THE HOMOTOPY FIXED POINT SETS OF SPHERES ACTIONS ON RATIONAL COMPLEXES

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Abstract

In this paper, we describe the homotopy type of the homotopy fixed point sets of $S^3$-actions on rational spheres and complex projective spaces, and provide some properties of $S^1$-actions on a general rational complex.

1. Introduction

An action of a group $G$ on a space $M$ gives rise to two natural spaces, the fixed point set $M^G$ and the homotopy fixed point set $M^{hG}$. It is crucially important that there is an injection

$$k: M^G \rightarrow M^{hG}.$$

Indeed, one version of the generalized Sullivan conjecture asserts that, when $G$ is a finite $p$-group, and $M$ is a $G$-CW-complex, then the $p$-completion of $k$ is a homotopy equivalence. This conjecture was proved in the case when $M$ is a finite complex by Miller [7].

For a finite group $G$, the rational homotopy theory of $M^{hG}$ has been studied by Goyo [5].

In [1, 2], the authors studied the homotopy type of $M^{hG}$ for a compact Lie group $G$ with particular emphasis when $G$ is the circle.

From now on, and unless explicitly stated otherwise, $G$ will denote a compact connected Lie group and by a topological $G$-space we mean a nilpotent $G$-space with the homotopy type of a CW-complex of finite type and $M^G \neq \emptyset$. Then the action of $G$ on $M$ induces an action of $G$ on $M_Q$.

We then start by setting a sufficiently general context in which $M_Q^{hG}$ has the homotopy type of a nilpotent CW-complex. Identifying the homotopy fixed point set

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with the space Sec(ξ) of sections of the corresponding Borel fibration

\[ ξ: M \to M_{hG} \to BG, \]

we have that if \( π_{>n}(M) \) are torsion groups for a certain \( n > 1 \), then \( M_{Q^{hG}} \) is a rational nilpotent complex with the homotopy type of a CW-complex [1].

In this paper, we explicitly describe the rational homotopy type of the homotopy fixed point sets of certain \( S^3 \)-actions.

**Theorem 1.1.** Given an \( S^3 \)-action on the rational n-sphere \( S^n_{Q} \),

(1) When \( n \) is odd, \( S^n_{Q}^{hS^3} \) has the rational homotopy type of products of odd dimensional spheres, precisely, we have

\[ S^n_{Q}^{hS^3} \cong_{Q} S^{a} \times S^{a+4} \times \cdots \times S^{a}, \]

where

\[ a = \begin{cases} 1, & n = 4k + 1, \\ 3, & n = 4k + 3. \end{cases} \]

(2) If \( n = 4k \), \( S^n_{Q}^{hS^3} \) is either path connected, and of the rational homotopy type of \( S^3 \times K_k \), where \( K_k \) has the minimal Sullivan model

\[ (Λ((x_s)_{1≤s≤k}, (y_r)_{2≤r≤2k}), d) \]

with \( |x_s| = 4s, |y_r| = 4r - 1, dx_s = 0 (1 ≤ s ≤ k), dy_r = \sum_{s+1−r} x_s x_r (2 ≤ r ≤ 2k), \) or else, it has 2 components, each of them has the rational homotopy type of

\[ S^{4k+3} \times S^{4k+7} \times \cdots \times S^{8k-1}. \]

(3) If \( n = 4k + 2 \), \( S^n_{Q}^{hS^3} \) is path connected, and of the rational homotopy type of \( S^3 \times S^7 \times T_k \), where \( T_k \) has the minimal Sullivan model

\[ (Λ((x_s)_{1≤s≤k}, (y_r)_{3≤r≤2k+1}), d) \]

with \( |x_s| = 4s + 2, |y_r| = 4r - 1, dx_s = 0 (1 ≤ s ≤ k), dy_r = \sum_{s+1−r} x_s x_r (3 ≤ r ≤ 2k + 1). \)

**Theorem 1.2.** Given an \( S^3 \)-action in the rational complex projective space \( \mathbb{C}P^n_Q \).
(1) If \( n \) is odd, \( \mathbb{C}P^n h^{S^1} \) is path connected, and has the rational homotopy type of one of the following spaces:

\[
\begin{align*}
\mathbb{C}P^1 \times S^7 \times S^{11} \times \cdots \times S^{2n+1}, \\
S^3 \times \mathbb{C}P^3 \times S^{11} \times \cdots \times S^{2n+1}, \\
S^3 \times S^7 \times \mathbb{C}P^5 \times \cdots \times S^{2n+1}, \\
\cdots, \\
S^3 \times S^7 \times \cdots \times S^{2n-3} \times \mathbb{C}P^n.
\end{align*}
\]

(2) If \( n \) is even, \( \mathbb{C}P^n h^{S^1} \) is path connected, and has the rational homotopy type of one of the following spaces:

\[
\begin{align*}
\ast \times S^5 \times S^9 \times \cdots \times S^{2n+1}, \\
S^4 \times \mathbb{C}P^2 \times S^9 \times \cdots \times S^{2n+1}, \\
S^4 \times S^5 \times \mathbb{C}P^4 \times \cdots \times S^{2n+1}, \\
\cdots, \\
S^4 \times S^5 \times \cdots \times S^{2n-3} \times \mathbb{C}P^n.
\end{align*}
\]

In [1, Corollary 2], they give a criterion of an elliptic \( S^1 \)-space. We first show that the condition \( M \) is a finite complex is necessary by the following example: there is a nilpotent \( S^1 \)-complex \( M \) which is not an elliptic space, such that each component of \( M h^{S^1} \) is elliptic. We also observe that an \( S^1 \)-finite nilpotent complex \( M \) is elliptic if and only if one of the component of \( M h^{S^1} \) is elliptic, complementing the mentioned result.

Finally, we show that the injection \( k \) is generally not a rational homotopy equivalence.

**Theorem 1.3.** For an \( S^1 \)-complex \( M \) which is simply connected with

\[
\dim \pi_n(M) \otimes \mathbb{Q} < \infty.
\]

Then

\[
k : M^{S^1}_Q \hookrightarrow M^{hS^1}_Q
\]

is a rational homotopy equivalence if and only if \( M \) is rational homotopy equivalent to a product of \( \mathbb{C}P^n \).

In the next section we prove Theorems 1.1 and 1.2. In Section 3 we prove Theorem 1.3.
2. $S^3$-rational spheres and complex projective spaces

Our results heavily depend on known facts and techniques arising from rational homotopy theory. All of them can be found with all details in [4]. We simply remark a few facts.

We recall that when $M$ is path connected, the Sullivan model of $M$ is a quasi-isomorphism

$$m: (AV_M, d) \to A_{PL}(M),$$

where $(AV_M, d)$ is a Sullivan algebra.

We also recall that a space $M$ is elliptic if both $H^*(M; \mathbb{Q})$ and $\pi_*(M) \otimes \mathbb{Q}$ are finite dimensional vector spaces over $\mathbb{Q}$.

For a $G$-space $M$, we have the corresponding Borel fibration

$$\xi: M \to M_{hG} \to BG,$$

where $M_{hG} = (M \times EG)/G$. It is a classical fact that the homotopy fixed point set

$$M^{hG} = \text{map}_G(EG, M)$$

is homotopy equivalent to the section space $\text{Sec}(\xi)$ of this fibration.

Each fixed point gives rise to a trivial section of the product bundle

$$M^G \to BG \times M^G \to BG.$$

Composing with the injection $M_G \times BG \hookrightarrow EG \times M/G = M_{hG}$ gives a section of the Borel fibration. Thus we have a natural injection:

$$k: M^G \hookrightarrow M^{hG}.$$

For any $G$-CW complex $M$, there is an equivariant rationalization $m: M \to M_Q$, that is, $M_Q$ is also a $G$-CW complex, $m$ is an equivariant map, and $(M_Q)^G \simeq (M^G)_Q$. Moreover, we have

**Proposition 2.1** ([1, Proposition 12]). If $M$ is a Postnikov piece, that is, $\pi_{\geq N}(M) = 0$ for some $N$, then

(i) $M^{hG}$ has the homotopy type of a nilpotent CW-complex of finite type.

(ii) $(M^{hG})_Q \simeq (M_Q)^{hG}$.

Note that if $M_Q$ is a Postnikov piece, then $(M_Q)^{hG}$ makes sense and is a rational space.

Now, we determine the homotopy type of the homotopy fixed point sets of certain $S^3$-actions.
Proof of Theorem 1.1. (1) CASE 1: \( n \) is odd.

We only prove the case \( n = 4k + 3 \), the case \( n = 4k + 1 \) is similar, so we omit it.

As in the proof of [1, Theorem 19], it is not hard to get the model of the corresponding Borel fibration

\[
\xi: (A, 0) \hookrightarrow (\Lambda (e) \otimes A, D) \to (\Lambda e, 0),
\]

where \((A, 0) = (\Lambda x/x^k, 0)\) and \(|x| = 4, |e| = n\). This fibration is trivial, so \( \text{Sec} (\xi) \simeq \text{Map}(\mathbb{H}^{P^k}, S^n) \).

By [1, Theorem 9], the model of \( S_n^Q hS^3 \) is \((\Lambda (x_1, x_2, \ldots, x_{n+1/4}), 0)\). It is exactly the model of \( S^3 \times S^7 \times \cdots \times S^n \). It follows that \( S_n^Q hS^3 \simeq Q S^a \times S^{a+4} \times \cdots \times S^n \).

(2) CASE 2: \( n = 4k \).

As \( \pi_{\geq 2n} (S^n) \otimes Q = 0 \), a model of the Borel fibration is

\[
\xi_{2n}: (A, 0) \hookrightarrow (\Lambda (e, e') \otimes A, D) \to (\Lambda (e, e'), d),
\]

where \( A = \Lambda x/x^{2k+4}, x, e, e' \) are of degree 4, \( n, 2n - 1 \) respectively, \( De = 0, De' = e^2 + \lambda x^{n/4}e, de' = e^2 \).

(i) If \( \lambda = 0 \), then \( \xi_{2n} \) is trivial and

\[
S_n^Q hS^3 \simeq \text{Map}(\mathbb{H}^{P^k}, S^n)_Q.
\]

A straightforward computation shows that this mapping space has a model of the form

\[
(\Lambda y_1, 0) \otimes (\Lambda ((x_s)_{1 \leq s \leq k}, (y_r)_{2 \leq r \leq 2k}), d)
\]

with \(|y_s| = 4s, |y_r| = 4r - 1, dx_s = 0 (1 \leq s \leq k), dy_r = \sum s+r=r x_s y_r (r > 1)\).

(ii) If \( \lambda \neq 0 \), then the fibration \( \xi_n \) has two non homotopic sections \( \sigma, \tau \) which correspond to the only two possible retractions of its model:

\[
\varphi_\sigma, \varphi_\tau: (\Lambda (e, e') \otimes A, D) \to (A, 0), \quad \varphi_\sigma (e) = 0, \quad \varphi_\tau (e) = \lambda x^k.
\]

By the same way in [1], we have that the model of \( \text{Sec}_\sigma (\xi_{2n}) \) is of the form

\[
(\Lambda ((x_s)_{1 \leq s \leq k}, (y_r)_{1 \leq r \leq 2k}), d).
\]

with \(|y_s| = 4s, |y_r| = 4r - 1\). The linear part of \( d \) is:

\[
d(y_r) = \lambda x_r
\]

for \( 1 \leq r \leq k \), which shows that the minimal model of \( \text{Sec}_{\sigma} (\xi_{2n}) \) is

\[
(\Lambda (y_{r})_{k+1 \leq r \leq 2k}, 0).
\]
Replace \( \lambda \) by \(-\lambda\), we have that the model of \( \text{Sec}_r(\xi_{2n}) \) is the same.

(3) **Case 2:** \( n = 4k + 2 \).

As \( \pi_{\geq 2n}(S^n) \otimes \mathbb{Q} = 0 \), a model of the Borel fibration is

\[
\xi_{2n}: (A, 0) \mapsto (\Lambda(e, e') \otimes A, D) \mapsto (\Lambda(e, e'), d),
\]

where \( A = \Lambda x / x^{2k+1} \), \( x, e, e' \) are of degree 4, \( n, 2n - 1 \) respectively, \( De = 0 \), \( De' = e^2 \), \( de = e^2 \). It follows that the fibration \( \xi_{2n} \) is trivial, we have

\[
S^n_{Q}^h \simeq \text{Map}(\mathbb{H} P^{2k}, S^n)_{Q}.
\]

The model of \( S^n_{Q}^h \) is

\[
(\Lambda(y_1, y_2), 0) \otimes (\Lambda((x_s)_{1 \leq s \leq k}, (y_r)_{3 \leq r \leq 2k+1}), d)
\]

with \( |x_s| = 4s + 2, \ |y_r| = 4r - 1, \ dx_s = 0 \) (\( 1 \leq s \leq k \)), \( dy_r = \sum_{s+r-1} x_s x_t \) (\( 3 \leq r \leq 2k + 1 \)).

The desired result follows. \( \square \)

Proof of Theorem 1.2. First, we assume \( n = 2k + 1 \). As \( \pi_{\geq 4k+4}(\mathbb{C} P^n_{Q}) = 0 \), it suffice to use the model of \( \xi_{2n+2} \)

\[
(A, 0) \mapsto (\Lambda(e, e') \otimes A, D) \mapsto (\Lambda(e, e'), d),
\]

where \( A = (\Lambda x) / x^{k+2} \), \( |x| = 4, \ |e| = 2, \ |e'| = 4k + 3 \), and

\[
De = 0, \quad De' = e^{n+1} + \sum_{j=1}^{k} \lambda_j e^j x^{n+1-2j}, \quad \lambda_j \in \mathbb{Q}, \ j = 1, \ldots, n.
\]

The retraction of this model of fibration is just \( \varphi(e) = 0 \). So we have \( \text{Sec}(\xi_{4k+4}) \) is connected, and the model of it is

\[
(\Lambda(e, (e'_r))_{1 \leq r \leq k+1}, \tilde{d})
\]

with \( |e| = 2, \ |e'_r| = 4r - 1, \ \tilde{d}(e'_r) = \lambda_{k+1} e^{2r} \) for \( 1 \leq r \leq k \) and \( \tilde{d}(e'_{k+1}) = e^{2k+2} \).

If \( \lambda_1 \neq 0 \) this is a model of

\[
S^2 \times S^7 \times \cdots \times S^{4k+3}.
\]

If \( \lambda_1 = \cdots = \lambda_i = 0 \) and \( \lambda_i \neq 0 \), this is a model of

\[
S^3 \times \cdots \times S^{4k-4i+3} \times \mathbb{C} P^{2k+1-2i} \times S^{4k-4i+3} \times \cdots \times S^{4k+3}.
\]
Finally, if all $\lambda_i = 0$, then it is a model of

$$S^3 \times S^7 \times \cdots \times S^{4k-1} \times \mathbb{C} P^{2k+1}.$$  

For $n$ even, the proof is similar, so we omit it.  

\[\square\]

3. The Inclusion $k : M^{S^1} \hookrightarrow M^{hS^1}$

We begin with some interesting observations on $S^1$-actions.

In [2, Example 12], there is an $S^1$-action on $M = K(\mathbb{Z}, n) \times K(\mathbb{Z}, n + 1)$, such that the model of it’s Borel fibration is

$$\eta_n : (Ax, 0) \hookrightarrow (Ax \otimes A(z, y), D) \hookrightarrow (A(z, y), d),$$

where $|x| = 2$, $|z| = n$, $|y| = n + 1$, $D(z) = 0$, and $D(y) = xz$. For $n = 2k$, there is only one retraction $\sigma: \sigma(z) = \sigma(y) = 0$. Thus $\text{Sec}(\eta_{2k})$ is path connected.

By the same method used in [1], a model of $\text{Sec}(\eta_{2k})$ is

$$(\Lambda((z_i)_{1 \leq i \leq k}, (y_j)_{1 \leq j \leq k+1}), d),$$

where $|z_i| = 2i$, $|y_j| = 2j - 1$ and $d(y_i) = z_i$. Since the minimal model of $\text{Sec}(\eta_{2k})$ is $(\Lambda y_{k+1}, 0)$, $\text{Sec}(\eta_{2k}) \simeq_{\mathbb{Q}} S^{2k+1}$ is an elliptic space. However, $M$ is not an elliptic space.

Next we complement [1, Corollary 2] with the following

**Proposition 3.1.** For an $S^1$-space $M$ which is a nilpotent finite complex, the following conditions are equivalent:

1) $M$ is elliptic.

2) Each component of $M^{hS^1}$ is elliptic.

3) One of the components of $M^{hS^1}$ is elliptic.

Proof. 1) $\Rightarrow$ 2): [1, Theorem 15].

2) $\Rightarrow$ 3): Trivial.

3) $\Rightarrow$ 1): By [2, Theorem 13], $\dim \pi_*(\text{Sec}_\sigma(\xi) \otimes \mathbb{Q}) \geq \dim \pi_*(M) \otimes \mathbb{Q}$. By $\text{Sec}_\sigma(\xi)$ is elliptic, $\dim \pi_*(\text{Sec}_\sigma(\xi)) \otimes \mathbb{Q}$ is finite, so $\dim \pi_*(M) \otimes \mathbb{Q}$ is finite. Then $M$ is elliptic.

\[\square\]

**Remark 3.2.** The theorem holds also for $G = S^3$. The proof is similar.

The rest of the section is devoted to showing Theorem 1.3.
Let $M$ be an $S^1$-space and $M^G \neq \emptyset$. Then the inclusion $M^{S^1} \hookrightarrow M$ induces a map of Borel fibrations:

$$
\begin{array}{ccc}
M^{S^1} & \longrightarrow & M \\
\downarrow & & \downarrow \\
\mathbb{C} P^\infty \times M^{S^1} & \xrightarrow{\gamma} & M_{hS^1} \\
\eta & \downarrow & \xi \\
\mathbb{C} P^\infty
\end{array}
$$

(3.1)

If there exists some $N$ such that $\pi_{>N}(M_G) = 0$ and $\pi_{>N}(M^{S^1}_G) = 0$. Then $k$ is identified with the corresponding

$$
M^{S^1} \hookrightarrow \text{Map}((\mathbb{C} P^\infty)^{(N)}, M^{S^1}) \rightarrow \text{Sec}(\xi_N) \cong M_{hS^1},
$$

which can be obtained by truncating in the diagram (3.1):

$$
\begin{array}{ccc}
M^{S^1} & \longrightarrow & M \\
\downarrow & & \downarrow \\
F_N & \xrightarrow{\gamma_N} & E_N \\
\eta_N & \downarrow & \xi_N \\
(\mathbb{C} P^\infty)^{(N)}
\end{array}
$$

Now let

$$
\begin{array}{ccc}
(A \otimes \Lambda V, D) & \longrightarrow & (\Lambda V, d) \\
\downarrow & & \downarrow \\
(A, 0) \otimes (\Lambda Z, d) & \longrightarrow & (\Lambda Z, d)
\end{array}
$$

(3.2)

be a model of the above diagram, where $(A, 0) = (\Lambda x/(\Lambda x)^{>N}, 0)$, $(\Lambda V, d)$ and $(\Lambda Z, d)$ are minimal Sullivan models of $M$ and $M^{S^1}$, respectively.

Then we have the following

**Theorem 3.3.** [1, Theorem 21] The composition

$$(\Lambda(V \otimes A^d), \tilde{d}) \xrightarrow{\phi} (\Lambda(Z \otimes A^d), \tilde{d}) \xrightarrow{\gamma} (\Lambda Z, d)$$
is a model of \( k : M^S_Q \to M^h_Q \). The morphisms above are defined by

\[
\phi(v \otimes \alpha) = \rho^{-1}[\psi(v) \otimes \alpha], \quad v \otimes \alpha \in V \otimes A^\#,
\]

\[
\gamma(z \otimes \alpha) = \begin{cases} 
  z & \alpha = 1, \\
  0 & \alpha \neq 1,
\end{cases} \quad z \otimes \alpha \in Z \otimes A^\#.
\]

Then we give some information about \( \psi \). First, let \((\Lambda x \otimes \Lambda V, D)\) be a model of the fibration \( \xi \), we can decompose the differential \( D \) in \( A \otimes \Lambda V \) into

\[
D = \sum_{i \leq 1} D_i, \quad D_i(V) \subset \Lambda x \otimes \Lambda^i V.
\]

**Proposition 3.4.** \([2, \text{Lemma } 14]\) The vector space \( V \) can be decomposed into a direct sum \( W \oplus K \oplus S \) where

1. \( W \oplus K = \ker D_1 \),
2. \( K \) and \( S \) have the same dimension admitting bases \( \{v_i\}_{i \in I}, \{s_i\}_{i \in I} \), and for any \( i \in I \), there exists \( n_i \geq 1 \) such that \( D_1(s_i) = x^{n_i} v_i \).

Let \( K = Q(x) \), the field of fractions of \( \Lambda x \), we obtain a morphism of (ungraded) differential vector spaces

\[
\tilde{\psi} : (K \otimes V, D_1) \to (K \otimes Z, 0) = (Z_K, 0).
\]

If we assume \( K \) concentrated in degree 0 and consider in \( V \) and \( Z \) the usual \( \mathbb{Z}_2 \)-grading given by the parity of the generators, then the Borel localization theorem claim that:

**Theorem 3.5.** \([1, \text{Theorem } 22]\) The morphism

\[
\tilde{\psi} : (K \otimes V, D_1) \to (Z_K, 0)
\]

is a quasi-isomorphism.

By Proposition 3.4, we have

**Lemma 3.6.** \( 1 \) \( \dim W = \dim Z \).
2. There are \( \{w_j\}_{j \in J}, \{z_j\}_{j \in J} \) which are homogeneous basis of \( W \) and \( Z \) respectively, and non negative integers \( \{m_j\}_{j \in J} \) such that

\[
\psi(w_j) = x^{m_j} z_j + \Gamma_j, \quad \Gamma_j \in R \otimes \Lambda^2 Z, \quad j \in J,
\]

and

\[
\psi(s_i) \in R \otimes \Lambda^2 Z, \quad \psi(v_i) \in R \otimes \Lambda^2 Z, \quad s_i \in S, \; v_i \in K, \; i \in I.
\]
**Theorem 3.7.** For an $S^1$-complex $M$ which is simply connected with 
$$\dim \pi_*(M) \otimes \mathbb{Q} < \infty.$$ 
Then the inclusion 
$$k : M^{S^1} \hookrightarrow M^{hS^1}$$ 
is a rational homotopy equivalence if and only if $M$ is rational homotopy equivalent to a product of $\mathbb{C}P^\infty$.

**Proof.** By Theorem 3.3, the model of $k$ is 
$$\alpha: (\Lambda(V \otimes A^\#), \tilde{d}) \to (\Lambda(Z \otimes A^\#), \tilde{d}) \to (\Lambda Z, d).$$

By [1, Theorem 24], $\pi_*(k) \otimes \mathbb{Q}$ is injective, so we only consider the surjective part.

By [1, Theorem 11], $(\Lambda(V \otimes A^\#), \tilde{d})$ is a model of $M^{hS^1}_Q$. Then we have 
$$H^k(V \otimes A^\#_j, \tilde{d}_1) \cong \text{Hom}(\pi_k(M^{hS^1}_Q), \mathbb{Q}),$$
where $k \geq 1$.

By Proposition 3.4, $V = W \oplus K \oplus S$. An easy computation shows that $(W \otimes A^\#) \oplus S \subset H^*(V \otimes A^\#, \tilde{d}_1)$. It is obvious that 
$$\alpha(w_j) = 0 \iff m_j \neq 0,$$
$$\alpha(w_j \otimes (x_i)^\#) = 0 \iff m_j \neq i,$$
$$\alpha(x_j) = 0.$$

If there exists some $j$ such that $|w_j| \geq 2$ or $S \neq \emptyset$, then $H(\alpha, \tilde{d}_1)$ is not injective, so $k$ is not a rational homotopy equivalence.

If $|w_j| = 2$, for each $j \in J$, and $S = \emptyset$, we have $(\Lambda W, d)$ is a model of a product of $\mathbb{C}P^\infty$. It is easy to show that $k$ is a rational homotopy equivalence. \hfill \Box

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