that \(L \oplus L' \mid \wp L \downarrow_N (L^\uparrow \wp) \downarrow_N\). Let \((L^\uparrow \wp) \downarrow_N = L \oplus L' \oplus W\). Let \(T\) be a \(kN\)-module satisfying the conditions (i) and (ii) of Lemma 4.2 for \(\wp L\). Then

\[((L^\uparrow \wp) \downarrow_N, \quad T)^\wp,N \]
\[= (L, \quad T)^\wp,N \oplus (L', \quad T)^\wp,N \oplus (W, \quad T)^\wp,N.\]

But by Lemma 4.3, \(((L^\uparrow \wp) \downarrow_N, \quad T)^\wp,N \simeq (L, \quad T)^\wp,N\). This implies that \((L', \quad T)^\wp,N = 0\), which is contrary to Lemma 4.4.

We are now ready to prove the main theorem of this paper.

**Theorem 4.6.** \(\wp\) induces a graph isomorphism from \(\Lambda\) onto \(\Theta\) which preserves edge-multiplicity and direction. Also \(\wp\) gives rise to a one-to-one correspondence between indecomposable modules in \(\Theta\) and those in \(\Lambda\) and the following hold:

1. Let \(M\) be an indecomposable \(kG\)-module in \(\Theta\). Then \(M \downarrow_N = \wp^{-1} M \oplus (\Sigma_i \oplus W_i), \) where \(W_i\) is \(\wp\)-projective for all \(i\).
2. Let \(L\) be an indecomposable \(kN\)-module in \(\Lambda\). Then \(L^\uparrow \wp = \wp L \oplus (\Sigma_i \oplus V_i),\)
where $V_i$ is $\mathcal{F}$-projective for all $i$.

Proof. It is a direct consequence of Lemmas 4.4, 4.5 and 2.4 that $\psi$ indeed induces a graph monomorphism. To show that $\psi$ is an epimorphism, let $M$ be an arbitrary element of $\Theta$ and let $M_0=M_1=\cdots=M_i=M$ be a walk in $\Theta$. If $t=0$, i.e., $M=M_0$, then $M_0=f^{-1}(fM_0)=\psi L_0$. Now, suppose then that there exists an element $L_{t-1}$ in $\Lambda$ such that $M_{t-1}=\psi L_{t-1}$. By Lemmas 4.4 and 2.4 we have $A(L_{t-1})^{fg}=A(M_{t-1})\oplus E$ and $A(\Omega^{-2}L_{t-1})^{fg}=A(\Omega^{-2}M_{t-1})\oplus E'$, where $E$ and $E'$ are split sequences. Recall that $M_t$ is a direct summand of the middle term of $A(M_{t-1})$ or $A(\Omega^{-2}M_{t-1})$. Therefore there exists some indecomposable direct summand $L$ of the middle term of $A(L_{t-1})$ or of $A(\Omega^{-2}L_{t-1})$ such that $M|L|E$. Since $Q\subseteq \text{vx}(M)\subseteq \text{vx}(L)$, $L$ lies in $\Lambda$. Consequently $M=\psi L$ and $\psi$ is an epimorphism.

Next we prove (1) by induction on the distance $t$ from $M_0$ to $M=M_t$. If $t=0$, i.e., $M=M_0$, then the statement (1) follows since $f$ is the Green correspondence. Suppose the statement (1) holds for $M_{t-1}$. We may assume that $M_t$ is a direct summand of the middle term of $A(M_{t-1})$ (otherwise replace $M_{t-1}$ by $\Omega^{-2}M_{t-1}$). Let $M_t\downarrow N:=\psi^{-1}M_t\oplus(\Sigma_t\oplus W_t)$ and let $M_{t-1}\downarrow N:=\psi^{-1}M_{t-1}\oplus(\Sigma_t\oplus W_t)$. By Lemma 2.5, $A(M_{t-1})\downarrow N:=A(\psi^{-1}M_{t-1})\oplus E'$, where $E'$ is a $Q$-split sequence. Note that $E'$ is an exact sequence terminating at $\Sigma_t\oplus W_t$. If $W_t$ is a direct summand of the middle term of $A(\psi^{-1}M_{t-1})$, then $Q\cong \text{vx}(W_t)$, since otherwise $W_t$ lies in $\Lambda$ but this contradicts that $\psi$ is a graph isomorphism which preserves edge-multiplicity. Therefore $W_t|Q$ is $\mathcal{F}$-projective. Suppose then that $W_t$ is a direct summand of the middle term of $E'$. Then since each $W_t|Q$ is $\mathcal{F}$-projective and $W_t|Q|(\Sigma_t\oplus W_t)|Q|Q(\Sigma_t\oplus Q2W_t)|Q$, it follows that $W_t|Q$ is $\mathcal{F}$-projective.

The statement (2) follows similarly by virtue of Lemma 2.4.

As an immediate consequence of the above theorem, we have

**Corollary 4.7.** Let $\Theta$ be a connected component of $\Gamma_s(kG)$ and let $Q$ be a minimal element in $V(\Theta)$. Then for any element $H$ of $V(\Theta)$, we have $H\subseteq N_G(Q)$.

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**References**


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