Holographic Method for Extracting Three-Dimensional Information with a CCD Camera

by

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Synopsis

A method for extracting three-dimensional information from holographic fringe patterns is studied. The holographic fringe patterns recording an object, which is treated as a collection of point light sources, are compared with standard patterns that are also holograms recording a point light source in the target space. We show theoretically that three-dimensional information can be extracted by three matching methods: the sum of products of each pixel, computational correlation using Fourier transformation, and optical correlation. We also simulate some extractions of the object by means of computational correlation and optical correlation. We experimentally confirm the effectiveness of this method applying to the actual objects, which consists of point light sources or line light sources.

Keywords: Holography, Three-dimensional measurement, Optical correlation, Pattern matching, Interferometer

1. Introduction

Progress of three-dimensional (3-D) imaging technology has released 3-D images from the restrictions of a CRT display. Today, a lot of 3-D displaying methods have proposed. Moreover, some interesting pieces of research on holographic 3-D motion picture have become active and are placed our hope as an ideal 3-D display technology in the future. In this situation, progress of 3-D display technology causes needs for 3-D measurement technology. The light-section method and the moire method are used in various fields as an optical measurement method. These methods can get the 3-D information of a target object, but it cannot display the 3-D images from its input. On the other hand, 3-D input method by a stereo camera is also popular. This method can display the 3-D images, but it is required to show the congruent points between the parallax pairs to get their 3-D information, and generally this process takes much time. The input images for 3-D display include 3-D information of the object. Our goal is to extract the 3-D information from the images that are obtained for 3-D display.

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We applied the ultrasonic imaging method\textsuperscript{4) } to the range of optics. It is based on an assumption that the object consists of a collection of point light sources, and on the fact that the phase information of a point light source depends on the spatial relationship between the point light source and an observational plane. Preparing the phase information for all spatial combination as standard patterns in advance, pattern matching processes extract the positional information of the object.

Yabe et al.\textsuperscript{5) } reported the matching simulation by specially designed hardware engines. In this paper, in addition to the simple computational matching method, two matching methods are studied: correlation matching by Fourier transformation and optical correlation matching, and then, we confirm our method by extracting actual objects.

2. Principle

2.1 Extraction of the object consists of point light sources

In Fig.1, interfering the reflected beam from the object in the target space with a reference beam at the CCD camera, an interference fringe pattern (hologram), which contains 3-D information of the object, is formed.

A holographic fringe pattern of a point light source depends on its spatial relationship as shown in Fig.2. So we divide the target space into three-dimensional voxel array, and examine the existence of a point object at every voxel.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{Optical layout for extracting three-dimensional information}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2.png}
\caption{Phase information from point objects}
\end{figure}
A complex amplitude of the object beam reflected by a point object, which corresponds to a point light source, is expressed as

\[ O_{(m,n)} = U_{(m,n)} \exp[j\delta_{(m,n)}] \]  

where \((m,n)\) is the coordinates of CCD's picture elements, \(U_{(m,n)}\) and \(\delta_{(m,n)}\) are amplitude and a phase at a picture element, Taking that \(A\) is amplitude of a reflected beam at a point object, \(r_{(m,n)}\) is a distance between the point object and a picture element at \((m,n)\), and \(\lambda\) is the wavelength of the laser beam, \(U_{(m,n)}\) and \(\delta_{(m,n)}\) are expressed as \(U_{(m,n)} = A/r_{(m,n)}\) and \(\delta_{(m,n)} = 2\pi r_{(m,n)}/\lambda\).

Using a parallel reference beam, its complex amplitude is expressed as \(R_{(m,n)} = R_{(m,n)} \exp[j\alpha_{(m,n)}]\). Then, the light intensity distribution on CCD is given by

\[ S_{(m,n)} = |O_{(m,n)} + R_{(m,n)}|^2 - |O_{(m,n)}|^2 - |R_{(m,n)}|^2 = 2U_{(m,n)}R \cos(\alpha_{(m,n)} - \delta_{(m,n)}) \]  

In eq.(2), as the light intensity of the object and the reference beam has been excluded, only the 3-D information of the object is extracted. We use this \(S_{(m,n)}\) as a standard pattern for extracting 3-D information.

Since \(S_{(m,n)}\) contains the 3-D information only for one point object, we have to prepare all \(S_{(m,n)}\) pattern for every point object in other voxels before applying to the object consists of plural points. Then matching a hologram of the target object, which is captured by CCD, with standard patterns, we can extract 3-D information of the object.

If we assume that an object surface consists of \(N\) point objects and we expand \(U\) and \(\delta\) in eq.(1) to \(U_i\) and \(\delta_i\), amplitude of an object beam is given as \(\Sigma U_i\) by integral theorem of Kirchhoff. With subtracting the object and reference beam distribution in the same way of eq.(2), we obtain eq.(3):

\[ G_{(m,n)} = |O_{(m,n)} + R_{(m,n)}|^2 - |O_{(m,n)}|^2 - |R_{(m,n)}|^2 = 2R \sum_{i=1}^{N} U_{(m,n)} \cos(\alpha_{(m,n)} - \delta_{(i,m,n)}) \]  

This \(G_{(m,n)}\) corresponds to the light intensity of a target hologram. Substituting eq.(2) into eq.(3), we obtain:

\[ G_{(m,n)} = \sum_{i=1}^{N} S_{(i,m,n)} \]  

We use eq.(4) to make holograms instead of experimental data for following simulations. We assume \(U_{(m,n)}\) constant because of \(r^2_{(m,n)} >> m^2 + n^2\).
2.2 Matching operation by sum of products

To extract 3-D positional information, we use a pattern matching method between a hologram of a target object $G(m, n)$ and a set of standard patterns calculated beforehand. As a pattern matching method, here we use the sum of products operation by a computer. First, we calculate standard patterns for each point object in every voxels according to eq. (2), and then, carry out sum of products operation between every standard pattern $S(i, m, n)$ and the target hologram $G(m, n)$. This sum of products operation is shown as

$$P_j = G(m, n) \oplus S(j, m, n) = \bigoplus_{i=1}^{n} S(i, m, n) \Theta S(j, m, n)$$

where the operator $\oplus$ means the sum of products operation. Defining the partial space occupied by target objects as $O$ and the whole target space as $V$, $i$ and $j$ are shown as $i \in O$ $j \in V$. When $j$ satisfies, $j \in O$, $P_j$ becomes a matching procedure of the same pattern, and it shows higher value compared with itself at $j \notin O$. Hence we obtain the 3-D information of the object by examining $P_j$.

However, it doesn't only waste enormous memory to hold all standard patterns for every voxels, but also waste time to calculate them even if they are processed by a high speed engineering work station (EWS). So the special hardware engine is proposed, but it doesn't have enough computation power for this purpose. Since realization of optical computer system will most likely to solve this quantitative problem, we will also discuss about an optical method: correlation by Fourier transformation.

2.3 Standard pattern enlargement

Standard patterns of two point objects whose $z$ coordinates share the same fringe pattern (Fig. 3), and the position of their fringe pattern indicates the $x-y$ coordinates of each point object. For example, the standard pattern that the coordinates of the point object is $(x_1, y_1, 0)$ is correspond to that of point object at $(0,0,0)$ which is moved by $(x_1, y_1)$. Then if all standard patterns which point objects are on the same $z$ coordinates are united, memory size for standard patterns is extremely reduced. A number of standard patterns for a certain $z$ coordinates are substituted with one large standard pattern. Size of the new pattern depends on the size of the target space and the CCD sensing area, but when we assume both sizes are close, it is 4 times as large as that of $x-y$ plane of the target space.
2.4 Matching operation by Fourier Transformation

Using united standard patterns, following two-dimensional correlation can be used for matching with the target hologram $G(x, y)$. Taking the target hologram as $g(x, y)$ and a united standard pattern as $S(x, y)$, cross correlation of them is expressed as

$$\phi_{gs}(x', y') = \iint g(x, y) \cdot s^*(x-x', y-y') dx dy.$$  \hspace{1cm} (6)

We can extract 3-D information of the object by use the correlation with each united standard pattern of every $z$ coordinates.

Some methods for operating correlation are available, here we consider two methods of them: correlation by fast Fourier transformation (FFT) and optical correlation utilized Fourier transform function of lenses.

Taking the Fourier transformed patterns of an target hologram $g(x, y)$ and a united standard pattern $S(z, y)$ as $G(u, v)$ and $S(u, v)$, the cross correlation is shown as

$$\phi_R(x, y) = F^{-1}\left\{G(u, v)S^*(u, v)\right\}. \hspace{1cm} (7)$$

Eq.(7) is used for the computational correlation.

3. Simulation

3.1 Optical layout and Nyquist's sampling theorem

For a practical experiment, resolutions of CCD's picture elements and frame memory are required to satisfy Nyquist's sampling theorem against the spatial frequency of $G$. Table.1 shows specification of the CCD and the frame memory. For reasons of our computer system, we use 512 x 480 (pixels) of a captured image with 8 bit depth.

A hologram of a point light source is called Fresnel zone plate (FZP), and its frequency $N$ and focal length $f$ are expressed as $N=r/r_0^2$, $f=r/r_0^2/2\lambda$ where $r$ is the radius of a fringe ring, $r_0$ is the radius of the first order bright fringe ring. The focal length of the FZP corresponds to a distance between the point light source and CCD. When the size of $x-y$ plane of the target space is same as the size of CCD's picture elements, radius of the maximum fringe ring becomes 8.8 mm. As the resolution of frame memory restricts the resolution of the system, the pixel interval of frame memory $x_p$ have to satisfy following expression:

$$f > \frac{x_p}{\lambda} = \frac{0.0172 \times 8.8}{0.000633} = 239 \text{ mm}. \hspace{1cm} (8)$$
Actually, higher resolution is required in diagonal direction of CCD, then we adopt the optical layout specified in Table 2.

**Table 1. Specification of CCD camera and frame memory**

<table>
<thead>
<tr>
<th>Picture elements</th>
<th>768 (H) x 493 (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensing area</td>
<td>8.8 x 6.6 (2/3 inch)</td>
</tr>
<tr>
<td>Cell size</td>
<td>11 (H) x 13 (V) μm</td>
</tr>
<tr>
<td>Frame memory</td>
<td>512 x 512 pixels</td>
</tr>
</tbody>
</table>

**Table 2. Arrangement of optical layout**

<table>
<thead>
<tr>
<th>Half mirror - Target area</th>
<th>300 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Half mirror - CCD camera</td>
<td>300 mm</td>
</tr>
<tr>
<td>Standard pattern size</td>
<td>17.2 (x) x 12.9 (y) μm</td>
</tr>
<tr>
<td>Target area</td>
<td>8.8 (x) x 6.2 (y) mm</td>
</tr>
</tbody>
</table>

On the other hand, a spatial resolution of point objects reconstructed by a recorded hologram is expressed as \( \Delta \theta = \frac{2A}{D} \), where \( D \) is the size of the hologram. Then point objects which form the target object have to be apart each other at least \( f \Delta \theta \). From Table 2, minimum interval of point objects is 0.12 mm.

### 3.2 Simulation 1: Correlation by Fast Fourier Transformation

In this simulation, we prepare the united standard pattern at \( z=0 \). The resolution of target object is set to be the same as the resolution of CCD, and standard patterns for 512 x 480 = 245760 point objects are united into one.

As a target object, we assume the one that consists of 3 point objects, which are set at \((3,0,0)\), \((0,0,0)\), and \((-3,0,0)\), and its interference fringe pattern is calculated according to eq.(4). (Fig.4)

Here, this target hologram and the united standard pattern are processed by cross correlation. The correlation output is shown in Fig.5. The output image size is 512 x 480 pixels, and the center of the image corresponds to the origin of the coordinate axis. The coordinates where a spike is exist indicates the coordinates of a point object. Fig.6 is the part of Fig.5 (at \( y=0 \)). This image is normalized their intensity range within \([0,1] \) by 256 steps. It shows sharp spikes at the points the target point objects have set.

![Fig. 5. Extraction of three point objects (correlation)](image)

![Fig. 6. Extraction simulation of three point objects (correlation)](image)
3.3 Simulation 2: Optical correlation

In the case of optical correlation, the phase distribution given by eq.(2) cannot utilize as a standard pattern, because of it's negative values. Since the input image to the optical operation must have no negative values, we add an offset value to make all values positive. With the same condition as the FFT simulation, the optical correlation is performed (Fig.7, Fig.8).

![Fig. 7. Extraction simulation of three point objects (optical correlation)](image1)

![Fig. 8. Extraction simulation of three point objects (optical correlation)](image2)

We can easily recognize three spikes in Fig.8, but its $S/N$ ratio is less than that of Fig.6. One reason is the offset value addition to the standard pattern. However, it is supposed to be easily overcomed by high pass filtering since the main ingredients of noise are low frequencies.

4. Extraction by optical correlation system

4.1 Optical setting

We use the modified Rayleigh's interferometer as an optical correlation system (Fig.9). Since the optical correlations are performed by utilized Fourier transform function of lenses, exchanging the united standard patterns for every $z$ coordinates, we can obtain the 3-D information of the object one after another.

At first, it is necessary to prepare Fourier transform holograms (FTH) of united standard patterns in advance. In Fig.9, the united standard pattern is set at $P_1$ as the origin corresponds to the intersection point with the optic axis in $P_1$ plane. A gap of the reference beam from the origin at $P_1$ is realized by parallel movement of the half mirror between $P_1$ and $L_1$. Then at $P_2$, the Fourier transform plane of $P_1$, a FTH of the united standard pattern is formed by the reference point light source B.
Fig. 9. Optical correlation system

Fig. 10. Extraction of three point objects (optical correlation)

This FTH is over exposed in its central area, which corresponds to low frequency area, because of the offset bias.

After developing the hologram, setting it at P₂ precisely and a target hologram at P₁, which captured by CCD, and the diffracted beam by the target hologram illuminates the FTH at P₂. The result of this correlation is observed at P₃.

4.2 Simulation: Extraction by optical correlation system

The object that consists of 3 point objects is used here again (Fig.4). Its interference fringe pattern is calculated according to eq.(4) and recorded on a film as a target hologram.

The target hologram is placed at P₁ and the CCD is set so that the output position corresponds to the center of the CCD. Fig.10 shows the output of this simulation.

Compare with the result of previous simulation, peaks of the image have certain width about 50 pixels (0.65 mm), however, each of them can be easily recognized as a point. It is most likely caused by the error of optical system.

Fig.11 is a part of Fig.10 (at y=y₀) where there is a peak value of the intensity. It is normalized their intensity range within [0,1] by 256 steps to make its peak area clear. It seems that less influence has suffered by the offset bias. It is probably caused by the over exposure of the FTH of the united standard pattern. The over exposed area, which corresponds to the low frequency area, cannot reconstruct the wavefront sufficiently. According to its effect, the influence by the offset bias is canceled. In brief, it performs likes a kind of high pass filter, and improves \( S/N \) ratio of output images.
4.3 Extraction of real target

We use the target object in Fig.12(a). It consists of 9 point objects lined along the shape of character 'V'. Each interval between the neighboring points is 0.75 mm and the size of whole object is about 4x4 mm.

Fig.13 shows the result of the computer simulation in the same manner of 4.2. The results are well agree with the input pattern.

Fig.14 shows the output of the optical correlation system. Nine peaks can be observed. Since each point objects actually are not ideal points and have certain size, the output spikes also have widen to a certain degree.

Then we change the object to the one consists of line objects (Fig.12(b)). Its size is 4x4 mm and the line width is 0.5 mm. Fig.15 and Fig.16 are results of the computer simulation and the optical correlation. They have partially lacked, but the shape of 'V' can be easily observed.

![Fig. 11. Extraction of three point objects (optical correlation)](image1)

![Fig. 12. Target object (a) Point objects (b) Line objects](image2)

![Fig. 13. Extraction of nine point objects (correlation)](image3)

![Fig. 14. Extraction of nine point objects (optical correlation)](image4)
5. Conclusion

We proposed a holographic method for extracting 3-D information. Preparing the Fresnel zone plates as a standard pattern for each voxels recording the hologram of a target object, 3-D information is extracted by the pattern matching operation between them. Noticing the similarity of standard patterns, number of the standard patterns is respectively reduced to one at each z coordinates.

As the pattern matching technique, sum of products, correlation by FFT, and optical correlation are examined, and it is confirmed this method can extract 3-D information. The sum of products method is easy to realize, but it takes enormous time to process. The FFT correlation method is faster than that of sum of products, but it isn’t sufficiently. The optical correlation method requires the special optical system for matching process, but it have finished the matching process at a minute. Furthermore, even if the resolution of a standard pattern is increased to improve accuracy, it doesn’t influence to matching response.

When a holographic television will be practical in the future, this method will be advantageous. Input images for the holographic television can be used as the images for 3-D information extraction. Furthermore, Since the images are converted to the 3-D information, they are easily transmitted. It means a kind of the data compression.

References


