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Osaka Metropolitan University
MEXT Joint Usage/Research Center on Mathematics and Theoretical Physics

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The 21th International Conference on the Teaching of Mathematical Modelling and Applications

Organized by

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Akihiko Saeki
Keiichi Nishimura
Akio Matsuzaki
Takashi Kawakami

September 10–15, 2023

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Organizers

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Kanazawa Institute of Technology

Keiichi Nishimura
Tokyo Gakugei University

Akio Matsuzaki
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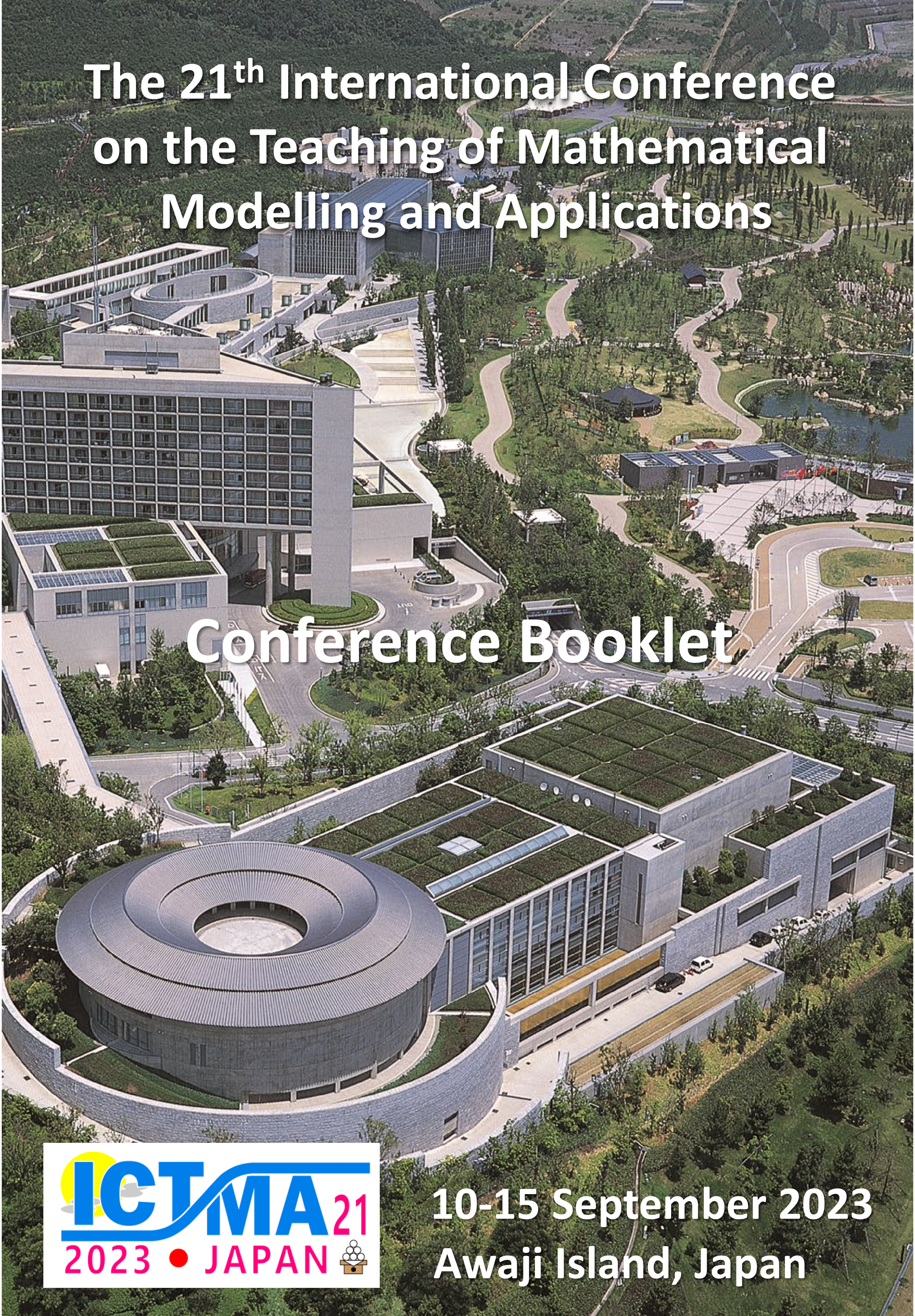
Takashi Kawakami
Utsunomiya University

The 21th International Conference on the Teaching of Mathematical Modelling and Applications



Conference Booklet

ICTMA 21
2023 • JAPAN

10-15 September 2023
Awaji Island, Japan

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Welcome to ICTMA21

Dear esteemed participants,

On behalf of ICTMA21, I would like to express my heartfelt gratitude for joining us at this prestigious international conference. We are thrilled to welcome you to our beautiful country, Japan.

This conference marks a significant gathering of individuals from diverse cultures and backgrounds, all united to share knowledge, exchange ideas, and forge new friendships. Japan, with its rich cultural heritage and captivating traditions, offers a unique backdrop for this intellectual exchange.

Renowned for its innovative technologies, cutting-edge research, arts, and culinary delights, Japan provides an inspiring setting that we hope you will fully embrace during your stay. As you immerse yourselves in the conference activities, we encourage you to also cherish encounters with the fascinating aspects of Japanese culture.

In tracing the historical journey of mathematics in the real world, we observe a shift from a strong emphasis on mathematization before World War II to a focus on pure mathematics in the 1960s. However, the 1990s saw a resurgence of interest in modelling and applications, aligning with international trends. Notably, Lesson Study, a teaching improvement process originating from Japanese elementary education, has garnered international recognition since the 2000s. Emphasizing collaboration as one of its key focuses, Lesson Study aligns well with our chosen conference theme of "Collaboration in mathematical modelling education." Through this theme, we underscore the importance of collaboration with students, teachers, researchers, and industry professionals.

Our goal is for this conference to provide all participants with valuable insights into the development of teaching mathematical modelling and applications, while also offering a unique opportunity to experience the warm culture and hospitality of Japan. We eagerly anticipate sharing inspiring and fruitful moments with each one of you. If you require any assistance during your stay, please do not hesitate to reach out.

With my best wishes,

Toshikazu Ikeda
Conference Chair
Local Organising Committee of ICTMA21

Toshikazu Ikeda

Local Organizing Committee

Conference Chair

Toshikazu Ikeda
Yokohama National University

Vice Conference Chair

Akihiko Saeki
Kanazawa Institute of Technology

Keiichi Nishimura
Tokyo Gakugei University

Secretariats General

Akio Matsuzaki
Saitama University

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Masafumi Kaneko
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Kyoto University of Education

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Saga University

Kazuhiro Kurihara
Tokiwa University

Hayato Hanazono
Miyagi University of Education

Yuichiro Hattori
Okayama University

Yoshiki Nisawa
Bukkyo University

Kosuke Mineno
Toin Yokohama University

ICTMA21 Schedule

	Sunday September 10th	Monday September 11th	Tuesday September 12th	Wednesday September 13th	Thursday September 14th	Friday September 15th
8:45		Registration 8:45-10:00				
9:30			Plenary lecture1 (Greefrath) 9:30-10:30	Plenary lecture2 (Czocher) 9:30-10:30	Session6 9:30-11:00	Plenary panel 9:30-11:00
10:00		Opening Ceremony 10:00-10:30				
10:30		Coffee break 10:30-11:00		Excursion (Option) 10:30- Lunch (Take away lunch boxes)		
10:45			Session3 10:45-12:15			
11:00		Keynote speech (Goos & Carreira) 11:00-12:30				
11:15					Session7 11:15-12:45	Pollak Award ceremony & Closing ceremony 11:15-12:00
12:00						
12:15			Lunch 12:15-13:30			
12:30		Lunch 12:30-13:45				
12:45					Lunch 12:45-13:45	
13:00	Registration 13:00-17:00; Early Career Research Day 13:00-16:30					
13:30			Session4 13:30-15:00			
13:45		Session1 13:45-15:15			Session8 13:45-15:15	
15:00	Executive Board Meeting 15:00-17:00		Coffee break 15:00-15:30			
15:15		Coffee break 15:15-15:45			Coffee break 15:15-15:45	
15:30			Session5 15:30-17:00			
15:45		Session2 15:45-17:15			Plenary lecture3 (Shimizu) 15:45-16:45	
16:30						
16:45					General meeting 16:45-17:45	
17:00	Welcome reception 17:00-		Poster session & Happy hour 17:00-18:30			
17:15		Special Lecture (Galluzzo & Cheung) 17:30-18:30				
17:45						
18:00					Conference dinner 18:00-	
18:30		Happy hour 18:30-19:30				
19:00						
19:30						
20:00						

Keynote, Plenary, Panel, & Lectures

Keynote (Joint Presentation)

CONCEPTUALISING THE RELATIONSHIP BETWEEN MATHEMATICAL MODELLING AND INTERDISCIPLINARY STEM EDUCATION

Merrilyn Goos
University of the Sunshine Coast, Australia

Susana Carreira
University of the Algarve and Institute of Education, University of Lisbon, Portugal

Plenary Lecture 1

TEACHER EDUCATION AND MATHEMATICAL MODELLING: PRE-SERVICE TEACHERS’ PROFESSIONAL COMPETENCE FOR THE TEACHING OF MATHEMATICAL MODELLING

Gilbert Greefrath
University of Münster, Germany

Plenary Lecture 2

IN THEIR OWN WORDS:
EXPLANATIONS OF STEM STUDENTS' REASONING DURING MATHEMATICAL MODELLING

Jennifer A. Czocher
Texas State University, USA

Plenary Lecture 3

LESSON STUDY AND ITS RELATIONS TO MATHEMATICAL MODELLING

Yoshinori Shimizu
University of Tsukuba, Japan

Plenary Panel

RELATIONS AMONG PROBLEM SOLVING/POSING, CREATIVITY AND MATHEMATICAL MODELLING

Chair: Jonas Bergman Ärlebäck
Linköping University, Sweden

Jinfa Cai
University of Delaware, USA

Gabriele Kaiser
University of Hamburg, Germany & Nord University, Norway

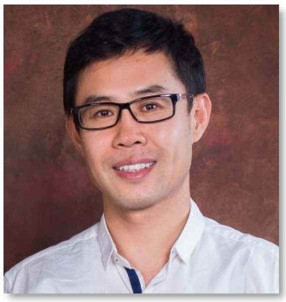
Roza Leikin
University of Haifa, Israel

Special Lecture

IMMC: CELEBRATING 10 YEARS OF INFLUENCING EDUCATIONAL CHANGE

Benjamin Galluzzo
Clarkson University, USA

Alfred Cheung
NeoUnion ESC Organization, Hong Kong, China



Early Career Researcher Day

EARLY CAREER RESEARCH DAY: REFLECTIONS ON PUBLISHING ON MODELLING RESEARCH

Gabriele Kaiser
University of Hamburg, Germany & Nord University, Norway

Şerife Sevinç
Middle East Technical University, Turkey

BEGINNING WALKING IN TWO WORLDS AS A TEACHER AND RESEARCHER OF MATHEMATICAL MODELLING IN THE CLASSROOM

Gloria Stillman
Retired, Australia

Scientific Programme

Conference booklet

Participants can download conference booklet (including abstracts) from the Google Drive:
https://drive.google.com/drive/folders/10OIq0kB3VjdI3JO5Q4DUz3hrDfTcCrN4?usp=drive_link

Early career research day (Sunday, 10 September)

Open only to those who have pre-registered and paid the ECRD registration fee. ECRD participants can download materials from the Google Drive. The Google Drive URL will be sent to the ECRD participants by email. If the URL is unsure, please contact Takashi Kawakami (editor@ictma21.jp) in the LOC.

Keynote speech (Monday, 11 September)

Sessions of 90 minutes including questions and discussions.

Plenary lectures 1-3 (Tuesday-Thursday, 12-14 September)

- Sessions of 60 minutes including questions and discussions.
- Plenary lectures 1 and 2 start at 9:30 in the morning; Plenary lecture 3 starts at 15:45 in the afternoon.

Plenary panel (Friday, 15 September)

Sessions of 90 minutes including questions and discussions.

Special lecture (Monday, 11 September)

Sessions of 60 minutes including questions and discussions.

Presentations (Monday, Tuesday, and Thursday, 11, 12, and 14 September)

- *Long presentation (total 45 minutes):* Presentation for up to 25 minutes allowing remaining time (up to 20 minutes) for questions and discussions
- *Short presentation (total 30 minutes):* Presentation for up to 20 minutes allowing remaining time (up to 10 minutes) for questions and discussions
- *Poster presentation (total 90 minutes):* The event will take place on Tuesday, 12 September, from 17:00 to 18:30 at the B1F lobby. Poster presenters are invited to stand by their posters during this time to explain and answer questions. Posters must be placed in their own location prior to the start of the

event and collected by participants themselves after the event.

General meeting (Thursday, 14 September)

A General Meeting of the Community will be held on Thursday afternoon, September 14th, from 16:45 to 17:45. The primary agenda of the General Meeting will be to elect eligible members to the IEC (International Executive Committee) and to discuss matters pertaining to the mission of the community. All conference-registered members are welcome to participate in the General Meeting.

Some Information

Making a presentation

- Please note that we do not provide laptops for presentations. Please provide a computer for your presentation. There will of course be projectors in each room. All projectors will have HDMI connections. However, if you need a different type of connection on your computer, please bring an HDMI adapter. LOC cannot provide adapters.
- Microphones and pin microphones are available at the venue and may be used as required.
- The chair of your session has been asked to keep time strictly within these limits to ensure smooth running of the conference. Please ensure that you keep to these times, particularly as there is only a short change-over time between sessions.
- Someone from the LOC and volunteer student staff will be on hand to assist if there are any problems with the presentation equipment.

Chairing sessions

- The chairperson for each session is specified in the programme. The role of the session chair is very important. The conference relies on the chair handling the session that they have been asked to chair with sensitivity and firmness.
- Chairs are asked to take turns within the same session (in the spirit of ICTMA21's theme of "collaboration"!). As a general rule, session chairs should rotate as follows:
  - *If there are two presentations in one session*
    When the 1st speaker presents, the 2nd speaker acts as a session chair.
    When the 2nd speaker presents, the 1st speaker acts as a session chair.

- *If there are three presentations in one session*
  When the 1st speaker and 2nd speaker present, the 3rd speaker acts as a session chair.
  When the 3rd speaker presents, the 1st speaker acts as a session chair.

- Please allow a full 25 minutes or 20 minutes for the presentation and the remaining time (up to 20 minutes or 10 minutes) for questions and discussion. Cards will be available in the room for you to indicate that 5, 2 and 1 minutes remain for the presentation. Please show these clearly from the audience so that the presenter is aware of the need to complete their presentation to time.
- During the ensuing questions and discussion, make sure that everyone who wants to contribute is able to do so making sure that no one person dominates the discussion.

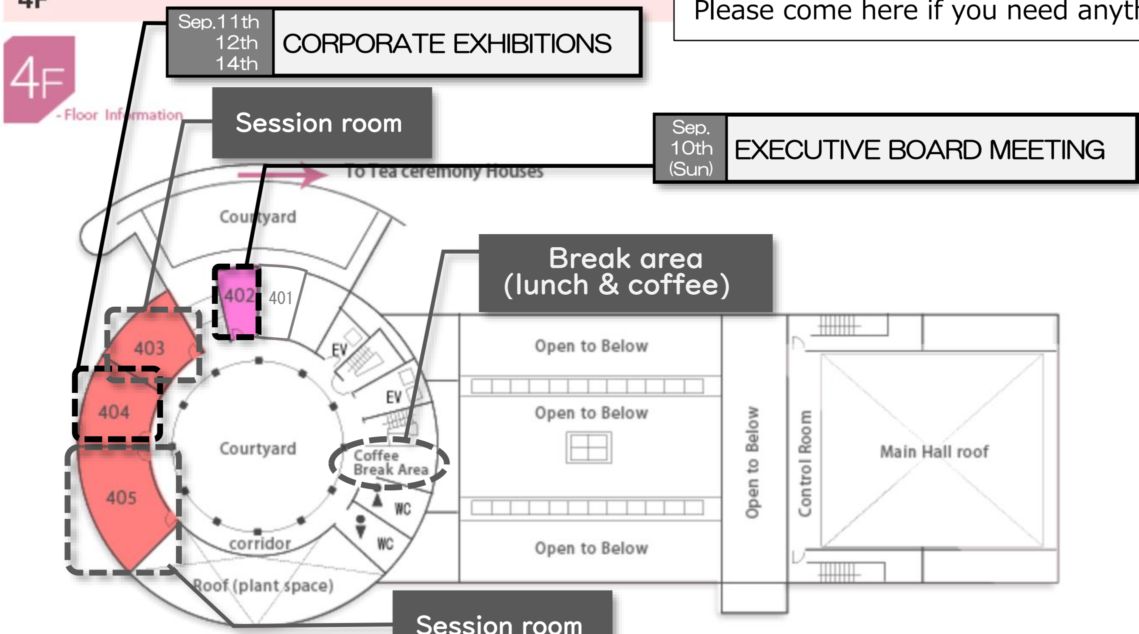
Publication

Regarding publication, there will be no proceedings of the conference in a strict sense. However, scholars, who have presented a paper at ICTMA21, are invited to submit a chapter for consideration in an edited volume after the conference devoted to the theme of the conference. Detailed instructions for authors are available on the ICTMA21 website (https://ictma21.jp/submission), and the deadline for contributions will be 15th November 2023. Notably, the book chapters go through a rigorous review process conducted by international expert referees coming from the ICTMA community. Editors of the book will be the conference organizers together with the ICTMA president and the permanent editor of ICTMA. The ICTMA series books are published under the Springer Series, titled "International Perspectives on the Teaching and Learning of Mathematical Modelling," with Series Editors being Gabriele Kaiser, and Gloria Stillman (https://www.springer.com/series/10093).

Floor Map

Rooms 401 and 402 are the LOC office rooms.
Please come here if you need anything.

4F



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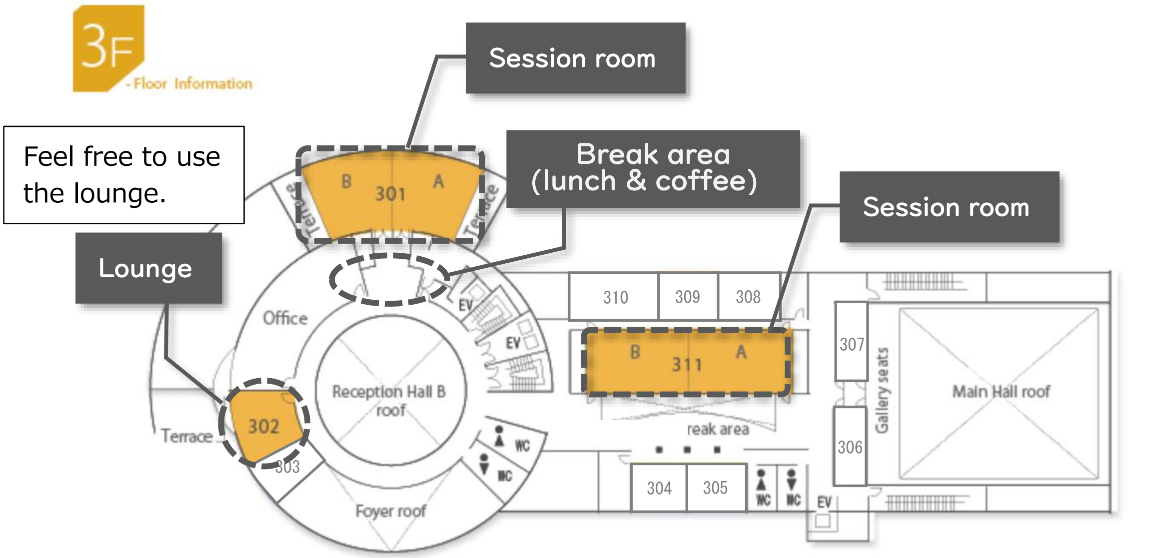

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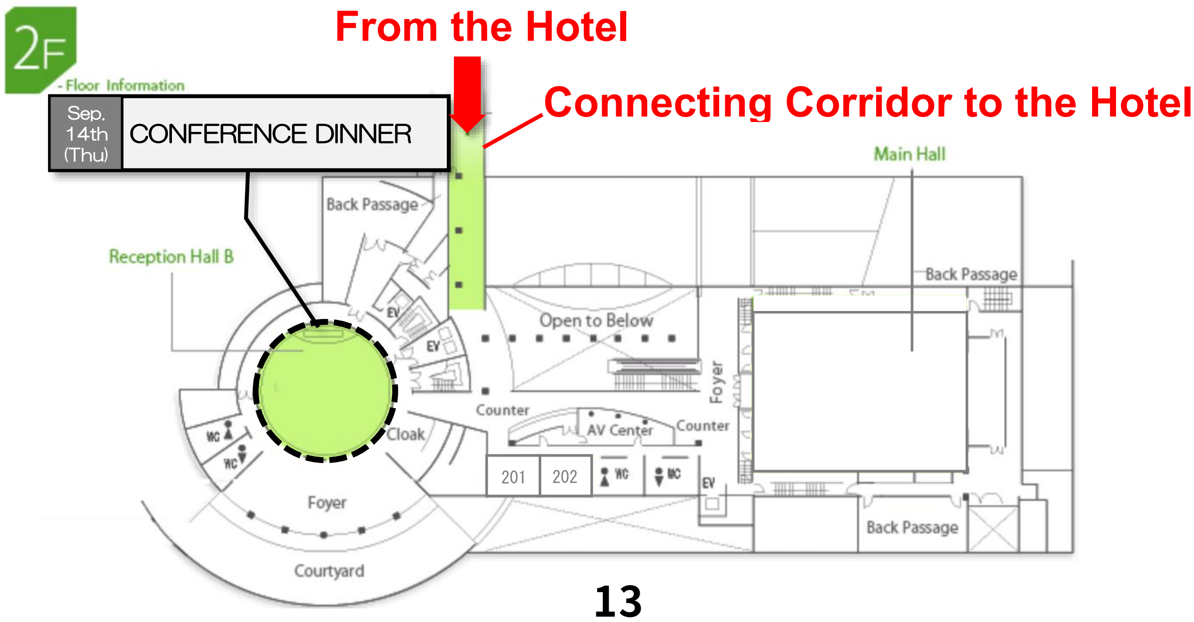

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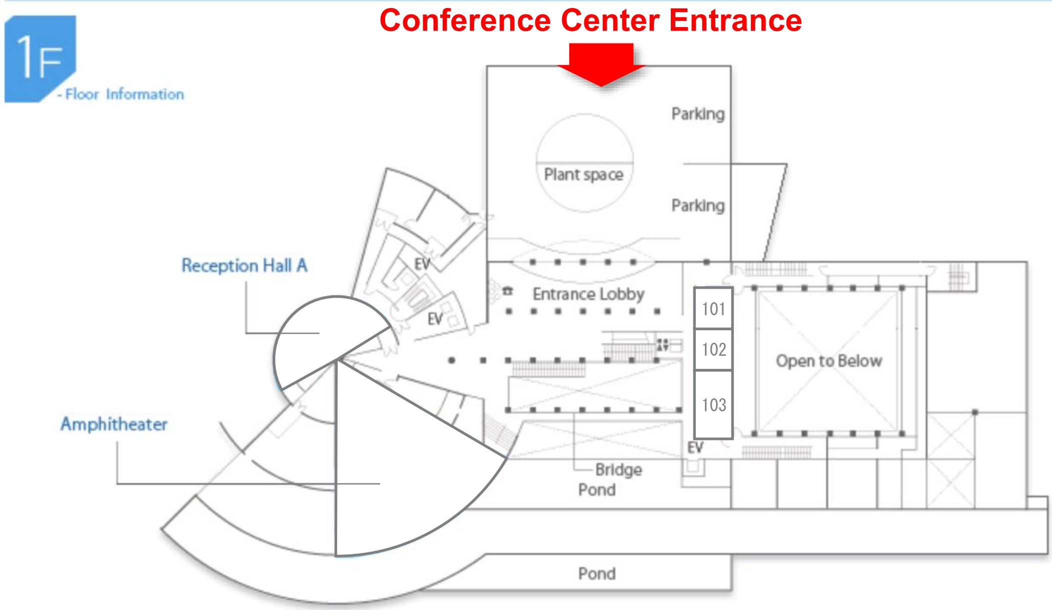
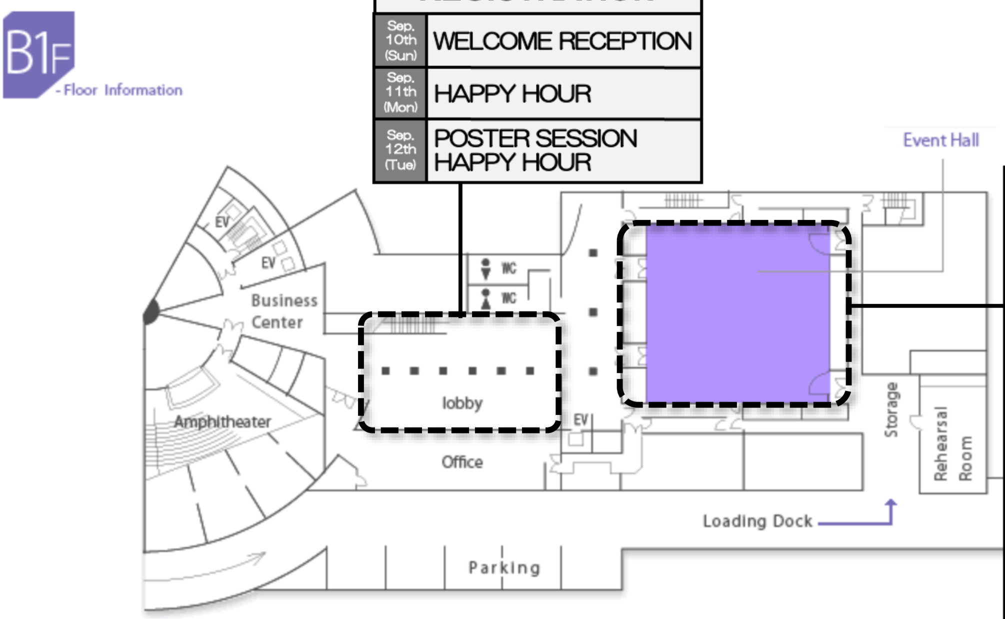
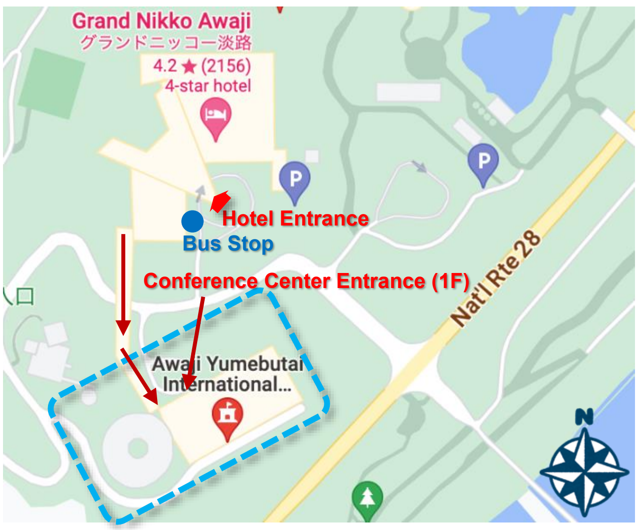


B1F

REGISTRATION	
Sep. 10th (Sun)	WELCOME RECEPTION
Sep. 11th (Mon)	HAPPY HOUR
Sep. 12th (Tue)	POSTER SESSION HAPPY HOUR



Sep. 10th (Sun)	EARLY CAREER RESEACHER DAY
Sep. 11th (Mon)	OPENING CEREMONY KEYNOTE SPEECH SPECIAL LECTURE
12th 13th 14th	PLENARY LECTURE
Sep. 14th (Thu)	GENERAL MEETING
Sep. 15th (Fri)	PLENARY PANEL POLLAK AWARD CEREMONY CLOSING CEREMONY



The hotel entrance is near the bus stop. If you enter via the hotel entrance, you can access the second floor of the conference center via the connecting corridor on the second floor of the hotel.

Programme

Sunday, September 10, 2023	
Time/Location	Activity
13:00–17:00/BF1 Lobby	**REGISTRATION**
13:00–16:30/B1F Event Hall	**EARLY CAREER RESEARCH DAY** E1 EARLY CAREER RESEARCH DAY: REFLECTIONS ON PUBLISHING ON MODELLING RESEARCH **Gabriele Kaiser**, *University of Hamburg, Germany & Nord University, Norway* **Şerife Sevinç**, *Middle East Technical University, Turkey* E2 BEGINNING WALKING IN TWO WORLDS AS A TEACHER AND RESEARCHER OF MATHEMATICAL MODELLING IN THE CLASSROOM **Gloria Stillman**, *Retired, Australia* Chair **Takashi Kawakami**, *Utsunomiya University, Japan*
15:00–17:00/Room402	**EXECUTIVE BOARD MEETING**
17:00–19:00/B1F Lobby	**WELCOME RECEPTION**

Monday, September 11, 2023	
Time/Location	Activity
8:45–10:00/B1F Lobby	**REGISTRATION**
10:00–10:30/B1F Event Hall	**OPENING CEREMONY**
10:30–11:00/B1F Lobby	**COFFEE BREAK (Coffee Break Area on B1F Lobby)**
11:00–12:30/Event Hall	**KEYNOTE SPEECH (JOINT PRESENTATION)** K1 CONCEPTUALISING THE RELATIONSHIP BETWEEN MATHEMATICAL MODELLING AND INTERDISCIPLINARY STEM EDUCATION **Merrilyn Goos**, *University of the Sunshine Coast, Australia* **Susana Carreira**, *University of the Algarve and Institute of Education, University of Lisbon, Portugal* Chair **Milton Rosa**, *Federal University of Ouro Preto, Brazil*
12:30–13:45/3F & 4F	**LUNCH (Take away lunch boxes in Coffee Break Areas on 3F and 4F)**
13:45–15:15 **SESSION 1**	
Room301A (Long/Short: 13:45–15:00) (Teacher Education) Chair Rita Borromeo Ferri Yongjian Sun	OA1 (Long: 45min.) PRE-SERVICE TEACHERS' INTERVENTIONS IN MATHEMATICAL MODELING ACTIVITIES **Yongjian Sun**, *East China Normal University, China* **Jing Cheng**, *East China Normal University, China*
	OA2 (Short: 30min.) A TEST INSTRUMENT FOR PROFESSIONAL MATHEMATICAL KNOWLEDGE ON MATHEMATICAL MODELLING – FIRST RESULTS **Rita Borromeo Ferri**, *University of Kassel, Germany* **Gilbert Greefrath**, *University of Münster, Germany* **Hans-Stefan Siller**, *University of Würzburg, Germany*
Room301B (Long/Short: 13:45–15:00) (Teacher Education) Chair Xinrong Yang Cyril Julie	OA3 (Long: 45min.) SYSTEM DYNAMICS SOFTWARE AND SCENARIOS FOR CONTINUOUS PROFESSIONAL DEVELOPMENT **Cyril Julie**, *University of the Western Cape, South Africa*
	OA4 (Short: 30min.) FACTORS AFFECTING CHINESE IN-SERVICE HIGH SCHOOL MATHEMATICS TEACHERS' IMPLEMENTATION OF MATHEMATICAL MODELLING IN CLASSROOM: AN INTERVIEW STUDY **Xinrong Yang**, *Southwest University, China*

Room311A (Long/Short: 13:45–15:00) (Teacher Education) Chair Yuki Watanabe Lluís Albarracín	OA5 (Long: 45 min.) DEVELOPING LOCAL REPRESENTATIVE SOLUTIONS TO FERMI PROBLEMS Lluís Albarracín , *Universitat Autònoma de Barcelona, Spain* **Jonas Bergman Ärlebäck** , *Linköpings Universitet, Sweden* **Serife Sevinc** , *Orta Doğu Teknik Üniversitesi, Turkey* **Irene Palomares** , *Universitat de València, Spain* **Daniel Orey** , *Universidade Federal de Ouro Preto, Brazil* **Milton Rosa** , *Universidade Federal de Ouro Preto, Brazil*
	OA6 (Short: 30 min.) DEVELOPMENT OF SECONDARY MATHEMATICS INSTRUCTIONAL STRATEGY FOR ACQUIRING CONCEPTUAL KNOWLEDGE Shota Higuchi , *Tokyo University of Science, Japan* **Yuki Watanabe** , *Tokyo University of Science, Japan*
Room311B (Short) (Societal Perspective) Chair Peter Frejd Kosuke Mineno	OA7 DESIGNING MATHEMATICAL MODELLING TASKS INVOLVING A SIMULATION WITH PHYSICAL EXPERIMENTS: ON THE TASK OF MARKER RECUPTURE METHODS Kosuke Mineno , *Toin University of Yokohama, Japan* **Akihiko Saeki** , *Kanazawa Institute of Technology, Japan* **Takashi Kawakami** , *Utsunomiya University, Japan*
	OA8 CAN MATHEMATICAL MODELLING WORK AS A CRITICAL THINKING-DEMANDING ACTIVITY? Liezle Boshoff , *Cape Peninsula University of Technology, South Africa*
	OA9 THE CO-PRODUCTIVITY CHAIN OF MODELLING – ON MODELS USED BY CITIZENS, OPERATORS AND CONSTRUCTORS Peter Frejd , *Linköping University, Sweden* **Pauline Vos** , *University of Agder, Norway*
Room405 (Short: 13:45–14:45) (Collboration & Modelling) Chair Rintaro Ueda Sakon Tangkawsakul	OA11 COLLABORATIVE DESIGN OF MATHEMATICAL MODELLING TASKS FOR MIDDLE SCHOOL STUDENTS Sakon Tangkawsakul , *Kasetsart University, Thailand* **Weerawat Thaikam** , *Nakhonsawan Rajabhat University, Thailand* **Nuttapat Mookda** , *Independent Scholar, Thailand*
	OA12 THE ROLE OF CARE IN MATHEMATICAL SCIENCES EDUCATION INCLUDING STUDENTS' VALUES Rintaro Ueda , *Tokyo Metropolitan Oizumi Sakura High School, Japan* **Keiichi Nishimura** , *Tokyo Gakugei University, Japan*
15:15–15:45/3F & 4F	**COFFEE BREAK (Coffee Break Areas on 3F and 4F)**
15:45–17:15 **SESSION 2**	
Room301A (Long) (STEM & Modelling) Chair Svitlana Rogovchenko Daniel Clark Orey	OA13 POSSIBILITIES OF STUDYING THE GOLDEN RATIO PROPORTION IN THE CHRISTS SCULPTURES OF MASTER ALEIJADINHO IN THE TOWN OF CONGONHAS, MINAS GERAIS, BRAZIL Daniel Clark Orey , *Universidade Federal de Ouro Preto, Brazil* **Milton Rosa** , *Universidade Federal de Ouro Preto, Brazil* **Kelly Cristina Santos Rocha** , *Universidade Federal de Ouro Preto, Brazil*
	OA14 MATHEMATICAL MODEL OF YEAST GROWTH: BIOLOGY STUDENTS USE MATHEMATICS TO UNDERSTAND BIOLOGICAL PROCESSES Svitlana Rogovchenko , *University of Agder, Grimstad, Norway* **Yuriy Rogovchenko** , *University of Agder, Kristiansand, Norway*

Room301B (Long) (Task Design) Chair Ruth Rodriguez-Gallegos Dominik Schlüter	OA15 ASSESSING AUTHENTICITY IN MODELLING TEST ITEMS: DEVELOPING AND PILOTING OF A THEORETICAL MODEL Dominik Schlüter , *Leuphana University Lueneburg, Germany* **Maik Rolfs** , *Leuphana University Lueneburg, Germany* **Michael Besser** , *Leuphana University Lueneburg, Germany*
	OA16 TASKS DESIGN TO ENCOURAGE AND FACILITATE MODELLING SKILLS AND ENGAGEMENT IN MATHEMATICS LEARNING Ruth Rodriguez-Gallegos , *Tecnologico de Monterrey, Mexico* **Rafael Ernesto Bourguet-Díaz** , *Tecnologico de Monterrey, Mexico*
Room311A (Long) (Modelling Empistemology) Chair Jonas Bergman Ärlebäck Bárbara Nivalda Palharini Alvim Sousa	OA17 MATHEMATICAL MODELLING AS A WAY OF SEEING THE WORLD Bárbara Nivalda Palharini Alvim Sousa , *State University of Northern Paraná, Brazil* Emerson Tortola , *Federal University of Technology - Paraná, Brazil*
	OA18 PRE-SERVICE UPPER SECONDARY TEACHERS' SPONTANEOUS VIEWS ON MATHEMATICAL MODELS AND MODELLING Jonas Bergman Ärlebäck , *Department of Mathematics, Linköping University, Sweden*
Room311B (Short) (Secondary/University Education) Chair Kerri Spooner Azita Manouchehri	OA19 TEACHING ACCOMODATIONS: FACILITATING CLASSROOM INTERACTIONS AROUND FERMI PROBLEMS Azita Manouchehri , *The Ohio State University, USA* **Eduan Soto-Martinez** , *The Ohio State University, USA*
	OA20 IMMC-SPAIN: AN OPPORTUNITY TO INTRODUCE MATHEMATICAL MODELLING IN THE CLASSROOMS Irene Ferrando , *Universitat de València, Spain* **Carlos Segura** , *Universitat de València, Spain* **Lluís Albarracín** , *Universtat Autónoma de Barcelona, Spain*
	OA21 STUDENT EXPERIENCES WITH CONNECTING CONTEXT AND MATHEMATICS WHILE MATHEMATICALLY MODELLING Kerri Spooner , *Auckland University of Technology, New Zealand*
Room405 (Short) (Technology) Chair Carolina Guerrero-Ortiz Yukiko Asami-Johansson	OA22 TRAINING PROSPECTIVE TEACHERS' DIDACTICAL PERSPECTIVES OF USING GEOGEBRA FOR MATHEMATICAL MODELLING Yukiko Asami-Johansson , *University of Gävle, Sweden* **Mikael Cronhjort** , *University of Gävle, Sweden* **Mirko Radic** , *University of Gävle, Sweden*
	OA23 WHEN DIGITAL TOOLS BECOME A SUPPORT FOR MODELLING ACTIVITIES: PRE-SERVICE TEACHERS' INSTRUCTIONAL TASKS Carolina Guerrero-Ortiz , *Pontificia Universidad Católica de Valparaíso, Chile*
	OA24 A THEORETICAL MODEL FOR DESCRIBING TECHNOLOGICAL, PEDAGOGICAL, MATHEMATICAL AND EXTRA-MATHEMATICAL KNOWLEDGE FOR TEACHING MODELLING Josefa Castillo-Funes , *Pontificia Universidad Católica de Valparaíso, Chile* **Carolina Guerrero-Ortiz** , *Pontificia Universidad Católica de Valparaíso, Chile*

17:30–18:30/B1F Event Hall	**SPECIAL LECTURE** S1 IMMC: CELEBRATING 10 YEARS OF INFLUENCING EDUCATIONAL CHANGE Benjamin Galluzzo, *Clarkson University, USA* Alfred Cheung, *NeoUnion ESC Organization, Hong Kong, China* Chair **Irene Ferrando**, *Universitat de València, Spain*
18:30–19:30/B1F Lobby	**HAPPY HOUR**
10:30–17:30/Room404	**CORPORATE EXHIBITIONS**

Tuesday, September 12, 2023	
Time/Location	**Activity**
9:30-10:30/B1F Event Hall	PLENARY LECTURE 1 P1 TEACHER EDUCATION AND MATHEMATICAL MODELLING – PRE-SERVICE TEACHERS' PROFESSIONAL COMPETENCE FOR THE TEACHING OF MATHEMATICAL MODELLING Gilbert Greefrath , *University of Münster, Germany* Chair **Jonas Bergman Ärlebäck** , *Linköping University, Sweden*
10:45-12:15 **SESSION 3**	
Room301A (Long) (ICT & Modelling) Chair Mustafa Cevikbas Laura Wirth	OB1 USING AN INSTRUCTIONAL VIDEO TO SUPPORT UPPER SECONDARY STUDENTS IN CREATING A MATHEMATICAL MODEL Laura Wirth , *University of Münster, Germany* **Gilbert Greefrath** , *University of Münster, Germany*
	OB2 FLIPPING UNIVERSITY-BASED MATHEMATICAL MODELLING SEMINARS ? INSIGHTS FROM PRE-SERVICE MATHEMATICS TEACHERS Mustafa Cevikbas , *University of Hamburg, Germany* Gabriele Kaiser , *University of Hamburg, Germany, Nord University, Norway* **Denise Mießeler** , *University of Hamburg, Germany*
Room301B (Long) (Technology & Modelling Pedagogy) Chair Benjamin Galluzzo Diana M. Fisher	OB3 MODELLING UNCERTAIN FUTURES: PROBLEMS, METHODS, TOOLS Peter Galbraith , *University of Queensland, Australia* **Diana Fisher** , *Portland State University, USA*
	OB4 STARTING MATHEMATICAL MODELLING FROM EXISTING MODELS Benjamin Galluzzo , *Clarkson University, USA* **Adewale Adeolu** , *Clarkson University, USA* **Rose Mary Zbiek** , *Pennsylvania State University, USA* **Amy Brass** , *Pennsylvania State University, USA* **Jie Chao** , *Concord Consortium, USA*

Room311A (Long) (Technology) Chair Akio Matsuzaki	OB5 DIFFERENCES IN INTERPRETATION OF COMPUTER RESULTS IN THE MODELLING CYCLE WITH ADDED COMPUTER MODEL: THROUGH THE MODELLING WORKSHOP ON THE PREMISE OF USING SCIENTIFIC CALCULATOR Yuzu Yamamoto , *Graduate School of Education, Saitama University, Japan* **Akio Matsuzaki** , *Saitama University, Japan*
	OB6 CHANGES OF MATHEMATICAL MODELS THAT CAN BE REPRESENTED BY CHANGING THE SLIDER TOOL IN GEOGEBRA: FOCUS ON THE TRAJECTORY OF BASKETBALL IN ONE SCENE OF THE MANGA "THE BASKETBALL WHICH KUROKO PLAYS" Mikino Tachihara , *Graduate School of Education, Saitama University, Japan* **Akio Matsuzaki** , *Saitama University, Japan*
Room311B (Short) (Teacher Education) Chair Emerson Tortola Shigekazu Komeda	OB7 IMPROVEMENT TO THE MATHEMATICAL MODELLING LESSON BY INEXPERIENCED SECONDARY SCHOOL MATHEMATICS TEACHER Shigekazu Komeda , *Saga University, Japan* **Shogo Ohbayashi** , *Saga University, Japan*
	OB8 MATHEMATICAL WORK IN A MODELLING TASK: CIRCULATIONS AND TYPES OF MODELS Jaime Huincahue , *Universidad Católica del Maule,Chile* **Paula Verdugo-Hernández** , *Universidad de Talca, Chile* **Carolina Henriquez-Rivas** , *Universidad Católica del Maule,Chile* **Gonzalo Espinoza-Vásquez** , *Universidad Alberto Hurtado, Chile*
	OB9 IMPLICATIONS OF LEARNING THROUGH FOR TEACHING USING MATHEMATICAL MODELLING Emerson Tortola , *Federal University of Technology - Parana, Brazil* **Jader Otavio Dalto** , *Federal University of Technology - Parana, Brazil* **Karina Alessandra Pessoa da Silva** , *Federal University of Technology - Parana, Brazil*
Room403 (Short: 10:45-11:45) (STEM & Modelling) Chair Yoshitaka Abe Osamu Inomoto	OB10 TRANSIENT PHENOMENA AS MATHEMATICAL MODELLING MATERIALS IN SCIENCE EDUCATION Osamu Inomoto , *Center for Fundamental Education, Kyushu Sangyo University, Japan*
	OB11 AN ANALYSIS OF THE THEORETICAL FRAMEWORK FOR NUMERICAL COGNITION IN ZAMBIA Yoshitaka Abe , *Hiroshima University Graduate School of Humanities and Social Sciences, Japan*

Room405 (Short) (Problem Posing & Modelling) Chair Songchai Ugsonkid Serife Sevinc	OB12 CHARACTERISTICS OF THE MODELING PROBLEMS POSED BY PROSPECTIVE MATHEMATICS TEACHERS Serife Sevinc , *Middle East Technical University, Turkey*
	OB13 FACILITATING PROBLEM POSING IN MATHEMATICAL MODELLING: THE CASE OF 12TH-GRADE STUDENTS Kosuke Yata, *Joto High School, Japan* **Akihiko Saeki** , *Kanazawa Institute of Technology, Japan* **Naohiro Minagawa** , *Naruto University of Education, Japan* **Takashi Kawakami** , *Utsunomiya University, Japan*
	OB14 ENHANCING PRESERVICE TEACHERS REAL-WORLD MATHEMATICAL PROBLEM-POSING ABILITIES: DESIGNING LEARNING TRAJECTORY Songchai Ugsonkid , Kasetsart University, Thailand **Sakon Tangkawsakul** , Kasetsart University, Thailand **Weerawat Thaikam** , Nakhonsawan Rajabhat University, Thailand
12:15-13:30/3F & 4F	**LUNCH (Take away lunch boxes in Coffee Break Areas on 3F and 4F)**
13:30-15:00 **SESSION 4**	
Room301A (Long) (Psychological Perspective) Chair Stanislaw Schukajlow Lisa-Marie Wienecke	OB15 THE RELATIONSHIP BETWEEN TEXT COMPREHENSION AND NOTE-TAKING WHILE WORKING ON REALITY-BASED TASKS Lisa-Marie Wienecke , *Leuphana University Lueneburg, Germany* **Dominik Leiss** , *Leuphana University Lueneburg, Germany*
	OB16 STUDENTS' PREFERENCES AND BELIEFS ABOUT THE OPENNESS OF MODELLING PROBLEMS Stanislaw Schukajlow , *University of Münster, Germany* **Janina Krawitz** , *University of Münster, Germany* **Katharina Wiehe** , *University of Gießen, Germany* **Katrin Rakoczy** , *University of Gießen, Germany*
Room301B (Long) (Task Design) Chair Shaliny Kannapiran Sevinç Göksen-Zayim	OB17 DESIGNING COLLABORATIVE MATHEMATICAL MODELLING TASKS FOR LOWER SECONDARY EDUCATION Sevinç Göksen-Zayim , *University of Amsterdam, The Netherlands* **Derk Pik**, *University of Amsterdam, The Netherlands* **Carla van Boxtel** , *University of Amsterdam, The Netherlands*
	OB18 COMPARISON OF MATHEMATICAL MODELLING IN SOLVING AWNING WINDOW PROBLEMS: MALAYSIAN CIVIL EXAMINATION PROBLEM VS OLD JAPANESE TEXTBOOK PROBLEM Shaliny Kannapiran , *Sekolah Kebangsaan Seri Tasik Kuala Lumpur, Malaysia* **Akio Matsuzaki** , *Saitama University, Japan*

Room311A (Long) (Secondary Education) Chair Rina Durandt Jill P Brown	OB19 IDENTIFYING AND TRACKING STUDENT CONCEPTIONS OF MATHEMATICS Jill P Brown, *Deakin University, Australia* Gloria A Stillman, *Retired, Australia*
	OB20 A COMPARATIVE CASE STUDY ON TEACHING MODELLING AT THE SECONDARY LEVEL WITH A UNIT DEVELOPED FOR THE TERTIARY LEVEL Rina Durandt, *University of Johannesburg, South Africa* Werner Blum, *University of Kassel, Germany* Alfred Lindl, *University of Regensburg, Germany* Rita Borromeo Ferri, *University of Kassel, Germany*
Room311B (Short) (STEM & Modelling) Chair Mitsuru Matsushima Kai-Lin Yang	OB21 DESIGN OF STEM TASKS BASED ON MATHEMATICAL REASONING IN A SEQUENCE OF MODELLING ACTIVITIES Kai-Lin Yang, *National Taiwan Normal University, Taiwan*
	OB22 HOW CHINESE CITIZENS UNDERSTAND THE INFLECTION POINT OF THE COVID-19 PANDEMIC: INSIGHT INTO FORMAL MATHEMATICS AND STREET MATHEMATICS Yi Wang, *Beijing Normal University, China* Lidong Wang, *Beijing Normal University, China*
	OB23 STATISTICAL PROBLEM SOLVING BASED ON CHILDREN'S VALUES IN MATHEMATICAL SCIENCES EDUCATION Mitsuru Matsushima, *Kagawa University, Japan* Daisuke Ishikawa, *The Ninth Haketa Elementary School, Japan* Keiichi Nishimura, *Tokyo Gakugei University, Japan*
Room403 (Short) (Task Situation & Modelling) Chair Masaaki Ishikawa Melanie Kämmerer	OB24 DIFFERENCES AND SIMILATIRIES IN STUDENTS' PERFORMANCE ON MODELLING TASKS WITH MUCH OR LITTLE PERSONAL INTEREST IN THE REAL-WORLD CONTEXT OF THE TASK – A COMPARISON Melanie Kämmerer, *University of Münster, Germany* Gilbert Greefrath, *University of Münster, Germany*
	OB25 WRITTEN AND SPOKEN RETELLING OF THE TASK SITUATION AS A STRATEGY FOR UNDERSTANDING OPEN MODELLING TASKS Anna Surel, *University of Münster, Germany* Gilbert Greefrath, *University of Münster, Germany*
	OB26 INFLUENCE OF LANGUAGE USED ON THE MATHEMATISATION：ANALYSIS BASED ON COGNITIVE LINGUISTICS Masaaki Ishikawa, *Aichi University of Education, Japan*

Room405 (Short) (University/Vocatinal Education) Chair Karolina Muhrman Claudio Gaete-Peralta	OB27 ACCUMULATION OF DEGREE-DAYS AND CATEGORY OF MODELLING: A CASE WITH STUDENTS IN CHILE **Claudio Gaete-Peralta** , *Universidad Bernardo O´Higgins, Chile* **Jaime Huincahue** , *Universidad Católica del Maule, Chile* **Jaime Mena** , *Pontificia Universidad Católica de Valparaíso, Chile*
	OB28 EXPERIMENTS IN TEACHING ADVANCED CALCULUS: AN EARLY INTRODUCTION TO DOUBLE INTEGRALS **John Gordon** , *The City University of New York-QCC, USA* **Dawnette Blackwood-Rhoomes** , *Tech Prep Solutions Inc., USA*
	OB29 VOCATIONAL MATHEMATICS EDUCATION IN DIFFERENT CONTEXTS **Karolina Muhrman** , *Linköping University, Sweden* **Peter Frejd** , *Linköping University, Sweden*
15:15-15:45/3F & 4F	**COFFEE BREAK (Coffee Break Areas on 3F and 4F)**
15:30-17:00 **SESSION 5**	
Room301A (Long) (Technology) Chair Barry Kissane Susana Carreira	OB30 CONNECTIONS ESTABLISHED BY 9TH GRADERS AFTER SIMULATING AND MODELLING THE EFFECT OF ATHEROSCLEROSIS **Susana Carreira** , *Universidade do Algarve & UIDEF, Instituto de Educação, Universidade de Lisboa, Portugal* **Nélia Amado** , *Universidade do Algarve, Portugal* **Marli Teresinha Quartieri** , *Universidade do Vale do Taquari, Brasil*
	OB31 USING A SCIENTIFIC CALCULATOR FOR MATHEMATICAL MODELLING AND APPLICATIONS IN SECONDARY SCHOOL **Barry Kissane** , *Murdoch University, Australia*
Room301B (Long) (Modelling in the 21st century) Chair Diana M. Fisher Michael Besser	OB32 MAKING STUDENTS BECOMING RESPONSIBLE CITIZENS IN 21ST CENTURY BY FOSTERING MATHEMATICAL REASONING ABOUT REAL-WORLD APPLICATIONS **Maike Hagena** , *University of Hamburg, Germany* **Dominik Schlüter** , *Leuphana University of Lueneburg, Germany* **Michael Besser** , *Leuphana University of Lueneburg, Germany* **Dominik Leiss** , *Leuphana University of Lueneburg, Germany*
	OB33 HANDS-ON WORKSHOP: PARTICIPANTS BUILD A SMALL HUMAN CARRYING CAPACITY FOR EARTH MODEL IN FREE STELLA ONLINE **Diana M. Fisher** , *Portland State University, USA*

Room311A (Long) (Epistemological Perspective) Chair Berta Barquero Tatsuya Mizoguchi	OB34 STUDENTS' MODELLING ACTIVITIES IN THE INQUIRY ON THE SLIDE RULE **Tatsuya Mizoguchi**, *Tottori University, Japan* **Kokoro Okada**, *Tottori University, Japan* **Berta Barquero**, *Universitat de Barcelona, Spain* **Marianna Bosch**, *Universitat de Barcelona, Spain*
	OB35 TEACHER EDUCATION IN MODELLING: MODELS, SIMULATION AND NEW EPISTEMOLOGICAL NEEDS **Berta Barquero**, *Universitat de Barcelona, Spain* **Marianna Bosch**, *Universitat de Barcelona, Spain* **Susana Vásquez**, *Universitat de Barcelona, Spain*
Room311B (Short) (Technology) Chair Jascha Quarder Stephanie Hofmann	OB36 HOW CAN ALEXA UNDERSTAND US? SPEECH RECOGNITION AND AI IN MATHEMATICS EDUCATION **Stephanie Hofmann**, *Karlsruhe Institute of Technology, Germany* **Martin Frank**, *Karlsruhe Institute of Technology, Germany*
	OB37 DIDATIC REVISION STRATEGY FOR DESIGNING MATHEMATICAL MODELING TASKS USING AI TECHNOLOGY **Dong-Joong Kim**, *Korea University, Korea* **Hee-jeong Kim**, *Korea University, Korea* **Won Kim**, *Korea University, Korea* **Young-Seok Oh**, *Graduate School of Education, Korea University, Korea* **Gima Lee**, *Graduate School of Education, Korea University, Korea*
	OB38 LEARNING THEORY BELIEFS ABOUT TEACHING SIMULATIONS AND MATHEMATICAL MODELLING WITH DIGITAL TOOLS **Jascha Quarder**, *University of Münster, Germany* **Sebastian Gerber**, *University of Würzburg, Germany* **Hans-Stefan Siller**, *University of Würzburg, Germany* **Gilbert Greefrath**, *University of Münster, Germany*
17:00-18:30/B1F Lobby	**POSTER SESSION & HAPPY HOUR**
10:30-17:30/Room404	**CORPORATE EXHIBITIONS**

Tuesday, September 12, 2023

Time/Location	Activity
17:00-18:30/B1F Lobby	**POSTER SESSION**
	PO1 EXAMINING EARLY-GRADE CHILDREN'S MATHEMATICAL MODELS IN PROPORTIONAL SITUATIONS: A GLIMPSE FROM TASK-BASED INTERVIEWS **Keiko Hino**, *Utsunomiya University, Japan* **Hiraku Ichikawa**, *Miyagi University of Education, Japan* **Hisae Kato**, *Hyogo University of Teacher Education, Japan*
	PO2 HOW DO STUDENTS CRITICALLY INTERPRET PLURAL WAYS TO DETERMINE THE MOST POPULAR ONIGIRI?- ATOMISTIC APPROACH FOCUSED ON INTERPRETATION, VALIDATION AND MODIFICATION- **Minami Asahara**, *Yokohama National University (Master's student), Japan* **Takashi Sumioka**, *Yokohama National University (Master's student), Japan* **Makoto Chonan**, *Yokohama National University (Master's student), Japan* **Kazuki Koike**, *Yokohama National University (Master's student), Japan*
	PO3 HOW TO USE ELEMENTARY MATH TEXTBOOKS TO PROMOTE MATHEMATIZATION **Hatsuda Hiroki**, *Yokohama National University(Master's students), Japan*
	PO4 FOSTERING STUDENTS' ABILITY TO COMPARE AND CRITICALLY EXAMINE PLURAL MODELS TO SOLVE REAL WORLD PROBLEMS-TEACHING EXPERIMENT BY USING A TENT PROBLEM- **Shota Kita**, *Yokohama National University(Master's student), Japan* **Masashi Ikemura**, *Yokohama National University(Master's student), Japan* **Kodai Aratani**, *Yokohama National University(Master's student), Japan* **Toshikazu Ikeda**, *Yokoaham National University, Japan*
	PO5 WHAT METAPHORS DO CHILDREN UTILIZE TO MAKE CONNECTIONS BETWEEN MATHEMATICS AND THE REAL WORLD? **Kensuke Koizumi**, *Gunma University, Japan* **Ryuta Tani**, *Tanaka Gakuen Ritsumeikan Keisyo Primary School, Japan* **Ryo Hanzawa**, *Seya Primary School, Japan*
	PO6 HOW DOES FIRST-YEAR SCITECH EXPERIENCE OF PROFESSIONAL DEVELOPMENT EFFECT MATHEMATIC EDUCATION? **Satoru Fujitani**, *Tamagawa University, Japan* **Motoko Fujitani**, *Joetsu University of Education, Japan*

Wednesday, September 13, 2023	
Time/Location	Activity
9:30–10:30/Event Hall	**PLENARY LECTURE 2** P2 IN THEIR OWN WORDS: EXPLANATIONS OF STEM STUDENTS' REASONING DURING MATHEMATICAL MODELLING **Jennifer A. Czocher**, *Texas State University, USA* Chair **Cyril Julie**, *University of the Western Cape, South Africa*
10:30–	**EXCURSION (OPTION)** **LUNCH (Take away lunch boxes at B1F Lobby)**

Thursday, September 14, 2023

Time/Location	Activity
9:30–11:00 **SESSION 6**	
Room301A (Long) (Modelling Process) Chair Akio Matsuzaki Elena Naftaliev	OC1 CONSTRUCTING MEANING FOR INTERACTIVE MATHEMATICAL MODELS WITHIN A DIALECTICAL LEARNING ENVIRONMENT Elena Naftaliev, *Achva Academic College, Israel*
	OC2 ADVANCING THE STUDY OF A MATHEMATICIAN'S MODELLING PROGRESS DISPLAYED WITH APPLIED RESPONSE ANALYSIS MAPPING Akio Matsuzaki, *Saitama University, Japan*
Room301B (Long) (Teacher Education) Chair Xiaoli Lu George Ekol	OC3 ANALYSING 'STUDENT-TO-STUDENT-TO-TEACHER' COLLABORATIVE LEARNING PRACTICES IN A PRE-SERVICE APPLICATIONS AND MODELLING COURSE. George Ekol, *University of Witwatersrand, South Africa*
	OC4 PRE-SERVICE TEACHERS' PERCEPTION ON THEIR ROLES IN THE TEACHING OF MATHEMATICAL MODELLING Xiaoli Lu, *East China Normal University, China* **Yan Zhu**, *East China Normal University, China*
Room311A (Long) (Literature Review) Chair Nina Unshelm Rose Mary Zbiek	OC5 CHARACTERIZING MATHEMATICAL MODELLING TASKS IN EMPIRICAL LITERATURE Rose Mary Zbiek, *Pennsylvania State University, USA* **Xiangquan Yao**, *Pennsylvania State University, USA* **M. Kathleen Heid**, *Pennsylvania State University, USA* **Matthew Victor Black**, *Pennsylvania State University, USA*
	OC6 A REVIEW OF RESEARCH ON MODELLING, BIG DATA AND EVIDENTIARY PRACTICES Nina Unshelm, *University of Wurzburg, Germany* **Vince Geiger**, *Australian Catholic University, Australia* **Hans-Stefan Siller**, *University of Wurzburg, Germany* **Sarah Digan**, *Australian Catholic University, Australia* **Andre Greubel**, *University of Wurzburg, Germany*
Room311B (Short) (Modelling Epistemology) Chair Blanca Cecilia Fulano-Vargas Koichi Nakamura	OC7 WHAT IS THE ROLE OF MODEL IN THE MODELINNG PROCESS Koichi Nakamura, *Tokyo Gakugei University, Japan*
	OC8 CONCEPTIONS, USES AND OBSTACLES IN THE TEACHING AND LEARNING OF MATHEMATICAL MODELLING Blanca Cecilia Fulano-Vargas, *Secretaría de Educación De Bogotá-Colombia, Colombia* **Nelson Enrique Barrios Jara**, *Secretaría de Educación De Bogotá-Colombia, Colombia*
	OC9 IDENTIFYING AUTHENTICITY AND MATHEMATICAL MODELLING COMPETENCIES AS STEPPING STONES FOR BLENDING DIFFERENT PERSPECTIVES Daiki Urayama, *International Pacific University, Japan*

Room405 (Short: 9:30–10:30) (Technology) Chair Andreas Back Britta Eyrich Jessen	OC10 THE ROLE OF STUDY AND TECHNOLOGY IN MATHEAMTICAL MODELLING Britta Eyrich Jessen , *University of Copenhagen, Denmark* OC11 LEARNING OF DIGITAL TOOL COMPETENCIES IN THE CONTEXT OF MODELLING Andreas Back , *University of Münster, Germany* **Gilbert Greefrath** , *University of Münster, Germany* **Stanislaw Schukajlow** , *University of Münster, Germany*
11:15–12:45 **SESSION 7**	
Room301A (Long) (Modelling Competency) Chair Janina Krawitz Dag Wedelin	OC12 TEACHING MATHEMATICAL THINKING: REASONING, MODELLING, PROBLEM SOLVING Dag Wedelin , *Chalmers University of Technology, Sweden* OC13 TAKING UP OWNERSHIP: WHAT CAN BE LEARNED FROM POSING MODELLING PROBLEMS? Janina Krawitz , *University of Münster, Germany* **Luisa Hartmann** , *University of Münster, Germany* **Stanislaw Schukajlow** , *University of Münster, Germany* **Werner Blum** , *University of Kassel, Germany*
Room301B (Long) (Primary/Secondary Education) Chair Yuan Yan Paul Brown	OC14 MATHEMATICAL MODELLING AT ALL YEAR LEVELS IN AUSTRALIA Paul Brown , *Curtin University, Australia* OC15 THE INVESTIGATION OF THE MODELLING COMPETENCIES OF TWO GROUPS OF GRADE EIGHT STUDENTS WHO RECEIVED DIFFERENT MODELLING INSTRUCTIONS Yuan Yan , *East China Normal University, China* **Yi Chen** , *Shanghai Wenqi Middle School, China*
Room311A (Long) (Data & Modelling) Chair Vince Geiger Takashi Kawakami	OC16 VALIDATION IN SIXTH-GRADE DATA-DRIVEN MODELLING Takashi Kawakami , *Utsunomiya University, Japan* **Katsuki Akizawa** , *Elementary School Attached to Utsunomiya University, Japan* OC17 STRENGTHENING TEACHERS' CAPABILITIES WITH BIG DATA Vince Geiger , *Australian Catholic University, Australia* **Hans-Stefan Siller** , *University of Wurzburg, Germany* **Sarah Digan** , *Australian Catholic University, Australia* **Andre Greubel** , *University of Wurzburg, Germany* **Nina Unshelm** , *University of Wurzburg, Germany*

Room311B (Short: 11:15–12:15) (University & Teacher Education) Chair Alina Alwast Xin Ma	OC18 IMPROVING UNDERGRADUATE STATISTICS EDUCATION: EDUCATIONAL LESSONS FROM PEDAGOGICAL EXPERIMENTS **Anushka Karkelanova** ,*University of Kentucky, USA* **Xin Ma**, *University of Kentucky, USA* **William Rayens** , *University of Kentucky, USA*
	OC19 PRE-SERVICE TEACHERS' NOTICING OF MATHEMATICAL MODELLING PROCESSES **Alina Alwast** , *University of Hamburg, Germany*
Room405 (Short) (Technology) Chair Sonja Bleymehl Ludwig Paditz	OC20 COMBINING BRUTE FORCE AND IT TO SOLVE A CLASSICAL SHOPPING PROBLEM **Ludwig Paditz** , *Dresden University of Applied Sciences, Faculty of Informatics and Mathematics, Germany*
	OC21 A STUDY ON MATHEMATICAL MODELING UTILIZING APOLLONIUS' CIRCLE **Takuma Takayama** , *Konan Junior High School, Japan*
	OC22 EXPLORATORY STUDY ON STUDENTS' UNDERSTANDING OF SIMULATIONS – HOW MATHEMATICAL MODELLING COMPETENCY NEEDS TO BE COMPLEMENTED TO PROMOTE UNDERSTANDING OF SIMULATIONS **Sonja Bleymehl** , *University of Education Ludwigsburg, Germany* **Christine Bescherer** , *University of Education Ludwigsburg, Germany*
12:45–13:45/3F & 4F	**LUNCH (Take away lunch boxes in Coffee Break Areas on 3F and 4F)**
13:45–15:15 **SESSION 8**	
Room301A (Long) (STEM & Modelling) Chair Mitsuru Kawazoe Yuriy Rogovchenko	OC23 MATHEMATICAL MODELING PROJECTS IN ADVANCED ORDINARY DIFFERENTIAL EQUATIONS COURSES FOR MATHEMATICS AND ENGINEERING STUDENTS **Yuriy Rogovchenko** , *University of Agder, Kristiansand, Norway* **Svitlana Rogovchenko** , *University of Agder, Grimstad, Norway*
	OC24 MODELLING TASKS FOR NON-STEM UNIVERSITY STUDENTS AND THE CHARACTERISTICS OF THEIR MODELLING PROCESS **Mitsuru Kawazoe** , *Osaka Metropolitan University, Japan* **Koji Otaki**, *Hokkaido University of Education, Japan*

Room301B (Short) (Data & Modelling) Chair Louise Meier Carlsen Camilla Hellsten Østergaard	OC25 COMPARING AND CONTRASTING STATISTICAL AND MATHEMATICAL MODELLING– A STUDY OF THE ROLE OF DATA Camilla Hellsten Østergaard , *University College Copenhagen, Denmark* **Britta Eyrich Jessen** , *University of Copenhagen, Denmark*
	OC26 ASPECTS OF DATA MODELLING AMONG JAPANESE JUNIOR HIGH SCHOOL STUDENTS: FOCUSING ON CRITICAL CONSIDERATION Yuichiro Hattori , *Okayama University, Japan* **Mayu Nomura** , *Hayama Junior High School, Japan*
	OC27 WHAT ARE THE POTENTIAL RELATIONS BETWEEN MATHEMATICAL MODELLING AND COMPUTING EDUCATION ? THE CASE OF SIMULATIONS Louise Meier Carlsen , *IT University of Copenhagen, Denmark* **Britta Eyrich Jessen** , *University of Copenhagen, Denmark*
Room311A (Short: 13:45–14:45) (Teacher Education) Chair Jader Otavio Dalto Katharina Wiehe	OC28 DIAGNOSTIC COMPETENCE OF PRESERVICE TEACHERS IN THE CONTEXT OF OPEN MODELLING TASKS Katharina Wiehe , *University of Münster, Germany* **Stanislaw Schukajlow** , *University of Münster, Germany*
	OC29 DIMENSIONS OF THE PEDAGOGICAL CONTENT KNOWLEDGE IN A FIRST MATHEMATICAL MODELLING PRACTICE: THE CASE OF LUCY Jader Otavio Dalto , *Federal University of Technology – Parana, Brazil* **Karina Alessandra Pessoa da Silva** , *Federal University of Technology – Parana, Brazil* **Adriana Helena Borssoi** , *Federal University of Technology – Parana, Brazil*
Room311B (Short: 13:45–14:45) (Cultural Perspective) Chair Tadashi Misono Milton Rosa	OC30 AN ETHNOMODELLING PERSPECTIVE OF EMIC, ETIC, AND DIALOGIC ANALYSIS OF ETHNOMODELS IN THE AFRO-DESCENDANT CARIBBEAN DANCE *PALO DE MAYO* OF COSTA RICA **Steven Eduardo Quesada Segura** , *Universidade Federal Ouro Preto, Brazil* Milton Rosa , *Universidade Federal Ouro Preto, Brazil* **Daniel Orey Clark** , *Universidade Federal Ouro Preto, Brazil*
	OC31 A SUGGESTION OF FRAMEWORK TO CAPTURE THE STRUCTURE OF REGIONAL PROBLEMS AND ITS META PROBLEM Tadashi Misono , *Shimane University, Japan* **Yuki Watanabe** , *Tokyo University of Science, Japan*
15:15–15:45/B1F Lobby	**COFFEE BREAK (Coffee Break Areas on B1F Lobby)**

15:45–16:45/B1F Event Hall	**PLENARY LECTURE 3** P3 LESSON STUDY AND ITS RELATIONS TO MATHEMATICAL MODELLING **Yoshinori Shimizu**, *University of Tsukuba, Japan* Chair **Stanislaw Schukajlow**, *University of Münster, Germany*
16:45–17:45/B1F Event Hall	**GENERAL MEETING**
18:00– /2F Reception Hall B	**CONFERENCE DINNER**
10:30–17:30/Room404	**CORPORATE EXHIBITIONS**

Friday, September 15, 2023	
Time/Location	Activity
9:30–11:00/B1F Event Hall	**PLENARY PANEL** P4 RELATIONS AMONG PROBLEM SOLVING/POSING, CREATIVITY AND MATHEMATICAL MODELLING **Jinfa Cai**, *University of Delaware, USA* **Gabriele Kaiser**, *University of Hamburg, Germany & Nord University, Norway* **Roza Leikin**, *University of Haifa, Israel* Chair **Jonas Bergman Ärlebäck** , *Linköping University, Sweden*
11:15–12:00/B1F Event Hall	**POLLAK AWARD CEREMONY & CLOSING CEREMONY**

Abstract

Keynote

CONCEPTUALISING THE RELATIONSHIP BETWEEN MATHEMATICAL MODELLING AND INTERDISCIPLINARY STEM EDUCATION

Merrilyn Goos[1] and Susana Carreira[2]

[1]*University of the Sunshine Coast, Australia;* [2]*University of the Algarve and Institute of Education, University of Lisbon, Portugal*

Around the world, STEM education is promoted by governments as a means of addressing social and economic challenges and creating a scientifically, mathematically, and technologically literate citizenry. In many countries, policies and reports by governments and business groups aim to incorporate STEM into the school curriculum, encourage young people to engage in STEM education, and advocate for STEM careers. Yet, STEM education research is still in an embryonic state and the field lacks a scientific evidence base that can inform theory, policy, and practice (Maass et al., 2019). It is also unclear how mathematical concepts and practices contribute to a better understanding of the other STEM disciplines; nor do we understand well enough how STEM education enhance students' learning of mathematics (English, 2016).

One way of exploring the role of mathematics in interdisciplinary STEM education is to examine synergies between STEM and mathematical modelling. For example, both approaches may involve making connections between mathematics and other disciplines (Goos, 2020). In STEM education, however, the disciplines connected to mathematics are science, technology, and engineering, while many real world situations to be modelled need not be related to such disciplines, despite the argument that mathematical modelling is inherently interdisciplinary (Stillman et al., 2023). Some perspectives see the engineering design process as foundational in STEM tasks (English, 2016), while others have proposed to articulate the design and the modelling cycles (Baioa & Carreira, 2021). In this presentation, we will consider the impact of the STEM education movement on interdisciplinary curriculum development, ask whether good interdisciplinary "M in STEM" tasks are also good modelling tasks (and vice versa), and compare the influence of STEM and mathematical modelling on school mathematics education.

References

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Plenary Lecture

TEACHER EDUCATION AND MATHEMATICAL MODELLING – PRE-SERVICE TEACHERS' PROFESSIONAL COMPETENCE FOR THE TEACHING OF MATHEMATICAL MODELLING

Gilbert Greefrath

University of Münster, Germany

Teaching mathematical modelling is considered challenging (Blum, 2011). However, it also offers special opportunities for mathematics teaching due to the openness and self-differentiating properties of modelling tasks. We know that pre-service teachers prefer tasks with low modelling content and have little knowledge about mathematical modelling. Thus, an important goal of teacher education is to enable pre-service teachers to acquire this specific professional competence for the teaching of mathematical modelling.

Professional competence is agreed to include affective-value-oriented aspects in addition to cognitive-oriented knowledge dimensions (Blömeke et al., 2015; Kunter et al., 2013). In this context, pedagogical content knowledge represents a central factor in determining the cognitive activation potential of teaching. These characteristics of professional competence can be transferred to the arena of mathematical modelling. In particular, the area of pedagogical content knowledge appears to be especially significant.

When designing modelling seminars for pre-service teachers with the participation of students in practical phases for the acquisition of professional competence, the question arises in particular as to which special facets of pedagogical content knowledge specific to modelling should be emphasised. The results show that, overall, the pedagogical content knowledge of pre-service teachers for teaching mathematical modelling can be significantly improved and that it can be very helpful for the acquisition of competence in some facets if the pre-service teachers develop modelling tasks for the practical phases themselves (Greefrath et al., 2022). Furthermore, we also take a look at the development of affective-value-oriented aspects. Going further, the question arises to what extent the use of technology can be considered in the acquisition of professional competences for teaching mathematical modelling and what new possibilities arise from this. Initial results will be reported on this.

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IN THEIR OWN WORDS: EXPLANATIONS OF STEM STUDENTS' REASONING DURING MATHEMATICAL MODELLING

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Teaching mathematics through mathematical modelling means moving beyond dressing up mathematical procedures with real-world terminology. It involves interacting with students' representational systems and ways of thinking about and with those systems – and those ways of thinking are often non-normative, incomplete, and self-contradictory to an objective observer, like a teacher or a researcher. Thus, one question the field has posed is *How should the observer respond to students' work?* There is a strong temptation to attend to accuracy and to correct students' modelling work because it does not match the intended solution. However, we know that consistent negative feedback dismissing students' real-world experiences teaches them to ignore their own intuitions and hinders them in learning to use mathematics to solve realistic problems.

In this talk, I start from the position that it is not possible to answer the question *How should the observer respond?* without understanding *Why did the students model the way they did?* This second question is partially answered. The field already has a number of high-level, descriptive answers to this second question. For example, interpretations of students' work might point towards metacognitive fault (the student was "not paying attention" to or "did not notice" some feature the observer thought was important), implicate obstacles to knowledge transfer (the student did not activate target mathematics content), or even indicate blockages to phases of a modelling cycle (the student did not include/ignore variables). I argue that descriptive framings, even when couched in suitable theories, fall short of explaining what the student might have been thinking that led them to their modelling decisions which limits our capacity to design contingent responses to students' modelling work. To enhance the field's ability to address students' patterns of reasoning, I argue we ought to be asking *What were they thinking?* A simple analogy is that a doctor should seek the underlying cause of symptoms to prescribe a proper treatment.

Awareness of the typical ways students think about mathematical content while modelling and the kinds of explanations that are sensible to students is an essential step towards equipping instructors with the pedagogical skills they need to contingently interpret student thinking and respond constructively to students' reasoning patterns. Using STEM students' own words, I will share some archetypal ways students think about mathematical content while modelling including: their desire for precision, accuracy and authenticity, the mathematical structures they anticipate, and the rationales they use to justify the mathematics selected for their models. My goal is to increase the sophistication we attribute to students' modelling decisions by deepening the field's understanding of students' mathematics – as they use it in modelling – as an alternative to focusing exclusively on accuracy of solutions to modelling problems.

LESSON STUDY AND ITS RELATIONS TO MATHEMATICAL MODELLING

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Lesson study that originated in Japan is an approach to improve teaching and learning mathematics through a particular form of activity in which a group of teachers works collaboratively to design, implement, observe, and reflect on the proposed "research lessons". Research and practice on lesson study has spread internationally with a focus on its capacity as a vehicle for the professional development of teachers. While recent advocacy of lesson study is focused on its function of professional development, key impacts of lesson study also include development of mathematics curriculum with the design of tasks and instructional sequences (Lewis, 2016).

If mathematical modelling is being introduced as a new product into the complex system of mathematics education in many countries, it has to fit with the existing parts and interfaces in this system (Pollak, 2015). For more than the decades, various approaches were taken for exploring the nature of teaching and learning of mathematical modelling in the classroom. Such approaches include teaching experiment, design research (Lesh et al., 2010), and case studies (Schukajlow et al, 2018).

By examining the phases of lesson study and the process of teaching and learning mathematical modelling in the classroom, this talk illustrates how a cyclic nature of lesson study can contribute to designing tasks and activities for the students and to develop an instructional sequence for teaching and learning mathematical modelling embedded in a teaching unit of mathematical topics.

Focusing on the process of mathematical modelling (e.g. "model-elicit-activities" or "being aware of the prerequisite of a model"), lesson study provide the community of teachers illuminating examples with the "evidences" of students' work, tasks to be appeared in the textbooks, and a shared vision on the new curriculum. The case of "Be Careful with the Blind Corner from the Car" (Nishimura, 2012), appeared in an "Open House" at a university-affiliated high school, is used for describing how task design and its implementation through lesson study procced. The example with other cases illustrates how cycles of lesson study can contribute to the task design and then to the curriculum development.

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Plenary Panel

RELATIONS AMONG PROBLEM SOLVING/POSING, CREATIVITY AND MATHEMATICAL MODELLING

Jonas Bergman Arleback[1] (Chair), Jinfa Cai[2], Gabriele Kaiser[3], and Roza Leikin[4]

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Mathematical problem solving and mathematical modelling have several commonalities as has been pointed out on various occasions (Blum & Niss, 1991; Niss & Højgaard, 2019). In the current discourse on mathematics education and mathematics education research, problem solving/posing, creativity and modelling play an important role and their fostering is emphasized in many curricular standards all over the world, being quite often strongly connected within the curricula.

However, although mathematical modelling and mathematical problem solving/posing share many commonalities in their emphasis on non-routine and open-ended solution processes, where creativity is concerned, both approaches have also many differences in their orientation and foci. Especially their relation to the real world as a context for the problems tackled and their requirements concerning authenticity and proximity to reality are quite different. The creativity-directed approach involves students in creative mathematical processing. The fostering of creativity through challenging mathematical problems and tasks play an important role within mathematics education and have been connected in the past especially to problem solving and complex problems in mathematical competitions. However, in the recent years the scope of fostering mathematical creativity has been widened, focusing on mathematical problems with different stages of complexity and a great variety of solution processes, also within real world contexts.

In all three approaches scaffolding activities play an important role as the teacher should function as a mentor supporting students to find their own solutions on their own pace, as independent as possible.

The panel will discuss the complex relations between these three important approaches concerning the following topics:

- What kind of tasks are important in the three approaches, how can they be characterised?
- How can the solution strategies be characterised?
- How can problem solving/posing competency and modelling competency be conceptualised, respectively? How can they mutually support each other for further advances?
- Which scaffolding activities have proven to be successful for mentoring students in their solution processes?
- Which kind of theoretical framework is appropriate, for example concerning the relation to real-world contexts or to metacognition?
- Under which conditions foster problem solving/posing and modelling tasks creative mathematical processing?

The panel will close with prospects for further research and effective practices.

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Special Lecture

IMMC: CELEBRATING 10 YEARS OF INFLUENCING EDUCATIONAL CHANGE

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The Consortium for Mathematics and Its Applications (COMAP) has had over 35 years of experience in running contests in mathematical modelling – at the undergraduate level with the Mathematical Contest in Modeling / the Interdisciplinary Contest in Modeling (MCM/ICM), at the secondary level with the High School Mathematical Contest in Modeling (HiMCM) and now at the middle school level with the Middle Mathematical Contest in Modeling (MidMCM). While these are international contests, we have never attracted more than 20 countries to participate, and entrants are primarily from China and the U.S. In an attempt to level the playing field, COMAP joined with NeoUnion ESC Organization (NeoUnion) of Hong Kong to create a modelling challenge based on COMAP's practice and the Olympiad model as well. And so, in the International Mathematical Modeling Challenge (IMMC or styled as IM^{2}C) each country or region is permitted to submit no more than two papers to the international judging round. This model has proved to be quite successful and pre-Covid we had 48 countries registering teams. Indeed, during the last ten years, the IMMC has played an increasing role in the education scene around the world. It has empowered students, teachers, and policy makers to place mathematical modelling at the center of curricula reform and educational change.

Our aim in this talk is to invite you to join us, not simply as a participating country, but also as a research partner with the shared goal to promote the value and importance of mathematical modelling as a critical skill all students should have the opportunity to learn. Of course, we are only at the beginning, but we now have an extensive database that we could (and should) make use of together. Currently we have access to student work from the first ten years of the IMMC, representing more than 400 teams from over 50 countries. We are excited to utilize this data to address questions such as: To what extent and in what sense does cultural context influence model development? What is the impact of developing a pre-selection competition in a country? What makes a math modelling task meaningful for an international audience? These are just a few examples; we look forward to sharing our data and hearing your ideas.

The second part of the talk will highlight one country's story. We will focus on what China does in IMMC and how attitudes towards modelling, including curricula, have dramatically changed over the past ten years. An emphasis on policy changes and influence, made possible through the IMMC, showing what a focus on mathematical modelling contests has made possible.

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Long & Short Presentation

PRE-SERVICE TEACHERS' INTERVENTIONS IN MATHEMATICAL MODELING ACTIVITIES

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The importance of teacher pedagogical interventions for student learning has been well recognized by substantial literature, however there has been little research on pedagogical interventions in mathematical modeling. Employing scaffolding theory (Stender & Kaiser, 2015) and the theory about teacher interventions in cooperative learning (Ferguson-Patrick, 2016), this paper presented the investigation of the intervention strategies used by pre-service mathematics teachers when they were offering assistance for students in doing mathematical modelling.

This study presented a modelling activity which included a total number of 18 pre-service mathematics teachers and 32 grade eight students who were divided into 6 groups, one group with 5-6 students. When the students were doing a modelling task in groups, the 18 pre-service mathematics teachers were playing different roles: usually a lead teacher who progressed the process of modelling, six teachers who sit in each student group to offer instant interventions and the other pre-service teachers who sit aside to observe the interventions. Since the students did three modelling tasks in the activity, all the 18 pre-service teachers had the chance to conduct interventions for the students to do different tasks. The entire modelling activity of 240 minutes was both audio and video taped.

In this paper, we analyzed the types of teacher interventions by adopting Stender and Kaiser's (2015) classification of teacher interventions (including the categories of motivative, feedback, general-strategic, content-oriented strategic, and content) and Ferguson-Patrick's focus on teachers' social role and organizational role when providing interventions. It is expected that different types of teacher interventions will be recognized in the modelling activities. Moreover, the characteristics of the different types of teacher interventions will be in-detailed analyzed; and the classification of teacher interventions on mathematical modelling might be influenced by the types of modelling tasks. Through the analytic results and discussions based on the results, this study might provide insights for the learning and teaching of mathematical modelling, especially in the context of China.

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A TEST INSTRUMENT FOR PROFESSIONAL MATHEMATICAL KNOWLEDGE ON MATHEMATICAL MODELLING - FIRST RESULTS

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The professional knowledge of mathematics teachers provides a rational basis for decisions in mathematics teaching. It also provides arranging the contents, necessary simplifications and the use of application possibilities (Vollrath, 1988). However, specific professional knowledge on mathematical modelling is required especially for the fostering of mathematical modelling competence of students at all ages. This can generally be considered as consisting of different areas of knowledge (Shulman, 1986). Although studies on mathematical modelling knowledge already exist, less attention has been paid to the aspect of professional mathematical modelling knowledge of pre-service teachers. Previous test instruments on the modelling-specific mathematical knowledge of pre-service teachers use either school-level mathematical knowledge of modelling (Yang et al., 2022), or knowledge that goes well beyond secondary school (Czocher et al., 2021; Haines et al., 2001). First test items on content knowledge at the level of in-depth school mathematical knowledge in the area of calculus were developed (Greefrath et al., 2022). In our test construction, an atomistic approach is chosen; the items address individual sub-competencies. In this study, we present the first results of piloting the test items. 101 pre-service teachers from three German universities participated in the pilot study. A scale for modelling-specific content knowledge for pre-service teachers consisting of a total of five multiple-choice test items for different sub-competencies was created. We present the test instrument and solution rates are reported and discussed against the background of the selected population - with surprising results.

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SYSTEM DYNAMICS SOFTWARE AND SCENARIOS FOR CONTINUOUS PROFESSIONAL DEVELOPMENT

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Software to assist with the investigation of behaviour over time of natural and social phenomena have been used for some time. Galbraith & Fisher (2021) present a concise but fairly comprehensive exposition on how the availability of free software opened opportunities to explore complex phenomena from a systems dynamic perspective. They suggest that "it is necessary to invest some effort into understanding the software…before their potential can be realised." (p. 620). This presentation is about this investment of effort to come to grips with a particular free version of system dynamics modelling software, Vensim PLE (Ventana Systems, https://www.ventanasystems.com/software/), to develop scenarios of continuous professional development (CPD) offerings for mathematics teachers.

Continuous professional of mathematics teachers is a complex social issue. An umbrella review (a review of reviews of the evidence); systematic reviews and single research studies on effective CPD reveal that at least 25 issues are at stake for the successful delivery of CPD. CPD providers in South Africa were surveyed to rate the 25 issues. This was done since qualitative data "allows decision makers and social actors to be more aware of the directions their decisions could lead the system; and what the key variables are for the implementation of public policies to achieve the desired (or agreed) scenario". (Corral-Quintana, et al., 2015, p. 83). Using the ratings of the CPD providers in South Africa, my own experience with CPD provision and some other literature scenarios for effective CPD were developed. I report, in a quasi-autoethnographic manner, on two different scenarios for the implementation of CPD that were developed.

The concluding argument revolves around the need for more personal-like accounts of laying bear the blind alleys we encounter when coming to grips with using software for mathematical modelling.

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FACTORS AFFECTING CHINESE IN-SERVICE HIGH SCHOOL MATHEMATICS TEACHERS' IMPLEMENTATION OF MATHEMATICAL MODELLING IN CLASSROOM: AN INTERVIEW STUDY

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Mathematical modelling has been widely accepted as a core component of mathematical literacy for students internationally nowadays. Due to such importance, mathematical modelling has been integrated into school mathematics curricula worldwide to promote connections between school mathematics and real-world situations (Kaiser, 2017). Similarly, the idea of mathematical modelling was started to be emphasized in China in 2003 to change high school students' mathematics learning experience, and it is now listed as one of the six key mathematical competences in the high school mathematics curriculum (MOE, 2017). However, after almost twenty years, like in many other places, there is still few mathematical modelling in everyday mathematics teaching in China at the moment (Blum, 2015). Without any doubt, teachers play the most decisive role in the successful incorporation of mathematical modelling in their everyday teaching (Stillman, 2010). In view of this, the study investigated what factors influence teachers to implement mathematical modelling in classroom.

Thirty in-service high school mathematics teachers with various teaching experience (1-32 years) from various places such as Shanghai and Chongqing in China attended this study. Each of them was interviewed on the topics of their experiences and difficulties or obstacles of implementation mathematical modelling in everyday teaching. Constant comparative analysis approach (Strauss & Corbin, 1990) was applied to analyse the transcribed documents.

Generally, five factors/obstacles were identified: 1) the confusion of teachers with respect to what mathematical modelling is, and the lack of teacher knowledge; 2) shortage of professional development opportunities at both pre-service and in-service stages; 3) shortage of recourses such as no enough or suitable modelling examples; 4) examination culture since mathematical modelling is not included in university-entrance examination and shortage of teaching time; 5) the lack of knowledge for students because students did not have plenty previous experience. These factors/obstacles will be further discussed with referring to societal and cultural factors such as traditions of teacher education within the Chinese context.

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DEVELOPING LOCAL REPRESENTATIVE SOLUTIONS TO FERMI PROBLEMS

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According to Niss and Blum (2020), mathematical modelling refers to the process of constructing mathematical representations or abstractions that capture essential features of real-world phenomena, situations, or problems. It involves translating a non-mathematical context into a mathematical framework that enables analysis, exploration, prediction, or decision making related to the original context. The modelling process begins by considering a real-world context and problem that motivates and engages the problem solver(s). Sriraman and Lesh (2006) suggested that Fermi problems (FPs) could serve as an effective starting point for engaging students in modelling because of their accessibility and ability to emphasise the connection between the real world and the mathematical domain. FPs often involve situations from everyday life, such as the number of cars travelling through a busy street in a day. Solving FPs requires strategies and estimations based on personal experiences (Ärlebäck, 2009), and provide an opportunity to explore how cultural aspects come into play during the modelling process. As FPs often involve estimating quantities that may vary significantly across different contexts or cultures, they provide a tool for investigating the influence of intercultural awareness (Baker 2011) in the solving of modelling tasks (Albarracin et al., 2022).

In this presentation we explain an empirical study of the solutions of a Fermi problem by prospective teachers from 4 universities in 3 different countries. A first phase of the research is to investigate the differences in the solutions from different countries, both in terms of mathematical methods used and in terms of the cultural elements that come to the fore. In the second stage, to enable students effectively to compare productions from all countries, it was necessary to develop what we call "cultural representative solutions". The aim of this presentation is to provide a methodological and theoretical basis for developing such 'representative solutions' as a research tool in understanding the cultural aspects of the models developed in the solutions to a FPs, detail the process of how data is analysed to define, design, and validate these "cultural representative solutions", as well as discuss the potential contribution of using such representative solutions in teaching and learning modelling.

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DEVELOPMENT OF SECONDARY MATHEMATICS INSTRUCTIONAL STRATEGY FOR ACQUIRING CONCEPTUAL KNOWLEDGE

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Japan's Ministry of Education, Culture, Sports, Science, and Technology (MEXT) (2018) has emphasized the significance of acquiring conceptual knowledge through mathematical activity. In Japan, mathematical modelling is a part of mathematical activity. Fujimura (2011) has pointed out that junior and senior high school students in Japan need more conceptual knowledge. Problem-solving prior to instruction (PS-I) (Loibl & Rummel 2014) is an instructional strategy for acquiring conceptual knowledge. Sinha and Kapur (2019) suggested that PS-I may not be effective for some learner profiles. MEXT (2018) pointed out that inadequate motivation to learn mathematics is one of the characteristics of Japanese students. As a result, they may stop trying to solve problems.

In this study, taking into account the characteristics of Japanese students, we designed an instructional strategy for acquiring conceptual knowledge based on PS-I.

The instructional strategy consists of six stages. The first stage is problem solving (part 1). Students work on a real-world problem that incorporates a fresh concept. The second stage involves evaluating the generated solutions. In the third stage, students organize the elements of existing knowledge. The fourth stage is problem solving (part 2). Based on the activities from stages 1–3, students work on the same problem. In the fifth step, they are informed about the correct solution. In the sixth step, students compare the correct solution with the generated solution and incorporate their understanding of the characteristics of each solution.

In December 2022, we conducted a mock session on quadratic functions with 22 pre-service mathematics teachers. We investigated whether the instructional strategy developed in this study would be effective in helping students acquire conceptual knowledge when compared with conventional teaching. According to the results, pre-service mathematics teachers believed that this lesson was sufficient to help students gain conceptual knowledge when compared with conventional teaching, thus signifying the value of this lesson. However, the mathematics domain may ascertain how useful However, how useful the teaching strategy from this study is may depend on mathematics domain. Going forward, we intend to conduct this class with high school students to collect more data. We plan to present our practice plan at the conference.

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DESIGNING MATHEMATICAL MODELLING TASKS INVOLVING A SIMULATION WITH PHYSICAL EXPERIMENTS: ON THE TASK OF MARKER RECUPTURE METHODS

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In order to properly utilize reasoning of sampling in the real world, which includes a variety of variables, it is necessary to focus on the factors of variation in the real world and to have a deep understanding of their methods. When designing a task for this purpose, the requirements are that the context be one of modelling from an authentic problem, that the student be able to trace a situation as close as possible to an actual real-world situation, and that the validity of the model be verifiable. To fulfill these requirements, we propose modelling involving simulation with physical experiments. The simulation with physical experiments has a role in verifying how the model works in reality (e.g., Carreira & Baioa, 2018). The purpose of this study is to provide a case study of 9th graders in Japan to see how the design of a mathematical modelling task that encompasses simulation with physical experiments contributed to a focus on factors of variation in the real world and a deeper understanding of the methods of reasoning of sampling.

Designing a modelling task is not easy, so in this paper, we focus on the sign recapture method, which is often treated in Japanese textbooks, and redesign it into a modelling task. The original task was to estimate the total number of fish in a lake, that is, the population, based on the ratio of colored to uncolored fish when some fish were captured, colored, returned to the lake, and recaptured a few days later. This is the basis of a model that is frequently used in biological population studies. This task was redesigned to incorporate a simulation with physical experiments using a fish tank, aiming to create a more realistic and authentic situation setting. Then, it revealed that the task of estimating the total number of fish lacked authenticity for the students and discarded many variables. In the redesigned task, a situation requiring estimation was first set up as an exotic fish problem. And based on the simulation using a fish tank, an activity was set up to understand the statistical estimation method and to critically interpret and validate the method in relation to the real world. Then, a model with a better estimation method was re-planned and implemented again through simulation.

In the practical lesson, the first problem with the existing model was that in reality, random sampling is not possible due to natural variation and migration by herds. Several methods based on multiple sampling were proposed as improvement measures. Two of the methods that students considered were suggested as characteristic examples of critical reflection: setting the number of colors to be applied and changing the number of colors to be applied. In both considerations, the students compared the mathematical model with reality while experimenting and refining their plans. They modified their models from a realistic and statistical perspective. This case study suggests that simulation with physical experiments of a realistic and authentic situation setting can help to better interpretation and validation of mathematical models in statistics.

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CAN MATHEMATICAL MODELLING WORK AS A CRITICAL THINKING-DEMANDING ACTIVITY?

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This research stems from a PhD research project that investigates how engineering undergraduates experience critical thinking (CT) – a study motivated by educators' struggle to understand engineering undergraduates' ways of engaging with CT. Part of the project called for an activity that provides an opportunity for students to think critically and this paper illustrates how the inherent structure of model-eliciting activities provides a suitable framework to design such an activity. Lu and Kaiser (2022) illustrated that mathematical modelling has the potential for being a creativity-demanding activity. This project further-develops this framing as the same question is being asked, this time about CT.

What is CT?

CT is a complex phenomenon which is difficult to define. Ahern et al.'s (2012) conception of CT does however provide a framework for designing activities that afford the development of *engineering* students' CT skills:

> CT is a movement from the concrete, from the factual to the abstract and back again – an ability not only to use knowledge and facts to create ideas, concepts and solve problems but also to use these developed concepts, theories, and ideas in the real world.

What is mathematical modelling?

Geiger et al. (2022) maintain:

> Mathematical modelling consists of identifying a problem within a real-world context, developing a relevant mathematical representation, determining a subsequent mathematical solution, interpreting the solution within the original context, and evaluating the solution's validity for resolving the problem.

Lu and Kaiser (2022) affirm mathematical modelling's potential as a *creativity*-demanding activity via its impact on the modelling cycle. Drawing on the definition of mathematical modelling, this paper argues that there is the potential for mathematical modelling to be reframed as CT-demanding.

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THE CO-PRODUCTIVITY CHAIN OF MODELLING – ON MODELS USED BY CITIZENS, OPERATORS AND CONSTRUCTORS

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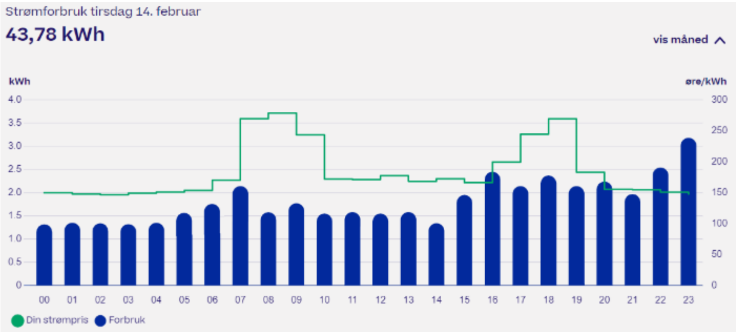
In this theoretical paper, we describe, analyse, and theorize how mathematical models play a role in vocational work and in daily life. Whilst many people do not engage in the modelling in itself, they productively engage with mathematical models in several ways.

We present several cases of public and vocational use of line and bar graphs (Kemp & Kissane, 2010). Figure 1 shows one such a line graph, namely of electricity prices fluctuating by the hour (in green). The hourly prices are published by a power company one day ahead. Reading the graph as prescriptive model (Niss, 2015), customers can plan to use less electricity when prices are high (blue bars). Their power consumption then offers descriptive feedback so the company can re-model their pricing. This interactivity is productive in two ways: citizens can try to reduce costs and companies can try to reduce power peaks.



Building on Skovsmose (2005), we distinguish across groups of people who are, in different ways, involved in the co-productivity chain of modelling, which involves data feedback and re-modelling. Civil or industrial institutions hire professional constructors of models (Frejd & Bergsten, 2016). These models are often hidden in technology but can materialize in spreadsheets or graphs. Operators (vocational workers) apply the models for safety, quality control, decisions, etc. Citizens benefit (or not) from processes regulated through mathematical models, and their behaviour is monitored and fed back into new models.

For the fostering of mathematical modelling in schools, we highlight that not all students will be constructors of mathematical models. We will describe competencies needed by operators using mathematical models in vocational work, and customers using models in daily life.

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COLLABORATIVE DESIGN OF MATHEMATICAL MODELLING TASKS FOR MIDDLE SCHOOL STUDENTS

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One of the crucial learning outcomes in mathematics, both in the Thailand curriculum and worldwide, is for students to develop mathematical literacy and the ability to apply mathematical knowledge to model real-world situations in various contexts outside the classroom. (Stacey & Turner, 2014; Klainin, 2015) So, they should be encouraged to apply their mathematical knowledge and experiences as tools to hone their mathematical thinking and competency (Blum & Borromeo Ferri, 2009; NCTM, 2014). To achieve the point, the objective of this study was to develop a prototype of the modelling tasks and establish principles to guide design tasks for middle school students. The research methodology involved three phases of design research: preparation and design, experimentation, and a retrospective study as described by Bakker (2018). The participants consisted of educators, teachers, and students. The instruments were open-ended tasks, which posed questions that encouraged the students to formulate, employ, interpret, and evaluate data to solve real-world problems. Semi-structured interviews were conducted and the results were assessed using descriptive statistics and content analysis. Results revealed that 1) The prototype consisted of three tasks, with each exhibiting three important characteristics as (i) real-world situations relevant to the students, (ii) tasks that encouraged the students to formulate, employ, interpret, and evaluate in order to solve real-world problems, and (iii) open-ended and challenging problems. 2) The principles for designing the tasks consisted of three principles as (P1) having a shared understanding with designers regarding mathematical literacy, modelling and how designing frameworks were valuable for modifying tasks (P2) selecting real-world situations relevant to students and posing questions that encouraged the students to apply their mathematical knowledge and experience, and (P3) design collaboration to ensure that the tasks were valid, reliable, and appropriate. 3) The result of tasks experimentation showed that most students were able to effectively use mathematics to solve problems. However, some faced obstacles and challenges when formulating the situations in mathematical contexts, and in interpreting and evaluating solutions in real-world contexts. The study findings can be used to develop the tasks that assess students' mathematical literacy in a contextually relevant manner.

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THE ROLE OF CARE IN MATHEMATICAL SCIENCES EDUCATION INCLUDING STUDENTS' VALUES

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Our research group focused on decision-making and consensus building using mathematical sciences (e.g., Yamaguchi et al., 2020). Mathematical sciences include mathematics, statistics, and applied mathematics at their core; however, they are also closely related to other disciplines such as science and engineering, depending on the authentic problem being dealt with. In mathematical sciences education, this decision-making process formulates and addresses real-world problems that require decision-making. Regarding the multiple options created by different formulations, it is necessary to clarify the premise, form a consensus, and make decisions (Yamaguchi et al., 2020). When building consensus, it is also necessary to pay attention to the values embedded in the mathematical model, which becomes the focal point of the dialogue.

Previously, the authors have clarified that the lack of "care for others" by students is one of the requirements for consensus building difficulty (Matsushima et al., 2023). "The care for others" is a relational involvement entailing the attention to, and recognition of, others and their material and spiritual need (Radford, 2021). The research question in this paper is the role of care in the practice of mathematical sciences education, especially in consensus building. The method is to analyse the aspects of consensus building through the lens of the concept of care in mathematical sciences in the practice targeting upper secondary school.

The practice under analysis is based on "Nurmi counterexample" in social choice theory. "Nurmi counterexample" is an example that yields different results depending on the models, such as majority rule, majority rule with runoff. The practice lesson was given for 11 hours from November 2022 to March 2023 to eight fourth-year students in a night crafts course at a public upper secondary school in Tokyo. In recent years, unreasonable school rules have become an issue in Japanese school education, and students are expected to participate in revising school rules. Using school rules revision as the subject topic, the question through the unit is, "What voting method and vote aggregation approach should be adopted to reflect the will of all?". The students critically examined the model of majority rule and developed an alternative model of scoring rules with various weightings. Then, among those models, they discussed what should be adopted and reached a consensus.

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POSSIBILITIES OF STUDYING THE GOLDEN RATIO PROPORTION IN THE CHRISTS SCULPTURES OF MASTER ALEIJADINHO IN THE TOWN OF CONGONHAS, MINAS GERAIS, BRAZIL

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This qualitative research was conducted by using data gleaned from 7 participants: 1 museum mediator, 1 tour guide, 1 sculptor, 1 plastic artist, 1 photographer, 1 mathematics teacher, and 1 tourist. The general objective of this study was to understand how the dialogical approach of ethnomodelling (Rosa & Orey, 2010) contributed to the study of the golden ratio that were present in these sculptures and which contributed to the development of the teaching and learning process in mathematics. The specific objectives were related to a) analyze the sculptures of 18th Century Aleijadinho's Christs to identify possible use of proportionality in his creation and b) understanding how a dialogical approach of ethnomodelling can help us in understanding the possibilities of proportional study in these sculptures.

According to these objectives, this study sought to answer the following research question: How can the dialogical approach of ethnomodelling contribute to the study of the golden ratio that may be present in Aleijadinho Christ Sculptures, in the chapels of the town of Congonhas, in Minas Gerais? In order to collect data we used: 2 questionnaires (initial and final), 3 semi-structured interviews, 1 focus group with two stages, and the researcher's field diary. For the presentation and analysis of the data, and for the interpretation of the results obtained in this study, we use an adapted methodological design of Grounded Theory (Glaser & Strauss, 1967) to conduct the open and axial codifications that made possible, respectively, the identification of the preliminary codes and 4 conceptual categories.

The results obtained in this study favored a new awareness and appreciation of these sculptures, as well showed a respect for aspects of local culture, through the dialogic approach of ethnomodelling. This approach helped the participants of this study in understanding the possibilities of the study of proportionality in these sculptures. This approach contributed to the improvement of cultural sensitivity of these participants, highlighting the importance of respecting and valuing Aleijadinho's mathematical knowledge and skills, as well as showing the presence of the golden ratio in some of his sculptures.

Finally, the results showed the possibility of inserting mathematical knowledge with the knowledge and actions of the participants through the development of a pedagogical action related to local history and culture with a theoretical foundation in the golden ratio, in the story of Aleijadinho, in ethnomathematics, in the sociocultural perspective of mathematical modelling, and in the dialogic approach of ethnomodelling.

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MATHEMATICAL MODEL OF YEAST GROWTH: BIOLOGY STUDENTS USE MATHEMATICS TO UNDERSTAND BIOLOGICAL PROCESSES

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In this paper, we analyse how biology undergraduates work on a mathematical modelling (MM) task guiding them to a discrete model for the yeast growth. This is only one of many tasks addressed by students in a collaborative research project where biologically meaningful MM tasks were offered to biology undergraduates to illustrate the use of mathematics and enhance students' mathematical skills and confidence. The study involved twelve first-year biology students who engaged in MM activities during four sessions alongside their standard mathematics course. Led by the second author, an experienced mathematician, sessions introduced students to different aspects of MM through small-group activities aimed at the promotion and integration of mathematical thinking and problem-solving skills into biology and deepening of students' understanding of mathematical principles in biology.

Logistic growth models are commonly used both in mathematics and biology to study and understand the growth patterns of microbial populations, including mathematical models of yeast growth. Mathematicians treat the logistic model of yeast growth as a mathematical abstraction, emphasizing the construction and analysis of continuous and discrete-time models which should capture the growth patterns observed in experimental data. They may explore the properties and behaviour of mathematical models including existence of equilibrium points, stability properties of solutions, bifurcations, and numerical simulations (Lewis & Powell, 2017).

In biology, the logistic model of yeast growth is considered in the context of understanding the underlying biological processes that drive population dynamics. Biologists are interested in studying how environmental factors, nutrient availability, competition, and other biological interactions affect the yeast growth. They may perform experiments, gather empirical data, and make observations to validate or refine their mathematical model. Biologists also investigate the mechanisms and molecular pathways involved in the yeast growth and analyse how they align with the predictions made by the mathematical model (Slonczewski et al., 2017; Stewart & Day, 2021).

We explore students' work on the task by comparing mathematical and biological approaches. The analysis of students' discourse and its projections on professional discourses in mathematics and biology is conducted in terms of the anthropological theory of the didactic (Chevallard, 2019).

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ASSESSING AUTHENTICITY IN MODELLING TEST ITEMS: DEVELOPING AND PILOTING OF A THEORETICAL MODEL

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Authenticity figures prominently in the discussions of the modelling community (Galbraith, 2013) and is often demanded for both learning tasks and test items. It is not only demanded on a theoretical basis normatively, but also empirical studies indicate that the use of *authentic* resources in mathematical tasks can increase motivation and performance of students (e. g. Vos et al., 2007). Central to the discourse around *authenticity* in mathematics tasks is the definition of Niss (1992) as well as the considerations of Palm (2009), who presents an operationalisation approach in the form of a didactical and pedagogical framework. In recent years, Vos' (2018) theoretical considerations have had an additional significant influence on the current international discourse. Her atomistic approach on authenticity is already being used for the analysis of learning tasks, but is not yet found in a theoretically and empirically robust model for assessing authenticity in modelling test items. This is the more astonishing, since many empirical studies (even PISA) claim at implementing "authentic modelling tasks" into standardised test situations and since it is therefore of special interest to empirically understand those studies frameworks on authenticity. This desideratum is taken up by the research project "AutMod" which aims at developing and piloting such a model and thus aims at investigating how to assess *authenticity* in modelling test items more closely.

In order to being able to set up the intended model on authenticity of modelling items in standardised test situations, (a) the current state of theory was reappraised, (b) standardised as well as informal discussions with experts were held and finally (c) an adapted theory-based model on authenticity was derived. For piloting this model, authenticity of overall 30 mathematical modelling test items from VERA (standardises performance test for evaluating the quality of math education in Germany) has been assessed by two independent raters based on a structured coding manual. In our presentation we will present the qualitative as well as quantitative analysis of the results of the piloting study.

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TASKS DESIGN TO ENCOURAGE AND FACILITATE MODELLING SKILLS AND ENGAGEMENT IN MATHEMATICS LEARNING

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This proposal focuses on the design of modelling tasks through two cases aimed at future Engineers. These cases aim to support the learning of mathematical notions such as Differential Equations (DE) in an undergraduate course of Engineering in a private university in Mexico, in such a way that it allows the student to know more closely approach this object of knowledge. This approximation allows the student to build meanings of the structure itself and the use of the DE notion in problems of social contexts. In previous research (Rodriguez & Bourguet, 2015), a high failure rate is reported on this topic but also an educational proposal to incorporate modelling tasks through system dynamics approach, this work is a continuation of this one. The intention is to complement the approach deeply developed by Fisher (2020, 2021). It is also intended to contribute toward this direction in the scientific training of engineers and citizens in general. The design of these tasks in specific contexts aims to improve the subject-object relationship in the construction of mathematical knowledge. In this research, we imagine with the students decision-making contexts of near and possible futures in post-pandemic times and in a digitalized society that offer hope and encourage entrepreneurship for economic development focused on solutions to social problems.

The purposes of context design are to encourage subject engagement and to facilitate harmonious closeness with the mathematical object (DE), in addition to develop modelling skills. The cases present different degree of difficulty and complexity, for example, it is asked to decide in a structured problem, about a decision to switch (or not) to a new type of employment, but also promote the inclusion of disabled people in the workplace. We also present a structure that we have envisioned for the design of tasks (cases) inspired by the case study methodology proposed by Harvard. The research will have a qualitative approach with a high emphasis on the part of the study of the evolution of critical and systemic thinking when students face this type of scenario during a Mathematics class. The tasks designed provide conditions to individually reflect on the meaning of the parameters, the structure of the differential equation and its solution within the framework of a socio-technological problem of a certain complexity.

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MATHEMATICAL MODELLING AS A WAY OF SEEING THE WORLD

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Modelling has been a way of promoting mathematics learning under the assumption of developing abilities and competences that, on the one hand, are shown to be powerful sources of understanding and insertion of students in the use of symbolic and formal mathematical systems, through the construction of mathematical models (Swan et al., 2007) and, on the other hand, enable a reading, even though partial and idiosyncratic, of reality through mathematics (Sousa & Tortola, 2021). In this context, mathematical models play an important role in assigning meaning to the facts of the world and to the mathematical concepts, and the research community has sought to clarify and connect frameworks, roles and uses of mathematical models (Doerr & English, 2003, Carreira & Baioa, 2018).

In this paper, we discuss modelling and mathematical models from a philosophical perspective that consider the concept of *use, language games*, and *ways of seeing* from Wittgenstein (2009), and a pragmatic perspective of knowledge that deals with the epistemic activity of constituting meaning based on the construction of meaning rules (Moreno, 1995, 2018). Concerned about mathematical models use in modelling tasks, we investigate the question "how the construction of models in modelling activities enable the constitution of a way of seeing the world?".

To support our reflections, we bring to the debate four modelling activities performed by different students group in Mathematics Degree. Data analysis were carried out from students' reports and teacher's class notes, and our methodology is based on qualitative research. Results point out to a reflection on how mathematical models uses promote and format ways of seeing the world through modelling. Through different practices, it is possible to reflect on different levels of schooling and the epistemic activity resulting from modelling process based on the articulations promoted between mathematics and reality, or sub-realities, when the students investigated topics of their interest, such as the Covid-19 pandemic, organizing a suitcase to stay within the allowed weight limit, launching a rocket, or even, locating a portal within a game.

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PRE-SERVICE UPPER SECONDARY TEACHERS' SPONTANEOUS VIEWS ON MATHEMATICAL MODELS AND MODELLING

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How teachers conceptualise and understand mathematical models and modelling can be a barrier (Burkhardt, 2006) or a facilitator (Blum, 2015) for the implementation of modelling in the classrooms. In other words, it is important to provide pre-service teachers with a nuanced and rich understanding of mathematical models and modelling, and their potential for mathematics teaching and learning. In order to support pre-service-teachers in developing such an understanding in an effective way, this paper investigates the spontaneous views on mathematical models and modelling held by 69 pre-service upper secondary teachers who were about to start a course focusing on the educational aspects of models and modelling in their third year of teacher training.

The analysis is based on a phenomenographic quantitative research paradigm (Marton, 1981; 1986) and uses open- and axial coding (cf. Strauss & Corbin, 1990) to analyse the 69 pre-service upper secondary students' written responses to six questions, with the aim of identifying and describing qualitatively different attitudes and beliefs about the notions, uses, functions, and roles of mathematical models and modelling. The six questions answered by the students were: (1) *What is a model?*; (2) *What is a mathematical model?*; (3) *What does it mean to engage in (mathematical) modelling?*; (4) *What are the functions and roles of (mathematical) models in different contexts and situations?*; (5) *Are (mathematical) models and modelling important content to know about and learn how to use? Why/why not?*; and (6) How much *emphasis and what role should modelling to have in the teaching and learning of mathematics in schools?* The results of the analysis will be discussed in relation to the existing literature in the field, and in particular in relation to and in contrast to the *cognitive*- and *affective shaped beliefs* and the four *types of ideal modellers* discussed by Maaß (2015).

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TEACHING ACCOMODATIONS: FACILITATING CLASSROOM INTERACTIONS AROUND FERMI PROBLEMS

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In recent years, the use of Fermi problems in mathematics education has gained popularity due to their perceived capacity for engaging students in reasoning and logical thinking (Barahmeh, Hamad & Barahmeh, 2017). Arlebäck (2009) positioned Fermi problems as a viable venue for bridging students' work on fermi tasks and mathematical modeling process, acknowledging that research is needed to better understand how students' work around Fermi tasks might be organized so modeling competencies are fostered in the course of their implementation. Relying on a teaching experiment methodology and using interactional analytical techniques (Schütte, Friesen & Jung , 2019) we examined, over two instructional sessions, teacher-student interactions in a pre-calculus course as students worked on a Fermi problem. The task asked: How many steps would it take to burn off an entire bag of potato chips?

Dirks and Edge (1983, cited in Ärlebäck 2009) argued that successful work on Fermi tasks requires the individuals to possess "sufficient understanding of the problem to decide what data might be useful in solving it, insight to conceive of useful simplifying assumptions, an ability to estimate relevant physical quantities, and some specific scientific knowledge" (p. 602). In reporting the results of using Fermi problems in secondary mathematics classrooms in Sweden, Ärlebäck highlighted the critical role that students' personal knowledge played in not only the assumptions they made once encountering Fermi problems but also in how they validated their answers. Ärlebäck (2009) cautioned that group composition and participants' preferences can strongly influence the variables they consider and assumptions they make, thus influencing the accuracy of models produced. In the sessions we examined, teacher interventions became central in focusing learners' work along some of the cognitive and social dimensions identified by Ärlebäck (2009). While in some cases the interventions were motivated by the teacher's perceived need to equalize the social capital of the participants in small groups, a majority of his comments aimed to provoke reflection, encouraging students to make connections among various answers obtained, addressing their implicit assumptions by introducing new constructs (age, gender, metabolism, height and weight of individuals), and confronting some false scientific assumptions they made (i.e sweating means more calories are burnt). As such the implicit assumptions that the learners constructed became a source for group deliberation.

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IMMC-SPAIN: AN OPPORTUNITY TO INTRODUCE MATHEMATICAL MODELLING IN THE CLASSROOMS

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In recent decades, the growing interest in the modelling approach to mathematics education has increasingly been transferred to the design of educational programmes. In the case of Spain, modelling appears as one of the mathematical competences to be developed by students at secondary level (grades 7-12) in the new education law (Ministry of Education and Vocational Training, 2022). As Garfunkel et al. (2021) point out, mathematical modelling can be integrated into teaching in several ways: in the "normal" classroom setting, or in the form of extracurricular activities with or without direct teacher support.

In this contribution we would like to describe the development of one of these extracurricular activities, present the design and analyse the results of the IMMC-Spain competition. This national competition for teams of students, which allows to choose the two teams participating in the International Mathematical Modelling Challenge organised by the Consortium for Mathematics and its Application, has as its central objective to promote the teaching of mathematical modelling and applications in secondary education. To date, four editions of IMMC-Spain have been held and more than 500 students have been involved in the competition. It is also worth highlighting the good results achieved in the international competition by some of the selected teams.

The key aspects of both the design of the competition and the modelling activity encouraged will be highlighted in the presentation. On the one hand, we will describe the process and criteria used for the design of the problems proposed in the IMMC. On the other hand, we will show the elements that characterise the solutions proposed by the students. We will also analyse the participation data and the performance of the teams. In this analysis, the gender perspective will be considered as a variable since, as mentioned in Boaler et al. (2019), a higher proportion of girls participate in this type of mathematical activity than in other individual competitions such as the Maths Olympiads.

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STUDENT EXPERIENCES WITH CONNECTING CONTEXT AND MATHEMATICS WHILE MATHEMATICALLY MODELLING

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Connecting mathematics and contexts is a fundamental requirement when mathematically modelling. However, connecting mathematics and contexts appears to be an unfamiliar experience for mathematics students. This study looked at the research question: What are the student experiences for connecting mathematics and contexts while participating in mathematical modelling? To gain insight into the student experience, tertiary student mathematical modelling experiences with modelling activities from three different New Zealand university modelling courses were looked at. The modelling activities across the three courses used different contexts specific to each course. Data was collected through twenty individual semi-structured interviews with the students. Reflective thematic analysis was used to identify themes in students' experiences with connecting mathematics and contexts within the mathematical modelling activities. The results of the study showed: connecting mathematics and contexts was new for students; students had different experiences depending on their mathematics background, first language, and previous experience with applied mathematics subjects; and for some students connecting mathematics with contexts made mathematics more relatable. The results of the study will be used to inform developing teaching practices that enable students to successful connect mathematics with contexts.

TRAINING PROSPECTIVE TEACHERS' DIDACTICAL PERSPECTIVES OF USING GEOGEBRA FOR MATHEMATICAL MODELLING

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Dynamic Geometry Software (DGS) can be used in teacher education regarding mathematical modelling with two purposes, to help prospective teachers (PTs) construct interactivity connected to mathematical representations (Arzarello, et al., 2012), and to let PTs analyse the benefits of DGS in the modelling activities (Hernández, et al., 2020; Kokol-Voljc, 2007). However, most empirical studies reporting the impact of DGS in teacher education focus on PTs' mathematical learning of modelling, and not on PTs' didactical analysis of the use of DGS in mathematics classrooms.

We report a case study where prospective secondary school teachers consider and discuss mathematical and didactical aspects of applying GeoGebra within modelling activities. The context is a course at the end of the first year of our teacher education programme. The course objectives include presenting current contents in secondary school mathematics courses, evaluating PTs' own teaching and their pupils' learning, and using educational tools to illustrate mathematical concepts. The study reports from the first of two seminars conducted by Zoom with nine PTs in two hours. The seminar was recorded. The seminar started with an overview of how the Swedish national curriculum describes the use of GeoGebra in upper/lower secondary school and discussed the underlying reasons for the use of DGS. Then the PTs presented their own experiences using GeoGebra and discussed their perceptions about possibilities and disadvantages using GeoGebra in mathematics lessons ("pupils can even explore a new concept−not only solve a problem by using GeoGebra". "Comparing to analogical work by drawing on paper, pupils can be confused when they put misleading data and get erroneous graphs", etc.). The PTs then discussed the task of creating maximum and minimum area of a rectangular pasture with a 100 m rope stretched against the shore of a lake. This task was constructed in such a way that it could be solved without using derivatives. First, the PTs intuitively guessed the maximum area, which was correct, and drew a graph with GeoGebra using a file available on the course website. They showed the maximum point on the curve of the quadratic graph and concluded that the minimum area is indeterminate. When asked to solve the same problem for an arbitrary numerical value, e.g., 103 m, they cannot guess the solution intuitively and realised that algebraic methods are needed to find the exact solution. In this way, the PTs received factors to consider the dynamics of DGS as a didactical tool, while the importance of giving pupils the opportunity to solve modelling tasks in a mathematical way was also highlighted.

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WHEN DIGITAL TOOLS BECOME A SUPPORT FOR MODELLING ACTIVITIES: PRE-SERVICE TEACHERS' INSTRUCTIONAL TASKS

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In recent years, digital tools have had an increasingly preponderant and diverse role in teaching and learning of mathematics, particularly in modelling. Exploring the implications of technology in the study of phenomena is relevant because not only the most of our activities are permeated by different digital tools, but also because the frequent complexity or inaccessibility to phenomena requires the mediation of technology to get closer to them. In the case of modelling, there are at least two ways of conceptualizing the role of technology. On the one hand, as internal support for the transit between the phases of the modelling process and on the other hand as an additional domain to the world of mathematics (Greefrath, Hertleif, Siller, 2018).

From the experience of analyzing the design of tasks and their implementation in the classroom (Guerrero-Ortiz, 2021; Guerrero-Ortiz & Camacho-Machín, 2022), some uses of technology have been identified, these suggest an alternative perspective to the schematic representation of known modelling processes. This work presents results of analyzing the implementation of modelling tasks in the classroom. The tasks were developed by secondary mathematics preservice teachers. In the tasks, access to the situation was facilitated through technological tools. Digital tools were also used as scaffolding the modelling process. In the students' work, the reality perceived by them was mediated by the characteristics of the technology; therefore, processes such as the interpretation and mental representation of reality can be different.

It is argued that the mathematical modelling in the classroom is not the same as that developed by expert modelers, just as the modelling process that teachers develop in the design of tasks is not the same as the one that students are encouraged to do as a result of working with the designed task (Guerrero-Ortiz & Camacho-Machín, 2022). Therefore, the approach to reality that each one experience is different. In a teaching context, students' experiences, the classroom environment, and the tools used shape how they perceive task contexts. Therefore, the characteristics of the tasks that involve the use of digital tools shape how individuals approach the proposed situation and how the modelling processes are developed.

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A THEORETICAL MODEL FOR DESCRIBING TECHNOLOGICAL, PEDAGOGICAL, MATHEMATICAL AND EXTRA-MATHEMATICAL KNOWLEDGE FOR TEACHING MODELLING

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A theoretical proposal that considers elements of the TPACK model such as the technology, pedagogy, and content knowledge and the proposal on teacher knowledge for teaching mathematical modelling (Wess et al., 2021) is discussed here. The TPACK model from Mishra and Koehler (2006) is constituted from three basic sources of knowledge: technological knowledge (TK), pedagogical knowledge (PK) and content knowledge (CK). While Greefrath (2021) suggests that the teaching of mathematical modelling requires specific pedagogical knowledge. He and his colleagues structure a model involving four dimensions of knowledge: interventions, modelling processes, modelling tasks and, goals and perspectives. The TPACK model considers pedagogical and technological content knowledge, without special attention to mathematics and not attention to mathematical modelling, while the proposal by Greefrath et al. (2021) considers pedagogical aspects for teaching modelling, without emphasis on the use of technology. To explain the knowledge that the teacher must have to teach modelling with technology, a theoretical model (TPMMK) is proposed here. This model highlights the interactions between mathematical knowledge, didactic knowledge, technological knowledge and extra-mathematical knowledge (Figure 1). The model can represent the knowledge for the teaching of interdisciplinary work with the use of technology. In area 1 we find mathematical modelling as an element linking mathematical and extra-mathematical knowledge, which may or may not require the use of technology (2.1 and 2.2). And by including knowledge for teaching mathematics and about modelling, we structure what we call an integrated knowledge TPMMK (3). This theoretical proposal is based on the literature and now is being validated with empirical data and through expert judgement (Escobar and Cuervo, 2008).

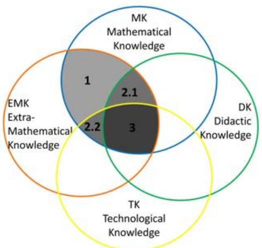

Figure 1. Dimensions of the TPMMK Conceptual Model

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USING AN INSTRUCTIONAL VIDEO TO SUPPORT UPPER SECONDARY STUDENTS IN CREATING A MATHEMATICAL MODEL

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Mathematical modeling is a challenge for many students since each step of the modeling process is a potential barrier (Galbraith & Stillman, 2006). To support students' modeling processes, various approaches have been developed, including the use of heuristic worked examples which display an expert's realistic solution process to a modeling problem. Based on research that text-based heuristic worked examples have been found to be beneficial in supporting students' modeling processes (Zöttl et al., 2010), we implement video-based heuristic worked examples because videos offer various advantages including that the application of strategies can be presented dynamically. Moreover, using videos to display the modeling problem offers the potential of helping students to make predictions (Cevikbas et al., 2023). Because understanding the problem is a major starting challenge and mathematizing the structured real-world problem is often one of the hardest parts for students (Galbraith & Stillman, 2006), we are focusing on the following research question: What changes in upper secondary students' creation of a mathematical model can be observed after they have worked with an instructional video on a modeling problem?

In order to gain insights into students' modeling processes, we videotaped four pairs of upper secondary students. Each pair collaboratively worked on a modeling problem, an instructional video on a second modeling problem, and a third modeling problem. The data from the solution processes and a subsequent interview focusing on how the video supported the working processes during the third modeling problem were analyzed using qualitative content analysis (Kuckartz, 2014). The results indicate that the instructional video offered students confidence in dealing with missing values and broadened their strategy repertoire in making assumptions, thereby supporting the students in the process of understanding and simplifying. We also found that the process of mathematizing was improved because some students chose a more adequate mathematical model than in the first modeling problem. Our findings suggest that instructional videos are a promising format for providing heuristic worked examples and may offer an innovative approach for supporting students in creating a mathematical model.

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FLIPPING UNIVERSITY-BASED MATHEMATICAL MODELLING SEMINARS – INSIGHTS FROM PRE-SERVICE MATHEMATICS TEACHERS

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Mathematical modelling empowers learners' critical thinking, prepares them for responsible citizenship in societal developments and promotes them to be effective problem solver (Blum, 2011). Mathematical modelling takes an important place in various national curricula (Borromeo Ferri, 2018). However, it is a complex subject that can be challenging to teach and learn and employing innovative approaches in modelling is necessary to enhance mathematical modelling competence (Cevikbas et al., 2022, 2023). The flipped classroom (FC) approach including the use of explanatory videos can be a promising pedagogy to enhance mathematical modelling education maximizing class hours for modelling activities and promoting student engagement, learning motivation, and achievement (Bergmann & Sams, 2012). In this study, we explore the value of using this innovative approach to teach and learn mathematical modelling from the perspective of pre-service mathematics teachers, including its benefits and challenges for learners and instructors. We flipped two mathematical modelling seminars at a large German university and gathered data through an online questionnaire including open-ended questions and semi-structured interviews from 42 pre-service teachers at master's level. The results of this qualitative study showed that the use of explanatory videos and the FC model can improve the understanding of mathematical modelling and promote engagement, collaboration, and modelling competence. However, there are also pitfalls associated with FC, including technological constraints, difficulties with self-directed learning, and inadequate preparation for in-class modelling activities. Pre-service mathematics teachers expressed positive attitudes towards flipping modelling instruction, but they highlighted that especially creating new content such as explanatory videos was a time-consuming activity for them. The results underscore the need to incorporate the FC pedagogy in teacher education programs to prepare pre-service mathematics teachers to effectively teach mathematical modelling in their professional life using technology-supported emerging approaches.

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MODELLING UNCERTAIN FUTURES: PROBLEMS, METHODS, TOOLS

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A previous ICTMA presentation addressed a gap in student knowledge of modelling identified through a problem set in the International Mathematical Modeling challenge 2019 (Garfunkel, et.al. , (2021). It involved estimating the future carrying capacity of planet Earth, and no student team entered in this world-wide competition had the resources to approach the problem adequately. Such problems contain non-linearities whereby solutions can only be approached through simulation. The earlier presentation illustrated the type of approach needed using System Dynamics. This presentation extends its predecessor by adopting the goal of pursuing possible solution(s) to the problem as set.

In doing so ancillary educational implications are considered within a methodological framework. These include, on the one hand, implications for teacher education in terms of an approach integrating mathematics with software. Additionally there is information available from classroom experience of the introduction of such material in secondary school classes, Fisher (2018).

Professional estimates of carrying capacity in terms of population have typically considered the depletion of interacting and depleting resources. This different approach engages issues that are of increasing concern to human society, involving adults and young people alike e.g., global warming.

Predictions of the future population of Earth have been made by several organisations e.g., the United Nations. These are based heavily on assumptions about future values of birth-rates treated seemingly in isolation from other factors. Articles and research reports continue to conjecture about the impact of increasing global temperatures on human welfare and industrial production. But the potential impact of interactions of these two quantities with each other and with population are not evident in the predictions. The approach here is to develop such scenarios, both in terms of assumptions made for models, and in the meaning, interpretation, and implications of outcomes.

The presentation will engage with material drawn from the following content: Modelling problems where simulation is essential; purpose and structure of system dynamics models; development of a basic model built around interactions between population, industrial growth, and global temperature; interpretation and implications of alternative scenarios; classroom experience with SD modelling.

Output from the model will be discussed relating to areas such as the following: potential impact of different levels of industrial growth; comparisons between population projections with and without assumed impacts from changing global temperatures; the relative effectiveness of different policies suggested for mitigating the effects of global warming.

The significance of such modelling relates to expressions of purpose found in many national curriculum statements: a goal of mathematics education being to equip students with competence that extends into the future and beyond the classroom - for the workplace, for individual undertakings, and as responsible citizens. We would include the word "global" in describing the last of these.

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STARTING MATHEMATICAL MODELLING FROM EXISTING MODELS

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The mathematical modelling process starts in the "unedited" or "messy" real world, is worked upon mathematically by modelers, and then is transformed back into the real-world context (e.g., Pollak, 2012). However, identifying a meaningful "starting point" or developing a quality mathematical modelling activity that is cognitively demanding is non-trivial (Borromeo Ferri, 2018, p. 41). Motivated by the need to provide an accessible, yet holistic math modelling experience for students new to modelling, we consider an alternative starting point for the mathematical modelling process that supports students in "translating between the real world and mathematics in both directions" (Blum & Borromeo Ferri, 2009). In this study we present students with an existing model, asking them to analyse the given model, and then develop a revised model that better represents the real-world context. Specifically, we are interested in the following: what does it mean for a modelling task to start with an existing model, and how does it impact students' modelling process?

This study is part of the M2Studio project, which aims to develop a new technology for learning mathematical modelling and a compatible curriculum. Here, we focus on the last activity in the curriculum, which provided students the opportunity to engage in the modelling process through an existing model. For this task, we presented a situation that one of our colleagues has faced; whether it is worthwhile to move to a cheaper apartment that is further away from her workplace. Our colleague (the client) shares her existing cost-focused model, but she is unsure of other factors that she should (or could) consider. The students are asked to revise her model to help her better understand her situation and ultimately make a decision. We note that the task, initiated with an existing model, satisfies characteristics associated with modelling tasks (Maaß, 2007).

In this talk, we present findings from two groups consisting of two students each. Data from video recordings reveal that students initially analysed the model by interpreting and validating aspects of the existing model to identify what needed to be improved. They then continued to move throughout the modelling process; creating sub questions and making additional choices to develop revised models. Notably, students in this study repeated this process, constantly interpreting and validating how their current model compared with the existing model and what further revisions may be needed to ensure they developed a model that better represents the client's situation.

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DIFFERENCES IN INTERPRETATION OF COMPUTER RESULTS IN THE MODELLING CYCLE WITH ADDED COMPUTER MODEL: THROUGH THE MODELLING WORKSHOP ON THE PREMISE OF USING SCIENTIFIC CALCULATOR

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One of the Scientific calculator's function "Regression Calc" displays the results of the regression calculations based on entered data. We conduct preliminary experiment to measure the distance from the projector lens to the projection screen using tape measure, and the brightness of the projection screen using lux meter. We collect the data obtained from the experiment, and inquiry the relationship between the measured distances and the brightness using "Regression Calc" (Yamamoto et al., 2022).

In the modelling cycle with added computer model (Greefrath, 2011), the experiment is real situation. A summary of the data on the distance from the projector lens to the screen and the brightness of projection screen is real model. The function of scientific calculator "Regression Calc" is computer model, and the approximation formulas and correlation coefficients obtained using "Regression Calc" are computer results. The research question is as follow: What are differences in the modeller's interpretations of computer results?

We plan and conduct a modelling workshop using scientific calculator and digital tools for 59 undergraduate Japanese students (Yamamoto et al., 2023). The students predicted the relationship between the distance from the projector lens to the screen and the brightness of projection screen. The experiment was conducted to measure the distance from the projector lens to the projection screen using tape measure, and the brightness of the projection screen using lux meter. Based on the date obtained from the experiment, the students explored the relationship using "Regression Calc" of scientific calculator. The students' descriptions about the relationship can be classified four types as follows: (i) proportion (5 students), (ii) inverse proportion (14 students), (iii) inverse proportion to the square (39 students), (iv) could not find relationship (1 student).

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CHANGES OF MATHEMATICAL MODELS THAT CAN BE REPRESENTED BY CHANGING THE SLIDER TOOL IN GEOGEBRA: FOCUS ON THE TRAJECTORY OF BASKETBALL IN ONE SCENE OF THE MANGA "THE BASKETBALL WHICH KUROKO PLAYS"

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The slider tool in GeoGebra can change variable numbers in functions easily. Namigata (2017) develops teaching materials on the subject of the manga "The Basketball Which Kuroko Plays" using GeoGebra. In this teaching materials, he focuses on the trajectory of the ball when a shooter shoots a three-point shot from a certain position, the trajectory of the ball when the shooter shoots a three-point shot changing the position, and the parabola focused on the ball velocity. Tachihara (2022) represents the trajectory of the ball using differential equations when the shooter shoots a three-point shot changing the position. We succeed to change an initial velocity of the ball that is a variable number of differential equations using GeoGebra and represent a mathematical model based on the three cases focused by Namigata (2017). We set a research question as follow: How can mathematical models be represented from dynamic graphs displayed on GeoGebra?

In the modelling cycle with added computer model (Greefrath, 2011), one scene of a three-point shot in the manga "The Basketball which Kuroko plays" is real situation, the two trajectories and the one parabola are real models. The parabolic equation that the shooter shoots a three-point shot changing the position $y = -\frac{g}{2{v_0}^2\cos^2\theta}x^2 + \tan\theta \cdot x + 3$ is mathematical model (Tachihara, 2022). The slider tool in GeoGebra that changes variable numbers is computer model. The dynamic graphs displayed in GeoGebra by changing variable numbers are computer results. Therefore, the maximum position of the range of a three-point shot and the range of the initial velocity of the ball are mathematical results. The ball's angle of incidence to the goal ring is set at 70 degrees (Namigata, 2017). Fixing gravitational acceleration g = 9.8 and $\theta = \frac{70\pi}{180}$, a new mathematical model $y = -\frac{9.8}{2{v_0}^2\cos^2\frac{70\pi}{180}}x^2 + \tan\frac{70\pi}{180} \cdot x + 3$ is represented. As a result, a range of the initial velocity of the ball is represented.

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IMPROVEMENT TO THE MATHEMATICAL MODELLING LESSON BY INEXPERIENCED SECONDARY SCHOOL MATHEMATICS TEACHER

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The aim of this study is to identify what is needed for inexperienced secondary school mathematics teacher who are conducting inadequate mathematics lesson as mathematical modelling to improve their own lesson towards the mathematical modelling lesson. All secondary school mathematics teachers in Japan have experience of conducting mathematics lessons on real-world problems, as textbooks contain teaching materials that cover real-world problems. However, these are "word problems", pointed out by Niss & Blum(2020), and lesson that deal with such problems as they are cannot be called mathematical modelling lesson. How, then, can mathematics teacher who conduct such lessons improve them towards mathematical modelling lesson?

In this study, “expansive learning" proposed by Engeström, Y.(2001) is used to help mathematics teacher who are teaching mathematics lesson with "word problems" to improve towards the mathematical modelling lesson. In this study, the following methods were implemented based on "the cycle of expansive learning" he proposed.

(1) Interviews were conducted with target teachers to clarify the challenges of dealing with real-world problems in mathematics lessons. (2) Based on the issues identified in (1), the target teachers were given experience in mathematical modelling suitable for them and asked to analyse the differences between the mathematical modelling process and teaching process of their own previous mathematical lessons of dealing with real-world problems. (3) Have them devise measures to overcome the issues identified in (1) and to compensate for the inadequacies of conventional teaching. (4) Have the target teachers design and implement the lesson that incorporate the strategies in (3). (5) Have the target teachers analyze and discuss the lessons implemented in (4).

In this study, four secondary school mathematics teachers with less than three years' experience and no previous experience of teaching mathematical modelling were targeted. The reason for selecting inexperienced mathematics teachers was that, due to their inexperience, we thought they had a strong desire to improve their lessons.

The results of the study showed that when the teaching process conducted by the four teachers were compared with their previous teaching process, improvement to the mathematical modelling lesson were observed for three of the teachers. However, one teacher had not been able to improve. The requirements of improvement to the mathematical modelling lesson were clarified from the results of interviews with the three teachers who showed improvement, their modelling experiences and their improved lesson.

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MATHEMATICAL WORK IN A MODELLING TASK: CIRCULATIONS AND TYPES OF MODELS

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The identification of routes for solving mathematical modeling tasks from an individualized approach allows understanding the learning processes in accordance with the characteristics of the subjects. There are successful examples in the literature that directly nurture the valorization of modeling tasks. Considering the task "The Assignment" by Reilly (2017), the present study performs an adaptation aimed at secondary level (15-17 years) and higher students of mathematics teacher training in Chile, with the purpose of characterizing the types of mathematical models and the circulations of epistemological and cognitive planes that are developed from a modeling task. The task is:

> You are building a house plan for your family, with the bedrooms and bathrooms you want. You have to worry about the economic cost of the interior of the house, specifically the floor and walls, how much would it cost? Next, you must design the floor plan of two types of houses (one small and one large), where one of them is the house you have initially designed. How many large houses could you build to spend the same, or less, on floor and walls? or less on floors and walls than you would spend on 50 small houses?

To analyze the task and its development, we consider the theoretical framework of Mathematical Working Spaces (MWS) (Kuzniak et al., 2022), which allows, from the articulation between the epistemological and cognitive levels, to characterize the existing circulations. With a qualitative methodology and interpretative approach, the task has been adjusted from the validation by five expert researchers in didactics of mathematics, and subsequently piloted in secondary school students. Then, a case study has been defined in students of secondary education and Initial Teacher Training.

The preliminary results show that the task has been understood by all the individuals to whom it has been applied. Particularly, in the case presented, the activation of the semiotic genesis is observed, where the student visualizes and designs a plan, then provides the values required for the construction of the house. Finally, he poses an equation, resorting to his referential and estimates a final value as a solution. The data analyzed lack rigor, and there are slight similarities with the a priori analysis performed. On the other hand, the mathematical models used by the students are identified in a figurative semiotics, being of interest to perform a more exhaustive data collection, in order to obtain the multiplicity of alternatives foreseen in the a priori analysis and thus complement the preliminary results in the final version of this report.

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IMPLICATIONS OF *LEARNING THROUGH* FOR *TEACHING USING* MATHEMATICAL MODELLING

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Education in mathematical modelling has been designed mainly on pragmatic and theoretical-epistemological aspects, which aim to structure cycles to guide the modelling practice (Boromeo Ferri, 2018). Based on these aspects, we have been investigating and configuring education designs organised around three axes: learning about modelling, learning through modelling and teaching using modelling. Teacher education based on these axes has been undertaken to prepare them to know aspects related to modelling and how to do it and how to define appropriate methods to teach using modelling (Borromeo Ferri, 2018).

In 2022, nine in-service teachers participated in a course in which they developed a modelling activity. The discussions about that activity fostered an approach directed towards the educational context. When we were analysing the discussions, we evidenced characteristics such as those pointed out by Barbosa (2010), who, based on modelling routes, categorised the students' mathematical, technical and reflective discussions in the construction of mathematical models. Notably, in a transversal way, we evidenced the pedagogical character in the discussions so that the teachers, when learning through modelling, related aspects that could be implemented to teach using modelling.

Inspired by these discussions, we investigated the questions: How do in-service teachers become aware of actions to teach using modelling when they develop an activity as modellers? What does this process of awareness reveal? The activity development, which was recorded in audio and video, together with the activity development reports and a collaborative planning prepared by the teachers made up the case study we analysed, according to the Content Analysis (Bardin, 2013).

The results indicate that the awareness to teach using modelling is present in what we call pedagogical discussions that were transversal to the other discussions pointed out by Barbosa (2010) and they were configured in the following categories: clarifying the definition of the problem, structuring data collection, considering a relevant mathematical approach within reach of students; and enabling the validation of the results. In addition, they indicate the need for the three education axes to be worked on in an articulated way so that learning about modelling is fundamental for teachers to recognise their actions through the cycles, and to be able to carry them out intentionally, pedagogically speaking, while developing modelling activities and, in this context, envisage actions that can provide subsidies for the constitution of a didactics to teach using modelling.

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TRANSIENT PHENOMENA AS MATHEMATICAL MODELLING MATERIALS IN SCIENCE EDUCATION

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In science and physics education, it is essential to formulate phenomena mathematically in order to quantitatively understand natural phenomena and their properties and regularities. Understanding through experimentation and observation requires a process of "discovering" laws based on experimental results. In this respect, the methodology of mathematical modeling is effective, and understanding can be deepened by repeating the process of formulating a hypothesis, planning and conducting experiments, formulating and interpreting the results, and discussing the results. The role of mathematical modeling in science and physics education is to deepen understanding of phenomena and to enhance observation and discussion (Michelsen, 2015). In mathematics education, on the other hand, mathematical modeling contributes to understanding the relationship between natural and social phenomena, for example, visualizing and interpreting equations and their solutions as phenomena (Burghes & Borrie, 1982). In addition, the importance of modeling physical phenomena represented by exponential functions is noted (Pisano & Bussotti, 2012).

This paper focuses on applications of the integral method in high school mathematics. In this unit, students will study initial value problems for the simplest differential equations $dy/dx = ky, y(0) = y_0$ for the variable $y(x)$, where k is a positive or negative constant. Differential equations of this type are widely found in natural and social phenomena and, depending on the sign of the coefficients, can either relax to equilibrium values or explode to infinity. A slight change in the right-hand side of this differential equation can also be used to describe biological phenomena, such as allosteric effects, for example. This is expected to enrich the content of mathematics education by connecting it with natural phenomena. As for physics education, it provides effective clues for mathematical understanding of transient phenomena such as thermal equilibrium, and enables students to "discover" the laws of phenomena and to deepen their physical interpretation through modelling procedures.

In this study, we propose a mathematical modeling method of transient phenomena from the viewpoint of physics education and its application to teaching materials. In particular, the author shows that experimentally "discovering" Newton's cooling law in thermal equilibrium by measurement, formulating it as a differential equation, and physically interpreting it can lead to a deeper understanding of transient phenomena compared to conventional learning. The experiments and discussions are effective in mathematics education and serves to utilize the research content.

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AN ANALYSIS OF THE THEORETICAL FRAMEWORK FOR NUMERICAL COGNITION IN ZAMBIA

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It highlights the importance of basic education in African countries, particularly due to their low academic performance. To deal with the challenges, the need for accurate assessment tools to understand the conditions of developing countries include Zambia and enhance instruction is emphasized (World Bank, 2018; Baba et al., 2021). While children in lower grades show partial understanding and varying mastery up to two digits, expanding the learning units to include three digits and decimals is crucial for further improvement (Uchida, 2012; Nakawa et.al., 2019). However, the literature on decimals or fractions is limited, with insufficient discussion regarding the knowledge children usage to comprehend them (Alcock et.al., 2016).

The objective of this study is to evaluate the reliability of the theoretical framework of numerical cognition in developing assessment tools specific to Zambia.

The integration of conceptual and procedural knowledge is identified as a potential challenge that may impact students' understanding of decimals (Hiebert & Lefever, 1986; Carpenter, 1986). The significance of structural knowledge for mastering decimals is also emphasised (Annette, 1998). The relationships among three types of knowledge are defined, and corresponding tasks that are frequently used in surveys of math competencies and correspond to concepts. (including T1; number identification, T2; object matching, T3; mental number lines, T4;pattern recognition, T5; composition/decomposition, and T6; calculations) (Fig.1).

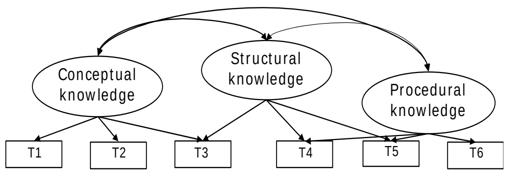

Fig. 1 Hypothesis theoretical framework

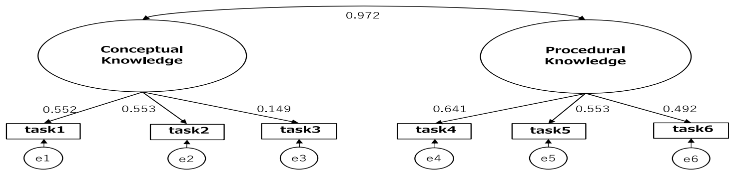

Fig. 2 Correlation among revised theoretical framework in natural number only

To evaluate this hypothetical theory by mathematical modelling. According to Ikeda (1999), mathematical modelling is an activity in which actual problems are mathematized to create a mathematical model, which is then interpreted and checked to then find suitable mathematical model.

I analysed the test results of approximately 150 Zambian elementary school students in grades 6 and 7 using the structural equation modelling. As a result, **the Fig1 did not fit well.** Alternatively, when **focusing on natural numbers only** and applying conceptual and procedural knowledge **(Fig.2), a better fit (p-value = 0.050)**. Consequently, it is concluded that the theoretical framework developed in this study is applicable only to natural numbers. I expect that limitations in adapting the framework to decimals are attributed to students' insufficient understanding and poor performance in specific tasks. To address these limitations, it is proposed to increase the sample size, modify tasks, and involve multiple schools. Furthermore, specific difficulties and typical errors were identified, leading to the need for redefining the theoretical framework based on these findings.

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CHARACTERISTICS OF THE MODELING PROBLEMS POSED BY PROSPECTIVE MATHEMATICS TEACHERS

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Mathematical modeling perspective is based on the premise of connecting mathematical world with the real-world (Blum & Niss, 1991; Lesh & Doerr, 2003). Hence, modeling process is based on contextual problems that do not require straightforward answers but construction of a model. Although modeling problems may differ regarding the modeling perspective they entail, they are contextual, ill-defined, and open-ended problems. For instance, models-and-modeling perspective suggests a genre of problems called MEAs that are often presented as realistic customer-driven situations and open to construction of multiple reasonable models (Lesh & Doerr, 2003). Another type of problems that modeling researchers studied are Fermi Problems given in estimation-based open-ended real-life situations (Albarracín, & Gorgorió, 2014; Ärlebäck, 2009). Furthermore, the modeling potentials of closed versus open real-world problems also took researchers' attention (Schukajlow et al., 2022). This body of research indicated that modeling problems shared some common aspects but also varied in some characteristics. In this study, 34 prospective mathematics teachers at a large public university in Turkiye worked in pairs to pose modeling problems addressing geometry topics. They were also asked to write at least one solution to the problem and reflect on the characteristics of the problems. Their modeling problems, solutions, and reflections were analyzed based on the cognitive demand, open-endedness, and contextual aspect of the problem. The study is still in the stage of on-going analysis; however preliminary results indicated that modeling problems posed by the prospective teachers involved multiple mathematical ideas while placing geometry as a central idea, and prospective teachers used relations between mathematics ideas to increase the level of cognitive demand. Furthermore, the open-endedness of the problem – being open to multiple correct solutions – helped them to pose problems requiring higher-level mathematical thinking. I believe that this analysis will expand our understanding of what potentials of modeling problems are considered by future teachers in posing modeling problems.

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FACILITATING PROBLEM POSING IN MATHEMATICAL MODELLING: THE CASE OF 12TH-GRADE STUDENTS

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Problem posing is crucial in mathematical modelling, but it has been found that it is not easy for students. For example, Hartmann et al. (2021) showed that approximately half of the problems posed by students were problems that had artificial connection to reality. We believe that this is influenced by the difficulty that students face in posing problems where they feel ownership of the problem. Therefore, the aim of the present study was to propose a strategy to reduce students' difficulty in problem posing in mathematical modelling, and to find out the effectiveness of problem posing using this strategy. We have prepared the following two strategies to pose problems in which students can have authentic connections to reality. The first strategy is to provide information about real-world situations that are close to students' lives and that facilitates the recall of specific situations such as personal, educational/occupational, and public situations (OECD, 2018). The second one is to provide tasks that allow students to pose authentic questions in real-world situations. This applies because ‘a question is authentic if it is one that people within the context would ask’ (Vos, 2018, p. 3).

We addressed the following question: *How do the two strategies support 12th-grade students' problem poising in mathematical modelling?* For this purpose, we designed and conducted a teaching experiment in which 37 12th-grade students repeatedly reformulated and solved textbook problems with real-world situations (i.e., using logarithmic function) by referring to information on real-world situations. First, the students reformulated the textbook problems by referring to information on food-poisoning bacteria. Secondly, they considered authentic questions based on new real-world situations (i.e., food poisoning in curries, plastic bottles, and packed lunches). Thirdly, they posed and solved the problems again, based on authentic questions that they had considered themselves. Finally, they modified the problems in new real-world situations presented by other students and advised by their biology teacher. We analysed the students' statements in the worksheets used in the teaching experiment.

The analysis showed that all students were able to pose authentic problems and that their mathematisation was richer in terms of specifying the assumptions necessary for the solution. Many students were also able to interpret and validate the results. These results indicate the effectiveness of two strategies in reducing students' difficulties in problem posing with mathematical modelling and in improving their mathematical modelling competencies.

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ENHANCING PRESERVICE TEACHERS REAL-WORLD MATHEMATICAL PROBLEM-POSING ABILITIES: DESIGNING LEARNING TRAJECTORY

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Fostering students' capabilities to utilize their knowledge, skills, and mathematical processes to solve real-world mathematical problems (RWMP) is a key objective outlined in the Thai Education Curriculum and also in many other countries (Klainin, 2015). Critical ways of encouraging students to solve RWMP related to preservice teachers' (PSTs) abilities to pose appropriate RWMP (Paolucci & Wessels, 2017). This research developed the hypothetical learning trajectory (HLT) described by Simon (1995) to enhance the RWMP-posing abilities of PSTs. Two research questions were addressed. 1) What are the key characteristics of the HLT that contribute to improving problem-posing abilities? 2) What are the RWMP posed by PSTs? The research methodology involved three phases of educational design research following Bakker (2018) as preparation and design, teaching experiments, and a retrospective study. The participants included collaborative educators and twenty-one PSTs engaged in problem-solving and problem-posing activities who were instructed on how to pose RWMP. Data were collected and analyzed using qualitative techniques such as interviews, observations, and content analysis. The results concluded that 1) the HLT consisted of two characteristics as a) implementing gradual problem-posing activities starting with simple strategy problem-posing, followed by solving worthwhile problems, practicing multiple approaches in problem-posing, and gradually incorporating free problem-posing with greater relevance to real-world contexts, and b) encouraging productive interaction among PSTs through group and discussion forums. The actual learning trajectories of most PSTs aligned with the HLT. Results also suggested that 2) most RWMP posed by PSTs focused on wrapper problems (Stillman, 1998) which had realistic context but lacked complexity and opportunities for assumption-making and seeking additional relevant information, both crucial criteria for modelling problems. These findings offer valuable insights for educators seeking to enhance the RWMP-posing abilities of PSTs by promoting effective learning regardless of their previous experience in mathematical modelling.

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THE RELATIONSHIP BETWEEN TEXT COMPREHENSION AND NOTE-TAKING WHILE WORKING ON REALITY-BASED TASKS

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Numerous empirical studies have demonstrated that the translation process between reality and mathematics while working on reality-based tasks is associated with cognitive barriers (Galbraith & Stillman, 2006) and that understanding real problem situations is a source of concern for students (Leiss et al., 2010). One promising strategy to overcome these difficulties is note-taking Graham et al., 2020; Rogiers et al., 2020). However, differential analyses on the design of notes and the specific influence of note-taking on the solution process while working on reality-based mathematical tasks represent a research desideratum.

The data on note-taking, including personal characteristics, were collected within the VAMPS-Project (Variation Tasks Mathematics Physics Language - funded by German Research Foundation). Tasks on three different topics (contexts) were varied at three levels of language complexity (Heine et al., 2018), resulting in a total of nine mathematical reality-based tasks. 424 students worked on each of the three contexts, generating 1 272 responses. These were then used to calculate correlations and multivariate regressions using SPSS and R Statistics.

Overall, unsolicited notes were made on just under one-third of the tasks completed. When counting any form of marking or formal structuring as a note, the figure is as high as 92.7 %. The initial results show that linguistically higher demands, increased reading enjoyment, and the use of different reading strategies are associated with increased note-taking. In contrast, there are only small effects related to the comprehension of the task text. Taking notes, however, is clearly related to the correct calculation path and thus to the correct solution. Another finding is that girls tend to take significantly more notes than boys and a higher socio-economic status also correlate with more notes of any kind. I am also pleased to be able to say results on the relationship between notes and performance at the conference.

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STUDENTS' PREFERENCES AND BELIEFS ABOUT THE OPENNESS OF MODELLING PROBLEMS

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One important characteristic of modelling problems is their openness (Niss & Blum, 2020). In an open problem, information that is essential for solving the problem is missing. Prior research revealed students' significant barriers in solving open modelling problems. Based on theoretical considerations and empirical findings (Galbraith & Stillman, 2001; Krawitz et al., 2018), we suggest the importance of the following processes in dealing with open problems: (1) noticing missing information, (2) identifying the unknown quantities needed to solve the problem, (3) making realistic assumptions about the missing quantities and (4) integrating open quantities in the mathematical model. In the randomized control trial carried out in the project OModA (Open Modelling Problems in mathematics teaching that is oriented toward self-regulation) we aimed to analyze the effects of teaching openness of modelling problems on students' learning outcomes. In our talk, we will focus on the effects on students' preferences for solving open problems and their beliefs about open problems and their relation to school grades. Students' preferences for solving open problems refer to the positive value of solving this problem type. Students' beliefs about open problems address the belief that a mathematical problem always includes all information needed for its solution. The research questions were: (1) Does teaching openness to modelling problems affect students' preferences and beliefs? (2) Are students' preferences and beliefs about open problems related to their school grades?

Two hundred and ninety-four German ninth graders (137 female) were randomly assigned within each class to two conditions. In the experimental condition (EC) students were taught to solve open modelling problems and in the control condition (CC) closed real-world problems. Before and after the teaching unit (4 lessons á 45 minutes) students ranked their preferences (4 items) and beliefs (1 item) on open modelling problems on a 5-point Likert-Scale and reported school grades.

As expected, teaching open modelling problems decreased students' belief that a mathematical problem always includes all information needed for its solution, compared to the CC. Results revealed a positive relationship between beliefs (but not preferences) and school grades in the pretest. Further, teaching openness to modelling problems tended to affect positively students' preference for solving open problems. One important practical implication is that teaching openness to modelling problems is a powerful approach to improving beliefs and preferences for this problem type. One theoretical implication of our study is that although open problems are demanding for students and it can be expected that these students prefer solving these problems, students on different levels of mathematical achievement have similar preferences to solve these problems.

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DESIGNING COLLABORATIVE MATHEMATICAL MODELLING TASKS FOR LOWER SECONDARY EDUCATION

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Collaborative learning is considered a potentially powerful way to learn mathematics (e.g. Dekker & Elshout-Mohr, 1998; Webb, 2009), but we still have limited knowledge about how group work contributes to mathematical modelling competencies. The research question we address is: How can group tasks for lower secondary education enhance collaborative mathematical modelling?

In an educational design research, a series of five modelling tasks have been developed building upon design principles for (individual) modelling tasks (Galbraith, 2006; Geiger, et al., 2021) and collaborative learning tasks (Webb, 2009). We used the following principles: (1) the problems are open-ended and sufficiently complex to provoke students to work collaboratively; (2) there is a genuine link with the real-world and a connection to students' perceptions; (3) the problems match the level of the students; (4) the different mathematical modelling (sub-)competencies are addressed in the tasks, including assumption making; (5) the tasks encourage the use of mathematical concepts, linking them to everyday language.

The participants were 37 groups of three students, aged 13 and 14, from grade eight coming from ten classes from five secondary schools. Each group performed one modelling task and then participating students individually completed a questionnaire about the task. Students' verbal interactions were analyzed using a previously developed coding scheme (Göksen-Zayim, et al., 2022).

The analysis showed that students generally worked well together on the modelling tasks and understood the problem. However, students hardly discussed the purpose of the task and possible approaches. Furthermore, in only half of the groups, students made more than one assumption and less than half constructed interpretations and validations, resulting in an incomplete solution of the modelling task. The results of the questionnaire showed that students considered the tasks interesting.

In conclusion, we found that the group tasks based upon our design principles can engage students in collaborative modelling, but they need more support on making assumptions and solving strategies.

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COMPARISON OF MATHEMATICAL MODELLING IN SOLVING AWNING WINDOW PROBLEMS: MALAYSIAN CIVIL EXAMINATION PROBLEM VS OLD JAPANESE TEXTBOOK PROBLEM

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The research question is as follow: What are the similarities and differences in solving awning window problems between Malaysian civil examination problem and Japanese old textbook problem based on modelling cycle (Blum & Leiß, 2007)?

Malaysian Civil Examination Problem (Ministry of Education, 2020)

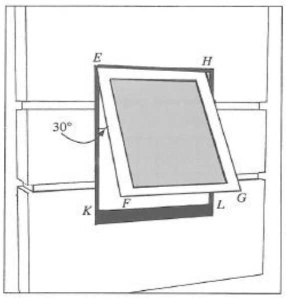
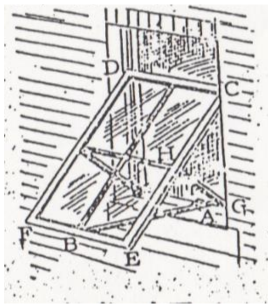


Diagram shows a window that has been installed in a house. The opening of the window is limited to 30° from fulcrums E and H. It is given the length of EF = EK = 55 cm.

a) Name the angle which corresponds to ∠FEK.

b) Hence, calculate the vertical distance from point F to the line EK when the opening of the window is maximum.

Japanese Old Textbook Problem
(The Lower Secondary Textbook, 1943)



There is a window which have a structure as the figure. The prop GH is furnished on midpoint of a window frame. If you push the stick AB, the frame CD slide down along the groove of the window. In this time, how does the bottomed frame EF move?

Figure 1. Awning window problems

In the case of Malaysian civil examination (left side of Fig.1) an awning window that has been installed in an architecture is a real situation & problem. In this problem mathematical model has already been made for the questions a) name the angle of opening of the window, and for question b), calculate the vertical distance when the opening of the window is maximum. Thus, the process of mathematical work leads to a mathematical result is mainly focussed. In the case of old Japanese textbook problem (right side of Fig.1) an awning window that has been installed in an architecture is a real situation & problem. The focus on the structure of the awning window explains the mechanism of opening and/or closing of the awning window. The mechanism of opening and/or closing of the awning window is a real model & problem and mathematical model & problem. Thus, modeller will solve the mechanism of opening and/or closing of the awning window mathematically in a series of process in modelling cycle. Hence the similarities are both problems are presented in the figures of awning window that are located on a real situation & problem. The differences are that in the case of old Japanese textbook it is needed to solve the problem in a series of process in modelling cycle while in the case of Malaysian civil examination it is needed to solve the problem through the process of mathematical work mainly.

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IDENTIFYING AND TRACKING STUDENT CONCEPTIONS OF MATHEMATICS

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A proposed enabler of successful mathematisation when mathematical modelling is believing that a valid use of mathematics is modelling real phenomena (Niss, 2010). We discuss analysis of data collected in the Victorian component of the Enablers Project[3] to answer the research questions: *What conceptions of mathematics are held by Victorian Year 11 students?* and *How are these conceptions changed by experiencing a collaborative modelling intervention as part of their mathematics lessons?* According to Wood et al. (2012, p. 70), by university entry, "students have already developed conceptions of mathematics". Houston et al. (2010) investigated undergraduate students' conceptions of mathematics categorising them as Number, Components, Modelling and Abstract, and Life. Modelling and Abstract are at a similar level and eventually combined if a student develops a Life conception, viewing "mathematics as an approach to life and a way of thinking" (p. 73). Higher levels subsume the conception(s) of lower levels, becoming broader. This categorisation was used for analysing developing conceptions of Grade 6 students (Brown & Stillman, 2017). Wood et al.'s short form of the Conceptions of Mathematics Questionnaire (CMQ) interrogates, via Likert scales, items related to what mathematics is as well as the role of mathematics in future studies and career. In the Enablers Project the CMQ was administered pre and post student participation in modelling lessons. We used selected items to ascertain students' conceptions of mathematics according to the Houston et al. categories. To visualise aggregated student responses, radar diagrams using responses to selected items targeting views underpinning a conception were constructed. A more robust determination of an individual's conceptions used information from the Metaphor Assessment Instrument (Cai & Melino, 2011) collected at the same time and responses in post modelling lesson implementation interviews. We found teachers and their students needed to adjust their view of students' variation in conceptions of mathematics. Classes were not as homogeneous in conceptions as teachers expressed in pre and post lesson interviews. The discussion will address: (1) usefulness of the techniques for studying students' conceptions of mathematics, (2) students' modelling experiences potential for changing in their awareness of conception variation and necessity to progress towards broader conceptions including modelling, and (3) implications for upper secondary.

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A COMPARATIVE CASE STUDY ON TEACHING MODELLING AT THE SECONDARY LEVEL WITH A UNIT DEVELOPED FOR THE TERTIARY LEVEL

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This paper reports on results from the CoSTAMM project ("**Co**mparative **S**tudies into **T**eaching **A**pproaches for **M**athematical **M**odelling"). An important goal of research is to explore which teaching methods are effective for teaching mathematical modelling at different levels of education. In the first three CoSTAMM studies, two teaching designs were investigated, a *teacher-directive design* and a *method-integrative design*, similar to the German DISUM project. A classical research design with an entrance test, a pre-test, a treatment (modelling unit with five lessons, altogether ten tasks), and a post-test was implemented at the *tertiary level* with engineering and chemistry students in South Africa (for results in the engineering context see Durandt, Blum & Lindl, 2022a). The main results from these studies show that both teaching approaches led to a significant competency growth, both in mathematics and in mathematical modelling, with advantages for the method-integrative groups, similar to the results in DISUM.

Since the mathematics required in the five-lesson modelling unit and in the pre-/post-test is essentially at the *secondary level* (grades 9/10; for details see Durandt, Blum & Lindl, 2022b), the idea was to implement the modelling unit and the two teaching designs at this level and to see whether similar results would emerge. This study (without an entrance test) was carried out in 2022 as a case study in four grade 9 classes in Germany (15-year-olds), where two classes followed a teacher-directive design (with a total of 34 students), and two classes followed a method-integrative design (with a total of 35 students). Apart from data on students' progress, data on teaching quality in the four classes were also collected. Linear mixed regression models were used to evaluate and to compare the results. The results show interesting similarities with the previous studies, but also some unexpected peculiarities. Globally speaking, the teaching unit and the two designs worked also at the lower secondary level, in a German context, and were effective for fostering modelling competency.

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DESIGN OF STEM TASKS BASED ON MATHEMATICAL REASONING IN A SEQUENCE OF MODELLING ACTIVITIES

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The importance of integrated STEM (science, technology, engineering and mathematics) education has been acknowledged to develop 21st century skills required in the global market. It indicates the urgent need to design integrated STEM tasks for STEM education. In this presentation, I will introduce how to design STEM tasks based on mathematical reasoning in a sequence of modelling activities. First, a sequence of modelling activities is introduced. Second, how the model development sequence can be modified to design integrated STEM tasks based on mathematical reasoning and illustrated with exemplary tasks. Last, implications and suggestions are provided for further investigation.

Model-eliciting, model-exploration and model-adaptation activities (MEAs, MXAs, MAAs) are recommended as a sequence of modelling activities for learning mathematics (Lesh, Cramer, Doerr, Post, & Zawojewski, 2003). Studies with MEAs have shown that students' thinking can progress during at least one or two full class periods. To further develop powerful representation systems, MXAs following MEAs are necessary. To consolidate the developed mathematical concepts and thoughts, MAAs provide students the opportunities to apply and generalize them.

Mathematics often serves as a tool to be applied in other disciplines. However, this technical aspect of mathematics is not enough for integrated STEM education, and multiple relationships between mathematics and other STEM disciplines should be included. By modifying the model development sequence based on mathematical reasoning (MR) where mathematical mental acts, such as explaining, conjecturing, structuring, predicting, problem solving, interpreting and validating, are involved (Harel, 2008), MEAs, MXAs and MAAs are respectively focused on connection of, interaction between, coordination of mathematics and other STEM disciplines. The modification will be illustrated with a sequence of STEM tasks, named as MR-STEM connection, interaction and coordination tasks.

The potentials and design guidelines of MR-STEM connection, interaction and coordination tasks will be discussed. The approach to the design of a sequence of STEM tasks is theory-based and practice-oriented. Further investigation of how mathematics and mathematical reasoning can be applied and developed in a sequence of modelling activities are needed to support multiple aspects of mathematics.

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HOW CHINESE CITIZENS UNDERSTAND THE INFLECTION POINT OF THE COVID-19 PANDEMIC: INSIGHT INTO FORMAL MATHEMATICS AND STREET MATHEMATICS

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Since the beginning of 2020, the COVID-19 pandemic has changed everything, bringing enormous global challenges to the mathematics education community (Bakker & Wagner, 2020). While doctors and other medical professionals are fighting the disease, people are struggling to remain safe and survive the pandemic.

Also noteworthy is what researchers in mathematics education can do, or are responsible for, during this time of crisis. Regarding the interaction between mathematics and society (Moreno-Aemella and Santos-Trigo, 2008; National Research Council, 2012), we should focus on the impact of mathematics education on social events and people's lives, especially focusing on the fact that mathematics education should aim to raise mathematically informed citizens who are able to understand mathematical or scientific claims as tentative, as deserving a fitting degree of confidence and skepticism – a stance that is neither gullible nor inflexible, but appropriately critical (Muğaloğlu et al., 2022).

This study investigates how people use mathematics to understand the inflection point of the COVID-19 pandemic, using data from 1,371 Chinese people, and explore the possible reasons behind their understanding. Questionnaire surveys were used to collect quantitative data and descriptive statistics were used to reveal people's interpretation of the inflection point numerically and graphically. Online interviews were employed to collect qualitative data that were organized and interpreted to provide more details regarding people's understanding. The results indicate that only a few people can precisely explain what the inflection point of the pandemic is, despite the fact that secondary school education is supposed to equip people with relevant knowledge and literacy and official media releases on the pandemic are another source for people to do so. In addition, citizens tend to apply more informal mathematics in the understanding of inflection point than formal mathematics.

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STATISTICAL PROBLEM SOLVING BASED ON CHILDREN'S VALUES IN MATHEMATICAL SCIENCES EDUCATION

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Our research group focused on decision-making and consensus building using mathematical sciences (e.g., Yamaguchi et al., 2020; Ishikawa, 2020). Mathematical sciences include mathematics, statistics, and applied mathematics at their core; however, they are also closely related to other disciplines such as science and engineering, depending on the authentic problem being dealt with. In mathematical sciences education, this decision-making process formulates and addresses real-world problems that require decision-making. Regarding the multiple options created by different formulations, it is necessary to clarify the premise, form a consensus, and make decisions (Yamaguchi et al., 2020). When building consensus, it is also necessary to pay attention to the values embedded in the mathematical model, which becomes the focal point of the dialogue. Through such dialogue, children can become aware of the close relationship between the existence of diverse values and mathematics. And it can update children's values (Ishikawa, 2020). This paper introduces one practice of mathematical sciences education.

This practice was performed using a statistical unit for sixth graders. In Japan, when a new batch of first graders enter elementary school, a welcome party is often planned and hosted mainly by the current batch of sixth graders, which is the highest grade. This practice is based on the sixth graders' values, "We want to hold a welcome party that can be enjoyed by both first and sixth graders." In this practice, the sixth graders planned a ring-toss game. To make it enjoyable for both, the sixth graders conducted an experiment to determine the distance to the target and collected data. They also asked participants how they felt about the ring-toss game during data collection. They determined the throwing distance using both quantitative and qualitative data. However, when they collected data, they found that the number of successful quoits among first graders varied. Therefore, it is necessary to discuss the number of times the ring is successfully thrown to clear the game. During the discussion, opinions were raised that the characteristics of the group should be understood based on data, and that the values of both first and sixth graders are important. In subsequent learning, based on the data of the entire class, they learned the concepts of average, median, mode, maximum, and minimum values, and captured the characteristics of the data.

From this practical example, we can conclude that children's values become the focus of mathematical sciences decision-making, as they are effective when examining the validity of the results of statistical problem solving, discovering new problems, and refining plans. Value-based modeling of real-world problems and statistical problem solving based on real data is an excellent way to master mathematical science methods. This engenders a real sense of usefulness and is useful for raising children with diverse values.

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DIFFERENCES AND SIMILATIRIES IN STUDENTS' PERFORMANCE ON MODELLING TASKS WITH MUCH OR LITTLE PERSONAL INTEREST IN THE REAL-WORLD CONTEXT OF THE TASK – A COMPARISON

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When working on a mathematical modelling task, there is always a real-world context that needs to be considered to solve the task. The real-world context could therefore be one aspect in the selection of appropriate tasks. While it is already known that interest in mathematics influences modelling performance, this is an open question for personal interest in the real-world context (Krug & Schukajlow 2014). Therefore, the PiMo project (Personal interest in modelling tasks) investigates the influence of the personal interest in the context of modelling tasks on the modelling process.

Here, it is investigated to what extent personal interest in the real-world context of a modelling task influences the performance and the procedure shown in the solution process of the modelling task. To answer this question a qualitative study with 10 students of seventh grade is carried out with a three-step design. For this purpose, three modelling tasks with similar characteristics in terms of mathematical content, authenticity and openness to the real-world contexts: soccer, running and arts & crafts were developed. First, the students state their personal interest in the given contexts. Then, for every student one task with much personal interest in the real-world context of the task (task I+) and one task with little interest (task I-) is selected. Second, students work in small groups on an example task to get familiar with modelling tasks. In the third step, about one week later, students work individually, while thinking aloud, on their task I+ and their task I- and a buffering item in between. This process is video recorded. The order of the tasks is altered over the whole sample.

The transcripts of the videos are analysed using a qualitative content analysis. Categories are the steps of the modelling cycle after Blum & Leiß (2007). After consensually coding the modelling steps, the performance on each step is evaluated using an assessment scheme, where the performance is ranked in four levels. Then, not only the overall score as well as scores in the individual modelling steps are examined, but also qualitatively the concrete procedure in the steps. The analysis is still in progress, but first results show that differences for the solution processes with much or little interest in the context of the task be observed.

The results of the PiMo project can supply teachers with information about how they can use the personal interest in real-world contexts of modelling tasks to support their students' modelling competences and enhance their performance.

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WRITTEN AND SPOKEN RETELLING OF THE TASK SITUATION AS A STRATEGY FOR UNDERSTANDING OPEN MODELLING TASKS

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When working on open mathematical modelling tasks, students have many difficulties, especially in the first steps of understanding, simplifying and structuring (Blum, 2015). However, these steps, which include the formation of a situation model for the task situation, are crucial for successful completion of the tasks (Leiss et al., 2010). One way to support these first steps is to have the students retell the task situation in their own words in order to engage more intensively with it. This can be done verbally or in writing. Both forms have their advantages and disadvantages and it is unclear which strategy is more helpful for the students. For example, a written approach is often more in-depth than a spoken approach, but it can also be more challenging, especially for students with weaker language competence, as it requires a certain level of writing skills (Grabowski et al., 2012).

In a qualitative video study, six seventh graders are observed working on two mathematical modelling tasks after retelling the given task situation verbally or in writing. For this, the students are given different instructions to retell the task situation verbally or in writing before they have to solve the task. Using the method of thinking aloud, the students' thought processes are made visible in order to analyse the different processing procedures. The extent to which the students' processing procedures and the retelling processes differ when working on open mathematical modelling tasks with written or spoken retelling is investigated. In order to be able to put possible differences also in the context of different levels of language competence, a short language competence test was conducted with the students beforehand.

A qualitative content analysis is used to analyse the transcripts of the videos. Since retelling as a strategy is meant to support the comprehension process, the focus of the analysis is on the first steps of the modelling process: the understanding, simplifying and structuring. First results on differences between the processing of open modelling tasks with written and spoken retelling will be presented at the ICTMA conference and discussed with regard to possibilities for supporting students in the processing of open mathematical modelling tasks.

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INFLUENCE OF LANGUAGE USED ON THE MATHEMATISATION：ANALYSIS BASED ON COGNITIVE LINGUISTICS

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Students tend to have difficulty in mathematising of real situation. In the study of word-problem, the possibility that the cause of this difficulty lies in the students' language ability has been pursued. For example, studies have focused on the relationship between reading comprehension and problem-solving skills (e.g.,Vilenius-Tuohimaa, Aunola, & Nurmi, 2008) and have analysed the effect of the complexity of written expressions on students' problem solving (e.g., Plath & Leiss, 2018). While these studies have developed internationally, they have not developed in Japan. In this study, we focus on the relationship between the Japanese and mathematical language and analyse how the ability to use Japanese affects the mathematisation of real situation. This study utilizes "cognitive linguistics" as a theoretical framework for consideration.

The purpose of this research is to clarify the effect of differences in the structure of Japanese and mathematical language at the stage of "mathematisation" where the translation from the language in the real world to the language in the mathematical world. To achieve the purpose, three sub-tasks were set in this presentation;(1) How has the issue of language used been discussed in Japanese word-problem and modelling research? (2) What implications does a linguistic approach have for word-problem research? (3) From the perspective of linguistics, how does the differences in structure between Japanese and mathematical languages affect learners' thinking?

The results corresponding to the sub-tasks are as follows;(1) In Japan, there was a time when the structural difference between Japanese and mathematical languages became a topic of discussion, but there were not many of them. The reason for this may be that studies have been case studies, and therefore a general methodology has not been developed to capture the effects of structural differences between Japanese and mathematical languages. (2) We pointed out that the field of linguistics, which has a high affinity with mathematics education and research, is cognitive linguistics, and organized the basic ideas. Furthermore, based on cognitive linguistics, we organized two relativist perspectives: "relativism of interpretation between individuals (conceptual relativism)" and "relativism of languages (linguistic relativism)". And we also theoretically argued that these relativism enable new perspective that are different from the existing framework of mathematical modelling research. (3) We conducted a survey and empirically verified that the perspectives of "conceptual relativism" and "linguistic relativism" are effective frameworks for capturing the influence of language on mathematisation.

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ACCUMULATION OF DEGREE-DAYS AND CATEGORY OF MODELLING: A CASE WITH STUDENTS IN CHILE

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The teaching of the definite integral is traditionally centred on a formal and mechanistic approach that distances this concept from an interpretation linked to processes of change (Contreras and Ordóñez, 2006). In this regard, Marcía (2020) provides theoretical references for the design of learning situations that allow students, from a development of category of modelling (Cordero et. al, 2022), to construct the integral defined as a model of accumulation degree-days in specific situations of change in Phenology. Based on the above, the aim of this research was to analyse the construction of the definite integral of university students when they faced an organised and sequenced learning situation from an argumentation of accumulation of degree-days in a specific situation of change.

To respond to the objective, a case study (Stake, 2007) was used with 10 students from a Chilean university, who developed a learning situation, designed based on the framework formulated by Marcía (2020), whose purpose was to determine the accumulation of degree-days of a particular type of watermelon harvested in the Azapa Valley (northern Chile) during the month of August 2023. For the development of the situation, students made use of: a) web pages to obtain data on the average temperature forecast for that month, and b) GeoGebra to construct a graph of average temperature $T(t)$ (time t in days) that models the data obtained and incorporates area sliders to determine the accumulation of degree-days. Data collection was obtained through written and audio-visual records and data analysis used interpretive techniques of content analysis (Flick, 2004).

The development of the situation contributed to the emergence of a category of modelling that allowed students to construct the integral defined as a model of accumulation degree-days. In this sense, the role of GeoGebra was fundamental as it favoured the students' construction of the function $T(t)$ and the visualisation of the quantities that accumulated.

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EXPERIMENTS IN TEACHING ADVANCED CALCULUS: AN EARLY INTRODUCTION TO DOUBLE INTEGRALS

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Differentiation and integration are the two major divisions of calculus. Traditionally, in the United States undergraduate education system, single integration theory is introduced through a first course in calculus, say Calculus I, via the critical topic of Riemann sums, which is then followed by proof and application of the First and Second Fundamental Theorems of Calculus (FTC) for well-prepared students, say those with a solid understanding of calculus as presented in Spivak, M. (2008), or a sketch and conceptual layout of the said proofs for students who might not have had such a deep, theoretical grounding in the essentials of calculus and who might have been educated in the subject through texts such as Varberg, D., (2007), or Stewart, J. (2003). This introduction is usually followed by several applications of the theory involving calculating the area under curves, volumes of revolution, finding the length of planar curves, numerical integration, and other applicable topics. After several other topics and perhaps more advanced calculus courses, say Calculus II, differentiation in two- and three-dimensional spaces is introduced followed by double and triple integration in two- and three-dimensional spaces, respectively, over rectangular and other regions.

This paper outlines strategies for experimentation with a much earlier introduction of double integrals over rectangular regions. The authors encourage introducing the techniques of double integration over rectangular regions immediately after students have been introduced to and practiced extensively on applications of single integration of functions of one variable in a first introduction to calculus. The idea is to present this topic mainly using conceptualization, analogies, and graphics, and then making immediate applications to concepts such as volumes of revolution and numerical integration of well-behaved functions of one variable. There are at least three reasons for this: (1) The audience for the calculus sequence (Calculus I, II, and III) in the United States (and elsewhere) tends to decrease significantly with each advanced level. Consequently, many students in a general audience who took an introductory first course miss the opportunity to see and make use of the power and application of advanced techniques like double integration. This strategy has the possible advantage of retaining some of these students for more advanced levels (2) Students who experience difficulties with concepts such as the computation of volumes of revolution might (and have been observed by us) have a less difficult time doing those computations using double integration and (3) “Repetition deepens impression,” which puts students having an earlier introduction at a distinct advantage when they see the concepts for a second time. For these reasons, we believe this is an experiment that mathematics educators should consider conducting.

The applications of double integrals to mathematical modeling are numerous and make an early introduction even more worthwhile. Areas of application include density and mass, moments and centers of mass, surface area, and probability. These are only a small fraction of possible applications which can engage curious students and pique their curiosity and interest in modeling.

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VOCATIONAL MATHEMATICS EDUCATION IN DIFFERENT

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The members of the International Community of Teachers of Mathematical Modelling and Applications (ICTMA) have during the last 40 years developed a large set of research literature mainly focusing on mathematical modelling (Geiger & Frejd, 2015). However, less focus has been paid on the A in ICTMA, namely applications. The dominating use of mathematics in most occupations is connected to applications of mathematics. Thus, the use of realistic applications should be a part of vocational mathematics education. However, the prevalent mathematics teaching practice in vocational education and training in upper secondary school in Sweden is based on textbooks that mainly focuses on formal academic mathematics where the link to the labour market is weak. Many students, therefore, do not see the relevance of using mathematics for their future vocation, which impacts both on students' motivation to learn mathematics and their mathematical skills (Muhrman, 2016). The overall consequence is that students do not get access to the labour market, since they do not meet the working life's requirements of mathematical knowledge. There is a national and international consensus among researchers of a need for research to explore teaching and learning methods that strengthen the relation between mathematics education and students' future occupations (e.g. FitzSimons, 2001). In particular, the problem of organising activities suitable for preparing students to solve and critically reflect on problems arising from realistic situations at the workplace is a current matter of research (Frejd & Muhrman, 2020).

This paper will report from an ongoing project where we aim to investigate how mathematics in vocational education and training can be organised to support students for their future profession. The project includes ten upper secondary vocational education schools in Sweden that work with teaching and learning interventions with applications of mathematics in different contexts (in a regular classroom and in a workplace authentic environment). Our goal is to explore the effects of these interventions on the students' mathematical skills and their motivation to learn mathematics. The study has a longitudinal design using pre- and post- tests and questionnaires and interviews to collect data before and after the interventions, and 12 and 18 months later, to investigate if the effect is permanent. During the conference we will present and discuss our first results from our project.

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CONNECTIONS ESTABLISHED BY 9TH GRADERS AFTER SIMULATING AND MODELLING THE EFFECT OF ATHEROSCLEROSIS

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In this study, we aim to know the types of connections, and their preponderance, established by 9th graders after carrying out a simulation of the accumulation of plaque in the arteries, and creating a mathematical model describing the reduction of blood with increased plaque thickness. Establishing connections, namely between real-world situations and the world of mathematics is one of the cognitive processes inherent to mathematical modelling and has been identified as one of the challenges that students face in modelling problems. In recent years, mathematical connections have been investigated (e.g., Amado et al., 2019), in line with an emphasis on their importance as curricular and educational targets (e.g., OECD, 2019). Businskas (2008) sees the construction of mathematical connections as a process through which relationships between ideas, definitions, concepts, or theorems (intra-mathematical connections); or relationships between these and other subjects or real contexts (extra-mathematical connections) are established. Mathematical connections are key to mathematical modelling, especially if entailing interdisciplinary work and links to other subjects (Borromeo Ferri & Mousoulides, 2018; Blum & Niss, 1991). This qualitative study involved 83 students, from four 9th grade classes. They worked in small groups, in a total of 24 groups. The task, of an interdisciplinary nature, asked: "How would you explain to a relative the effect of fat plaques on the volume of blood flowing in the arteries?". To support their answer, the students had materials to carry out a simulation and collect data: a cylindrical cup (representing an artery), flexible EVA sheets (representing fat plaques); syringe for measuring volume; coloured water. The task was done in the mathematics class, where both the science and the mathematics teachers participated. After completing the task, the students received a sheet containing images (alluding to the experience) and words (mathematical and non-mathematical concepts). The assignment was: "Draw all the links you find between pairs of items. Number each link and explain what the link means". Data were analysed using a deductive content analysis, and different types of mathematical connections were categorized. Results showed that students established both intra- and extra-mathematical connections, but the latter occurred more frequently and with more detailed justifications, namely referring to the pathology of atherosclerosis. Some intra-mathematical connections were not justified by the students, suggesting insufficient knowledge of some mathematical concepts, namely the quadratic function.

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USING A SCIENTIFIC CALCULATOR FOR MATHEMATICAL MODELLING AND APPLICATIONS IN SECONDARY SCHOOL

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Attention to applications of mathematics in secondary schools has morphed over recent decades into attention to mathematical modelling, perhaps motivated by students themselves and perhaps by others keen to see how mathematics might be useful. By the nature of secondary schooling, the mathematics involved is generally relatively unsophisticated. In this paper, the appropriateness of modern scientific calculators as tools for students and teachers will be assessed and exemplified.

Many formulations of the mathematical modelling process use the term 'solve' to describe the mathematics involved. In this sense, solving is a key part of both illustrative applications of mathematics and of mathematical modelling itself. (*reSolve* Project (2023); Galbraith and Holton (2023)) Usiskin (2011) makes clear that early steps in learning about mathematical modelling involve careful exposure to mathematical applications, which inevitably require mechanisms of dealing with mathematical representations and their elaborations, including sensitivity analyses.

For secondary school students in many countries, modern scientific calculators provide a rich yet affordable toolkit for most of the mathematical processes required for first steps in modelling and applications. The toolkit includes representation and evaluation of user-defined functions, tabulation of functional values, numerical solution of equations, numerical calculus, spreadsheets, elementary statistical analysis and random processes. While other technologies provide more powerful support (such as graphical support through graphics calculators and algebraic support via computer algebra systems), the real circumstances of many students do not permit everyday access to these in secondary schools, either because they are too expensive for universal use or because they are effectively disallowed by curriculum authorities. So the scientific calculator is an ideal technology for getting started, to be followed at a later stage by more sophisticated technologies.

The modern scientific calculator is a suitable technology to support early steps in mathematical modelling and applications of mathematics in the secondary school, because its functionality has been designed explicitly to support such work, in the wider economic and curriculum contexts in which many teachers and students are located. Examples to illustrate both the nature and scope of this support will be provided in the paper.

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MAKING STUDENTS BECOMING RESPONSIBLE CITIZENS IN 21ST CENTURY BY FOSTERING MATHEMATICAL REASONING ABOUT REAL-WORLD APPLICATIONS

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Promoting students becoming responsible citizens in 21st century is accepted being a central aim of education (see e. g. European Commission, 2019) and implemented in political documents for teaching and learning all over the world (see e. g. NCTM, 2000). With respect to teaching and learning mathematics at school, this special goal is addressed by highlighting the importance of mathematical reasoning about real world applications for students' everyday life: "The ability to reason logically and present arguments in honest and convincing ways is a skill that is becoming increasingly important in today's world" (OECD, 2018, p. 15). Based on these considerations, it is of special interest fostering students mathematical reasoning about real-world applications – and it is assumed that building up both competencies on mathematical modelling (CMM) in particular (Kaiser, 2020) as well as competencies on critical thinking (CCT) in everyday life in general (Lai, 2011) can support students' learning. However, empirical evidence about the impact of CMM and CCT on students' mathematical reasoning about real-world applications is rare. The present study harks back to this desideratum and analyses empirically, whether a special training on both CMM and CCT can foster students' competencies on mathematical reasoning about real-world problems.

In 2015/2016, overall 380 seventh-grade students (average age: 13 years) from 9 German middle schools took part in the research project FASAF (principal researchers: D. Leiss, A. Neumann, K. Schwippert). The research project aimed at fostering students' competencies on mathematical reasoning about real-world applications by offering to different types of weekly training, each lasting for 6 months: a training dealing with CMM and CCT separately (experimental group A; EG A; n = 82) and a training dealing with CMM and CCT simultaneously (experimental group B; EG B; n = 75). Additionally, a waiting control group (CG; n = 223 students) was part of the project. Tests on mathematical reasoning about real-world applications have been administered in a classical pre-post-design. Comparing EGA, EG B and CG with respect to these performance tests offers insights into the effect of those different trainings. These results will be discussed at ICTMA; implications for teaching and learning mathematical reasoning about real-world applications as well as about making students' becoming responsible citizens will be highlighted.

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HANDS-ON WORKSHOP: PARTICIPANTS BUILD A SMALL HUMAN CARRYING CAPACITY FOR EARTH MODEL IN FREE STELLA ONLINE

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Complex systems are ubiquitous in our world, from climate change, to human physiology, to how the teenage mind operates. Having students or professionals build computer models to capture the behavior of complex systems has been shown to aid them in understanding how complex systems operate and test policies to attempt to find leverage points that could modify the behavior of these systems (Hung, 2008). If we want to provide opportunities for students to develop the skill of analyzing complex systems we must win the hearts and minds of the teachers, who ultimately control what is taught in the classroom (Tondeur et al., 2019). Research shows that providing hands-on experience with new technologies can be valuable in reaching teachers at a level that can expand their belief system to include broader instructional strategies in their teaching (Beaudin et al., 1997).

In the 2019 IMMC competition the question to estimate the human carrying capacity of the earth secondary school students did not perform well (Garfunkel et al., 2021). A major issue was, in attempting to mathematize this problem, attention to non-linear dependencies, interconnections, and feedback was needed, suggesting that simulation would be necessary to perform the analysis (Galbraith & Fisher, 2021). Secondary school students have shown that they can build and analyze complex systems using System Dynamics (SD) software to develop original models (Fisher, 2018).

In order to provide teacher educators with experience building at least one model using the free, web based SD software that K-12 students could have used to analyze the 2019 IMMC question a short hands-on workshop has been prepared. Participants will build a World Population-Temperature model and potentially a second model to gain exposure to a tool that is readily available in the K-12 schools using either laptops, tablets, or Chromebooks. Multiple articles have already occurred in the ICTMA Proceedings publications explaining the SD method and giving numerous examples.

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STUDENTS' MODELLING ACTIVITIES IN THE INQUIRY ON THE SLIDE RULE

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There has been a growing focus on inquiry learning in educational contexts internationally (Artigue & Blomhøj, 2013). Although there is also an emphasis on inquiry in the recently revised Japanese Course of Study, it seems different from the international context, with more attention paid to a better application of the knowledge and skills to be learnt than to the inquiry process itself. This may be attributed to the assumed didactic paradigm in Japan, closer to the paradigm of visiting works than to the one of questioning the world, as claimed by the Anthropological Theory of the Didactic (ATD) (Chevallard, 2019). We use the proposal of *study and research paths* (SRPs) as a model of inquiry, which is proposed in the framework of the ATD. SRPs emphasise the dialectic between the generation of questions to be addressed by the inquirers and the search and construction of models to answer them. This dialectic of questions and answers provides a first description of the structure of the inquiry process, as well as the milestones for the paths foreseen during the inquiry (Bosch, 2018).

Our contribution considers university students' modelling activities carried out in an SRP about the functioning of "slide rules" for multiplying and dividing decimal numbers. The research question we focus on is about: how can the university students' modelling activities in the process of inquiring on the slide rule be characterised through a question-answer dialectic? The SRP has been implemented in a teacher education programme in Japan in spring 2023. It consisted of five sessions of group activities. Session 1 was an exploration about what can be done with a slide rule. In Sessions 2 and 3, students inquired into the following questions: How does a slide rule work? Which scales are combined to carry out what calculations? How is each scale constructed? Session 4 was devoted to students' self-analysis of their inquiry process. The didactic tool of the *question-answer maps* (QA maps) (Winsløw et al., 2013) was used for the analysis of the inquiry and modelling processes followed. Finally, Session 5 consisted of a poster session where the groups of students presented their final results.

Our analysis in this teacher education activity is primarily carried out based on the QA maps initially proposed by the students. On the one hand, we focus on the models used by students to explain the different calculations, mostly using logarithms, which is the basic concept of the slide rule. On the other hand, we look at how the question-answer dialectic differed from one group to another, especially with regard to the modelling paths taken and their productivity.

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TEACHER EDUCATION IN MODELLING: MODELS, SIMULATION AND NEW EPISTEMOLOGICAL NEEDS

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Teacher education at all school levels might take into account the epistemological and didactic needs of the teaching profession. These needs are not always well known, as they often correspond to important constraints created by the new curricular demands to teach mathematics as a modelling tool in inquiry approaches. The collaboration between teachers and researchers to elaborate epistemological and didactic tools to overcome these constraints is an open question in our field. In our paper, we address this question from the perspective of the Anthropological Theory of the Didactic (ATD) through the proposal of *study and research paths* (SRPs) and *SRPs for teacher education* (Barquero et al., 2018; Bosch, 2018). The experiences carried out with SRPs and SRPs-TE by our team at Universitat de Barcelona will serve as a basis for a reflection on the role of epistemological tools based on the notions of systems, models and simulation in helping teachers manage didactic processes based on modelling.

More concretely, we choose two case studies of SRPs implemented at secondary and university levels, respectively. On the one hand, an SRP about "How sure are some padlocks? (Vásquez et al., 2021) will be used to introduce two essential properties of modelling: those of *recursivity* and of *reversibility*. We present examples of students' modelling processes to illustrate the above mentioned properties and analyse their role in the construction and interpretation of models. The implementation of this SRP, after 4 consecutive years, brings light to the tools shared by the team of teachers to build a common infrastructure to talk, manage and analyse the students modelling processes. The second SRP is generated by the question "What is school segregation and how is it measured?". It aims to show the complex relationships between systems and models in a different situation: when one finds already-made models (like a formula) and aims at studying and developing them. In this case, the modelling process starts with the consideration of an external model that needs to be tested with different systems, some of them artificial or simulated. The particular role of simulations in the production and development of models thus appears as an important focus of the analysis.

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HOW CAN ALEXA UNDERSTAND US? SPEECH RECOGNITION AND AI IN MATHEMATICS EDUCATION

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The rapid development in the field of artificial intelligence (AI) has led to amazing advances in natural language processing (NLP) in recent years. From intelligent voice assistants like Siri and Alexa to chatbots like ChatGPT, we are surrounded by NLP technologies on a daily basis. Given these technological developments, it seems essential to have a basic understanding of the underlying mathematics in order to comprehend and critique how these systems work (Long & Magerko, 2020).

In this talk, we will use the example of speech recognition to show that the topic of NLP offers many links to school mathematics and is thus suitable for taking up highly relevant, authentic applications in mathematics education. It is shown how a suitable didactic reduction of the underlying mathematical models enables the learning of basic strategies of machine learning - a collection of methods in the research field of AI - as well as the application of school mathematical concepts in the field of NLP. We present the didactic reduction and a possible instructional progression that enables students aged 16 and up to acquire the mathematical foundations of single word recognition.

Four steps were identified as essential in single word recognition: preprocessing of the speech signal, feature extraction, classification and the evaluation of the classification. The first step is primarily concerned with the processing and visualization of the data. The speech signals are available in time period and can be converted into an amplitude spectrum using the Fourier transform. This can be done using suitable digital tools as a black box. Alternatively, however, the Fourier transform can also be worked out with students using the example of triads as a decomposition of the signal into individual sinusoidal oscillations (Wohak & Frank, 2021). In feature extraction, typical features in the spectrum are to be identified. For specified time steps, the local maxima as the typical features can be extracted and stored in a vector. Subsequently, a first mathematical model for classification is developed. For this purpose, the feature vectors of signals for which the word meaning is known are combined to classes of the same word meaning. If a signal is to be classified whose word meaning is still unknown, the Euclidean distance to the feature vectors of every class is calculated. Then, class by class, the mean value of the distance is calculated and the signal is assigned to the class with the smallest mean value. As a model improvement, the distance between the feature vectors can be determined using Dynamic Time Warping to account for the temporal variability of the speech signals (Pfister & Kaufmann, 2017). Finally, the classification result can be evaluated with the learners. Here, the assessment tool of the confusion matrix, which is common for machine learning methods, can be used and the reference to four-field tables, which are typically covered in school lessons, can be made.

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DIDATIC REVISION STRATEGY FOR DESIGNING MATHEMATICAL MODELING TASKS USING AI TECHNOLOGY

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Mathematical modeling, which involves abstracting mathematical concepts from and applying them to real-life contexts, not only enhances students' mathematical competency but also fosters an appreciation for the usefulness of mathematics (Gravemeijer & Doorman 1999). Thus, it has been adopted in the national mathematics curriculum of many countries. However, transforming real-life problems into modeling tasks suitable for secondary school mathematics classes presents challenges due to complexity, intricate data analysis, and the added teacher burden (Borromeo Ferri, 2018). AI technology can provide a solution to these challenges, assisting teachers in data-related difficulties and modeling task formation. To explore how teachers utilize AI technology to design mathematical modeling tasks, we developed an expanded Didactic Revision (DR) strategies framework. The development of this framework was created based on previous studies (Ahn & Yu, 2023; Kim et al., 2021; O'Neil & Schutt, 2013), and then revised by math education experts and AI math experts.

Our framework includes 8 distinct DR strategies: five from previous study (Kim et al., 2021) and three new ones identified in this study: (1) DR-S strategy: revising complicated computations for **S**impler problems; (2) DR-B strategy: **B**reaking a task into smaller subtasks; (3) DR-A strategy: revising **A**ssumptions in the context of the task for articulation; (4) DR-W strategy: using **W**ording appropriate for student levels; (5) DR-N strategy: revising **N**otations, such as symbols that can cause student misunderstandings or mistakes; (6) DR-D strategy: specifies criteria for **D**ata collection and processing so that students can collect and process the **D**ata needed to construct a mathematical model; (7) DR-PM strategy: provides criteria for **P**ro**M**pts to discuss with a chatbot (e.g., ChatGPT) allowing students to monitor their task-solving progress; and (8) DR-PG strategy, which encourages students to use **P**ro**G**ramming tools to explore collected data or to construct and compute a mathematical model. In our presentation, we will provide how this applies to data in which preservice teachers design modeling tasks with AI tools including GhatGPT, Orange3. Our study is significant because the framework provides pedagogical guidance for teachers designing mathematical modeling tasks using AI technology, particularly crucial in this era emphasizing both mathematical modeling and AI.

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LEARNING THEORY BELIEFS ABOUT TEACHING SIMULATIONS AND MATHEMATICAL MODELLING WITH DIGITAL TOOLS

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In simulations and mathematical modelling, as well as in the use of digital tools, a student-centred approach is accepted as an important teaching method in mathematics education (Schukajlow et al., 2012; Thurm & Barzel, 2022). A substantial influence on instructional design is attributed to beliefs of mathematics teachers (Voss et al., 2013). Beliefs are part of teachers' professional competence (Baumert & Kunter, 2013) and can be categorised into constructivist and transmissive beliefs from a learning theory perspective. Teachers with stronger constructivist beliefs more often integrate tasks that need to be actively worked on by the learners. Teachers with stronger transmissive beliefs understand learners as passive recipients who absorb knowledge from the teacher (Voss et al., 2013). Transmissive beliefs therefore often don't support the independent, process-oriented approach in simulations and mathematical modelling with digital tools, while constructivist beliefs favour it.

This chapter describes domain-specific (i.e. in the area of using of digital tools in simulations and mathematical modelling) beliefs by concretising global constructivist and global transmissive beliefs of mathematics teachers. We present the conception of a mathematics education course offered at the Universities of Münster and Würzburg. It aims to promote professional competence for teaching simulations and mathematical modelling with digital tools and therefore also intends to build up described constructivist beliefs and reduce described transmissive ones. In an intervention study with two measurement points, we investigate to what extent participation in the course can initiate changes in the mentioned beliefs. For this purpose, data from N = 204 pre-service mathematics teachers were collected and quantitatively evaluated using two self-developed belief scales (statements with a Likert scale each) and, among other things, t-tests. Initial results show significant effects on the beliefs in favour of the intervention: They indicate that the domain-specific constructivist beliefs were built up and the transmissive ones were reduced and that this can be attributed to the course participation. Based on the evaluations, its effectiveness and implications for teacher education are discussed.

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CONSTRUCTING MEANING FOR INTERACTIVE MATHEMATICAL MODELS WITHIN A DIALECTICAL LEARNING ENVIRONMENT

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Modelling activities in mathematics have changed in the last decades because of technological development. Using technology to design interactive materials provides a captivating, engaging tool which encourages learners to explore mathematical models and to devise their own models. Students need experience with mathematical models to understand the point of mathematical modelling, that is, its “language” (Schwartz, 2007). Once such models exist in the cognitive “baggage” of learners, they also become a tool for active mathematical modelling (Wilensky, 1999). While students learn about dynamic processes and about the mathematical models of the processes, it is especially important that the materials be represented in a similarly dynamic way by real-world models, such as animations, and interactive mathematical models (Schwartz, 2007; Naftaliev, 2017).

This presentation focuses on the investigation of interactive materials that encourage a dialectic process of building meaning for complex and unknown mathematical models by learners. The design of the materials focused on generic combinations of accelerated motion. It was intended to support the development of awareness of kinematic phenomena (such as rate of change and constant and changing speeds) and the analysis of their mathematical models. The students identified visual and kinematic conflicts and so reflected on their current mathematical understanding and modified it using different resources and facing the constraints designed in the materials.

The animation served as a tool for students to reflect on their currently dominant spontaneous structures mainly because it presented a surprising or contradictory situation to students, who did not yet understand the aspects of motion, and when the models in the diagram were not significant enough for them. But the animation served as a tool for modifying the spontaneous structures because it acted as an exploration and inquiry tool; running the animation was useful for noticing phenomena and verifying conjectures. The dialectic process emerged from the need to resolve the tensions between the learners' mathematical understandings and the information presented in the interactive materials. The animation was a tangible real-world model that helped build meaning for the linked mathematical models, and the linked models acted as tools to characterize the different aspects of the motion process.

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ADVANCING THE STUDY OF A MATHEMATICIAN'S MODELLING PROGRESS DISPLAYED WITH APPLIED RESPONSE ANALYSIS MAPPING

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Processing in modelling are different from each modeller because their experiences are different, and changing models are influenced by some situations. As previous studies I have already researched different subjects and some case studies (for example, graduate students or working adult and so on). Research question is as follow: How does an expert in mathematics make progress modelling? I reported a part of the modelling of a mathematician (Matsuzaki, 2019), and continue my study to display all of his modelling in this presentation. I have studied to chase modelling progress with applied response analysis mapping (Matsuzaki, 2011, 2018). Applied response analysis mapping is a method to display processing in modelling progress. One of characteristics of this method is reflected situations, which are used for broad meanings that are imaged based on each modeller's experiences. Components of the modelling based on prior experiences of modeller are identified as follows: components based on real experiences (CRE) and components based on mathematical experiences (CME). Links between CRE and CME during modelling have been confirmed by this method.

Problems for subjects are related to electronics: How much brightness is needed to read a book? I asked for subjects the following method to answer the problems: to decide conditions, pose models and solve problem. Adding to I asked the mathematician to talk about situations by think-aloud methods through solving problems. We can see the modelling progress displayed with applied response analysis mapping reflects some components (CRE & CME) of experiences of the mathematician, and that processing is affected by situations.

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ANALYSING ‘STUDENT-TO-STUDENT-TO-TEACHER’ COLLABORATIVE LEARNING PRACTICES IN A PRE-SERVICE APPLICATIONS AND MODELLING COURSE

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In South Africa, as in many other countries, mathematical modelling and its applications in real life situations are recognised in the secondary school curriculum (DBE, 2011). However, modelling is not yet uniformly taught in all secondary schools in the country, and it is not assessed in the national examinations. As universities train more secondary mathematics teachers to teach modelling, it is expected that more awareness of, and interest in the topic will be created in the schools and in the communities outside the school. The current study contributes to building the modelling competencies of pre-service teachers (PST) and preparing them for secondary mathematics teaching. The purpose of the study is to analyse features of ‘student-to-student-to-teacher’ (S-S-T) collaborative practices that occur frequently when the PSTs solve an open-ended mathematical modelling task, and whether such practices enhance or limit PSTs modelling solutions. The research question I address is: *What features of collaborative practices dominate PSTs solutions of an open-ended modelling task, and how do those practices enhance or limit the quality of their solutions?*

Participants in the study are 52 PSTs in their final year who have already been introduced to the principles of mathematical modelling (Blum and Leiss, 2007) for 10 hours of two-two hours of lectures, and two-three hours of tutorials per week for two weeks. After two weeks, the PSTs were assigned an open modeling task, the “Giant shoes” problem (see Blum & Ferri, 2009, p.45). They were to consult among themselves, but submit an individual report of about 1000 words, describing the collaborations that they enlisted from peers toward solving the task. The teacher/researcher was also available to the PSTs for consultation; however, apart from providing the minimum guidelines given to the PSTs at the beginning, and responding to general questions of clarification, the teacher/researcher remained largely in the background.

Collaborative learning has been associated with positive effect on students’ mathematics learning (e.g., Göksen-Zayim et al., 2022). Discussions in small groups can also promote the development of modelling competencies (Durandt et al. 2022; Geiger et al., 2022; Göksen-Zayim, et al., 2022; Maaβ, 2006). One of the difficulties for teachers/researchers is to observe the learning that occurs when students work collaboratively in small groups. In this study, I use collaborative learning framework (Göksen-Zayim, et al., 2022) to analyse PSTs written solutions and based on the analysis, to identify practices that enhance, and those that limit PSTs modelling solutions.

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PRE-SERVICE TEACHERS' PERCEPTION ON THEIR ROLES IN THE TEACHING OF MATHEMATICAL MODELLING

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This study was situated in a programme for mathematics teacher education to prepare the professional competencies of teaching mathematical modelling for pre-service mathematics teachers at a prestigious university for teacher education in China. In the programme, 36 pre-service mathematics teachers received the training of how to teach mathematical modelling at secondary schools, which followed Borromeo Ferri's (2018) four competence dimensions for teaching modelling including theoretical dimension, task-related dimension, teaching-related dimension, and diagnostic dimension. Especially, at the end of programme, the pre-service teachers were involved into a modelling activity, working with 63 Grade eight students to tackle with three modelling tasks in groups, during which they were able to transfer the competence they obtained from the programme to practice.

This paper presented the pre-service teachers' roles played in the modelling activity. According to the design of the activity, the pre-service teachers should take one of three roles when the students were working on one modelling task, and the students worked on three tasks in total. The three roles were: (1) the main lecturer for the whole class who organised the procedure of modelling and demonstrated related concepts, skills and etc, through following the instruction which mainly designed by the main lecturer; (2) an inventor for a student group who sit with a group of 5 to 6 students to provide inventions when necessary to offer assistance when the students doing a modelling task; and (3) an observer for a student group who sit aside the students to observe their work, his/her peer's intervention, and the interactions between students and the inventor. Since there were three modelling tasks, each pre-service teachers could take at least two roles in the whole modelling activity. As such, the pre-service teachers could be categorised by their roles in the modelling tasks. Six types were identified: the pre-service teacher who taking all the three roles, the roles of main lecturer and inventor, the roles of main lecturer and observer, the roles of inventor and observer, the role of inventor only, and the role of observer only. Interviews were delivered to the different types of pre-service teachers to gain their perceptions about the different roles they took when working with school students on mathematical modelling, such as perceptions on the effectiveness of teacher demonstration and intervention, on the relationship between demonstration and intervention, and on the categories and functions of teacher intervention etc. The interview data were analysed through a qualitative text analysis approach (Kuckartz, 2014). The results may reveal not only pre-service teachers' reflection on their performances on the modelling instruction, but also a systematic approach to promoting the professional competence of teaching modelling for pre-service mathematics teachers.

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CHARACTERIZING MATHEMATICAL MODELLING TASKS IN EMPIRICAL LITERATURE

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Mathematical modelling is a process that uses mathematics to represent, analyze, make predictions, or otherwise provide insight into real-world phenomena. Researchers have developed various diagrammatic representations to characterize the process (e.g., Blum & Leiß, 2007). Although different in detail, these diagrams share the common perspective that modelling involves simplifying and structuring, mathematizing, working mathematically, interpreting, and validating. These modeling activities occur in specific task environments and therefore are shaped by the tasks (Zaslavsky, 2005). Tasks as a dimension of teacher competence (Borromeo Ferri, 2018) underscores the centrality of tasks to modelling and learning to model and how a specific modeling task might prompt or hinder modelers' activity. Implicitly if not explicitly, tasks communicate to students how teachers and curriculum developers conceptualize modelling. As a result, it is important to examine the features of mathematical modeling tasks used in research and practice.

This study examined mathematical modelling tasks that were used as instruments for data collection in empirical studies published in major mathematics education journals. We identified 109 mathematical modeling tasks from research articles on modelling that were published from 2011 to 2020 in top-tier mathematics education research journals. Noting that many articles included little information about implementation of the modelling tasks, our analysis focused on the opportunities to model afforded by the task statements rather than on potential solutions. For each modelling task, we considered each phase of the modelling process as we analyzed whether the task requires modelers to make assumptions and identify variables; whether data, formula, or worksheet are provided in the task statement; whether it specifies mathematical content, representations, or tools to be used; and whether the task statement requires modelers to refine and validate their solutions.

Results reveal that few tasks demanded modelers to engage in full modeling cycles and few tasks demand modelers to refine and validate their solutions. Moreover, most task statements did not specify mathematical content, representation, and tools to be used. In most cases, the striking differences among modelling tasks were in their demand for modelers to make assumptions and identify variables. Informed by these results, we developed different profiles of modelling tasks. Results from this study not only suggest the existence of different opportunities to model afforded by modelling tasks in the literature but also inform the design of fully open-ended modelling tasks.

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A REVIEW OF RESEARCH ON MODELLING, BIG DATA AND EVIDENTIARY PRACTICES

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We live in an age of disruption, including the COVID-19 pandemic, climate change, food security, and poverty (Maass et al., 2019). These events have increased focus on sustainability in government policy, industry, and education internationally (see UN General Assembly, 2015). For example, energy, resources and environmental change are elements of Australia's Science and Research Priorities and sustainability is a cross-curricular priority in the Australian Curriculum. Similarly, the curriculum framework for German federal states includes sustainability as a key element, currently implemented in secondary mathematics level 1 and under development for secondary level II.

Solutions to sustainability problems often rely on the application of Science, Technology, Engineering and Mathematics (STEM), which often includes the use of modelling or applying models to very large data-sets in order to understand or make predictions about a phenomena. These problems and proposed solutions are communicated to the public through mathematics and statistics – typically via mass and social media. Thus, a capacity to interpret information and and evaluate findings from the analysis of data, often communicated as scientific, mathematical and/or statistical evidence, is key to a STEM literate citizenry and workforce (Gal & Geiger, 2022).

The capabilities associated with STEM literacy and STEM careers include the ability to pose questions, select relevant data-sets, evaluate the quality of data, and analyse,and critically evaluate findings (e.g., Duncan et al., 2018) – which we term evidentiary practices. These are key to making prudent decisions for individuals and societies. Little is known, however, about teaching approaches that can develop students' evidentiary practices, especially when using very large data-sets.

In this session, we will present the findings of a review aimed at synthesising relevant research literature concerned with how evidentiary practices are applied in studies that focus on mathematical modelling and very large data sets. Our presentation will address:

1. The nature of evidentiary practices
2. The connections between evidentiary practices and mathematical modelling
3. Findings related to the use of models and modelling to understand and/or analyse large data sets within educational settings.

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WHAT IS THE ROLE OF MODEL IN THE MODELINNG PROCESS

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Research on the modelling process has examined students' difficulties with modelling tasks and specific modelling routes (Blum et al. 2009). In this study, the role of the model is discussed using the example of the Can problem as one perspective from which to examine the modelling process.

Can problem and modelling process The Can problem is "Find the height and radius of the can that minimises the surface area so that less material is used to make a cylindrical can with a volume of 300 ml? "If the height of the cylinder is h and the radius is r, then $S=2\pi r^2+2\pi rh$, $V=\pi r^2h$. Taking the volume as 300ml and the surface area as a function of the radius, $S=2\pi r^2+600/r$. This is the first model: Minimum surface model. This model gives us to a minimum surface area at h=2r. This result allows us to observe real cans in everyday life. We find another problem, "Aren't real cans designed to minimise the surface area?" We collected data on cans and investigated some relationships, diameter and height, and surface area and volume. There is no relationship between diameter and height as $h\neq 2r$. We can get the formula $S=5.81V^{0.66}$ as the relationship between surface area and volume of real data. We find the second model, $S=5.81V^{0.66}$: Real can model. To test the adequacy of this model, data from real drum can is used, the height of is 89cm, the diameter is 58cm. At the same time, the relationship between surface area and volume is investigated under h=2r conditions, where the surface area is minimised. Expressing the surface area as a function of the volume gives $S=\sqrt[3]{54\pi}\times V^{2/3}=5.54V^{2/3}$. This is the third model that Ideal can model. Real can model helps us to create the third model. We are surprised to see Real can model and Ideal can model, these models are very similar to each other, order and coefficient, although the hypotheses are different, $h\neq 2r$ and h=2r. We meet again a new problem, "How much does the surface area of a real can increase compared to the surface area of an ideal can of the same volume?" and "How does the surface area of a can of constant volume change when its height and diameter change?". We make hypotheses, h=2tr, in order to specify more accurately capture the relationship between height and radius. We obtain the fourth model, $R(t)=S(t)/S_{min}=1/3(1+2t)\times t^{-2/3,}$ the Material Efficiency model (Figure 1). The second and third models give rise to new problems and contribute to the elaboration of hypotheses.

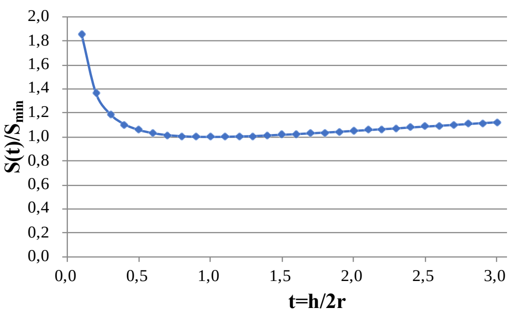

Figure 1 Material Efficiency

Discussion We see the role of models, they help to collect and examine the data, to find a new hypotheses, elaborate hypothesis, and new problems. The review of the tentative model provides opportunities to facilitate the various phases of modeling activities.

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CONCEPTIONS, USES AND OBSTACLES IN THE TEACHING AND LEARNING OF MATHEMATICAL MODELLING

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This paper presents the advances of the research entitled "Conceptions, uses and obstacles in the teaching practices to strengthen the process of mathematical modelling in basic students of Bogotá, Colombia South America". The objectives are a) determine the conceptions and uses in the teaching of mathematical modelling by teachers in the town of Fontibón, Bogotá, Colombia; and b) recognize the obstacles in the learning of mathematical modelling from the perspective of teachers.

When constructing the theoretical framework in relation the epistemological analysis of conceptions theoretical construct on mathematical modelling process (Blomhøj, 2019; Blum and Leiß, 2007; Frejd, 2014 Fulano and Barrios, 2021); the uses the teaching of mathematical modelling (Villa-Ochoa, 2019) and obstacles in the learning (Brosseau, 2007; Piaget, 1970).

The method the present research is qualitative, with a descriptive scope, through a structured interview applied to seven mathematics teachers from official schools; five women and two men with an average age of 45 years. The categories conception, use and obstacles are found in an internal consistency matrix validated by experts.

The results on the conceptions category, teachers use mathematical modelling as a process that involves solving problems inside and outside of mathematics. Regarding the category of obstacles, the ontogenetic ones associated with difficulties related to the immaturity of the student and the use of technological tools are based first; second, in thinking skills there are problems associated with recognizing and relating variables, generalizing, formulating hypotheses and validating the model; Third, in the affective domain, both intrinsic and extrinsic motivation difficulties are evident.

The conception of mathematical modelling is associated with problem solving, teachers use it as a tool to strengthen thinking skills. In addition, there are obstacles related to ontogenetic aspects, thinking skills, affective domain and administrative aspects that affect modelling learning.

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IDENTIFYING AUTHENTICITY AND MATHEMATICAL MODELLING COMPETENCIES AS STEPPING STONES FOR BLENDING DIFFERENT PERSPECTIVES

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The presence of diversity in the field of mathematical modelling within the realm of mathematics education has generated both advantages and disadvantages. The existence of diverse perspectives on mathematical modelling has resulted in a lack of consensus regarding the specific methods to integrate it into the curriculum (Ferri, 2013). A wide range of approaches to mathematical modelling has been identified, and the current landscape encompasses various perspectives such as Realistic, Educational, Models and modelling, Socio-critical, and Epistemology (e.g., Abassian et at., 2019; Ferri, 2013; Kaiser & Sriraman, 2006), offering intriguing considerations such as the frequency of use of these perspectives across different countries.

It is plausible that the advancement of curriculum development can be achieved through the networking or integration of fundamental concepts and assumptions from several perspectives. However, integrating these central ideas should be approached with meticulous consideration, as advocated by Scheiner (2020). Embracing a singular stance entails endorsing the notion of incommensurability put forth by Scheiner (2020), which consequently leads to divergent trajectories in mathematical modelling research. To foster a cohesive framework, it is imperative to consider the integration of multiple opposing perspectives to facilitate a convergent viewpoint.

Ledezma & Sala (2022) try to perform the task of networking theories, combining the cognitive perspective in mathematical modelling with onto-semiotic approaches. However, a comprehensive integration of multiple perspectives in mathematical modelling has not been accomplished. Addressing this gap, Kaiser & Sriraman (2006) proposes that adopting a Hegelian inquiry system may alleviate incommensurability resulting from conflicts among diverse perspectives. Moreover, Abassian et al. (2019) delve into several key aspects of mathematical modelling: goals, definition of a mathematical model, description of the mathematical modelling cycle, design of the task, examples of key researchers, and research focus, which are discussed in depth. However, when blending multiple opposing perspectives, it is pivotal to consider the core concepts of authenticity and mathematical modelling competencies (Kaiser & Brand, 2015) within each perspective. This study concludes that these concepts are vital to blend these perspectives, presenting an analysis across them.

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THE ROLE OF STUDY AND TECHNOLOGY IN MATHEAMTICAL MODELLING

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In recent years an emphasis has been put on mathematical modelling drawing on technology (Siller et al., 2022). Their study show that technology is often used to find information and data, exploring possible solutions, formulating problems, visualisations and calculations. This often links to students' pragmatic uses of technology. However, Jessen & Kjeldsen (2022) argue that the productive use of technology requires epistemic uses of technology done by students as well and demonstrate in two cases how such activities have furthered students learning of mathematics through modelling and the development of modelling competency.

These findings align with the claim of Artigue and Blomhøj (2013) who argue that modelling can be seen as an approach to inquiry based mathematics education, and Chevallard (2008) who argues that the process of study is an overlooked and sometimes ignored part of inquiry based approaches to education. In this study we revisit existing papers on mathematical modelling to analyse the role of study with an emphasis on how technology provide students with resources to be studied. This may be teaching resources, but also students' own productions such as calculations, data collections or visualisations needed to be scrutinised in order to fully grasp and solve a certain modelling problem.

The papers analysed will cover different approaches to mathematical modelling and feed into the discussion of how theoretical frameworks for mathematical modelling impact task design for the teaching and learning of mathematical modelling (Barquero & Jessen, 2020), though adding a new perspective regarding how to support students' autonomy in the modelling process and how this strengthens the authenticity of the activities.

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LEARNING OF DIGITAL TOOL COMPETENCIES IN THE CONTEXT OF MODELLING

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The use of digital tools in teaching can foster the development of competencies in mathematics (Hillmayr et al., 2020) and especially in mathematical modelling (Cevikbas et al., 2023). Furthermore, studies show that digital tools can be useful in most sub-competencies of mathematical modelling. However, there is still a need for research on how exactly digital tools can be useful for improving modelling competencies (Greefrath et al., 2018). This leads to the relevant question for teachers how digital tools should be introduced and how they should be practised in mathematics classes.

The students could improve their digital tool competencies via an intramathematical or a contextual learning environment. The open question is: Which way of instruction is better for improving digital tool competences and modelling competencies? We present the research project *Modi+*, in which, we analyse the effect of the type of instruction in the dynamic geometry software *GeoGebra* (intramathematical vs. in context) on the use of the digital tool and mathematical modelling. Before the instruction in digital tools, secondary school students (N = 150) were tested in their *GeoGebra* competencies. After the instruction students were tested in their *GeoGebra* competencies again and solve modelling problems. During the instruction students of one group learned to use *GeoGebra* by working on intramathematical tasks and students of the other group learned to use it by working on tasks in the context of geometrical modelling.

In the talk, we will illustrate the development of the digital tool competence test and introduce its quality criteria. Further, we will give insights in the first results on the effect the type of learning to use a digital tool has on the acquisition of digital tool competencies. We expect that students, who were in the intramathematical group, increases stronger their digital tool competencies than students, who were in the context group.

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TEACHING MATHEMATICAL THINKING: REASONING, MODELLING, PROBLEM SOLVING

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We describe how we teach *mathematical thinking* to first year engineering students at Chalmers. We see our more recent focus on mathematical thinking as a development of our earlier teaching where we have previously for many years taught mathematical modelling and problem solving to engineering students in their second year, as well as to mathematics teacher students (Wedelin & Adawi, 2014; Wedelin & Adawi, 2015). A basic motivation is to complement the ordinary university courses in mathematics, as well as our students' mathematical background from secondary school.

For this purpose, we find the notion of mathematical thinking attractive as it starts with everyone's natural ability to think mathematically. Mathematical thinking can therefore be applied by anyone from the beginning, which is a different point of view than seeing mathematics as a body of knowledge that first needs to be learned before we can do anything mathematical. This enables us to maintain a link between common sense and mathematics, to capture creativity in mathematics, and is helpful in understanding how mathematics and other scientific fields have evolved.

Over the years, we can see that in our own development we started out with a desire to teach mathematical modelling, discovering that it was useful to explicitly combine with concepts of problem solving as known in the mathematical literature. In this new course, we additionally include selected concepts and insights of mathematical reasoning which we have found are also useful to complement ordinary mathematics education. So in summary, we conceptualize mathematical thinking in terms of three main components: *reasoning*, *modelling* and *problem solving*.

It is clearly not possible to develop your mathematical thinking without engaging with actual problems, applied and theoretical. So while we have established a structure and a language to talk about mathematical thinking, the main focus must be to actually work with different kinds of problems. In order to clarify the purpose of mathematical thinking, we have chosen to group our problems in four main categories: *keeping track*, *investigating the abstract*, *investigating the world*, and *designing*. We also link to mathematical knowledge by highlighting different kinds of models and their associated mathematics, e.g. *functions and equations*, *optimization*, *probability and statistics*, and so on.

We discuss how our teaching is received by our students, and finally provide some general experiences and reflections.

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TAKING UP OWNERSHIP: WHAT CAN BE LEARNED FROM POSING MODELLING PROBLEMS?

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The central ideas of mathematical modelling include formulating conditions and assumptions to establish models and evaluating their impact on solving real-world problems (Niss & Blum, 2020). However, these activities are often overlooked, superficially addressed or cause difficulties when students solve modelling problems (Galbraith & Stillman, 2001). Hence, students require support in developing the modelling competencies that are necessary to master these activities. Prompting students to pose their own modelling problems could be a promising teaching approach for developing modelling competencies. Problem posing comprises the reformulation or generation of problems and involves processes such as problem analysis, condition variation, and problem evaluation (Baumanns & Rott, 2022). These processes may enhance modelling competencies such as making assumptions and validating the solution. However, research investigating problem posing approaches for modelling is still limited.

In the MoPro project (Modelling with Problem Posing), we aim at investigating the effects of problem posing on students' modelling competencies and motivation. First, we conducted a qualitative study to determine the processes and the difficulties that occur when students generate and reformulate modelling problems. Second, we developed and evaluated a test to assess competencies for generating and reformulating modelling problems. In a next step, we will investigate the effect of a teaching approach where students pose and solve modelling problems in comparison to an approach where students only solve modelling problems.

In this presentation, we focus on the results of the qualitative study on problem posing processes. Sixteen ninth- and tenth-graders, working in pairs, were prompted to reformulate and generate modelling problems. Through qualitative content analysis, we identified different processes that took place while students reformulated and generated modelling problems, and we evaluated the importance of these processes for building modelling competencies. For example, we observed students engaging in setting up and varying situational and numerical assumptions and discussing how these assumptions influence the authenticity and the difficulty of the problem. The findings contribute to a better understanding of the processes involved in reformulating and generating modelling problems. They also provide a basis for developing teaching approaches that encourage learners to take ownership of their own problems.

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MATHEMATICAL MODELLING AT ALL YEAR LEVELS IN AUSTRALIA

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In Australia, a new mathematics curriculum (Australian Curriculum, Assessment and Reporting Authority [ACARA], Version 9.0, 2022) is being introduced. It covers primary and secondary school mathematics classes from Year 1 to Year 10 (students aged from five years to about 16 years). At every year level, the new curriculum specifies mathematical modelling as a mandated topic, although the extent to which some Australian states will vary this requirement is not yet known. In Australia, individual states are free to adopt the ACARA curriculum in full or adapt it to their own requirements, perhaps by providing student work samples which contain cultural content pertinent to the local area.

The ACARA (2022) curriculum includes compulsory topics and examples, together with non-compulsory "elaborations" which may be of value to particular groups such as First Nations peoples or high-achieving students. The use of technologies is sometimes specified, especially computer-based technologies such as spreadsheets.

The presentation will consider the background of the "proficiency strand" topics which have been supplanted by the introduction of mathematical modelling as a separate topic. In addition, the declared theoretical underpinnings of the revised curriculum will be considered.

The presentation will discuss responses to the new curriculum as a case study. It will examine the work of publishers, including Oxford (Garvey, 2023), who produce hardcopy textbooks for Years 7 to 10, and Mathspace (n. d,), an online interactive tutoring system which caters for students from Year 4 and upwards. These publishers have been amongst the first to respond, supplying resources in time for the first Australian states to implement the revised curriculum in 2024.

The revised curriculum envisages mathematical modelling introduced at Year 1. This is in the context of "use mathematical modelling to solve practical problems involving equal sharing and grouping; represent the situations with diagrams, physical and virtual materials, and use calculation strategies to solve the problem" (ACARA, 2022, AC9M1N06). In Year 2, mathematical modelling is used "to solve practical additive and multiplicative problems, including money transactions, representing the situation and choosing calculation strategies" (ACARA, 2022, AC9M2N06). These objectives may seem ambitious, or as stretching the meaning of mathematical modelling, and it will be interesting to follow how systems, publishers, schools and teachers implement the curriculum – especially for the most junior classes.

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THE INVESTIGATION OF THE MODELLING COMPETENCIES OF TWO GROUPS OF GRADE EIGHT STUDENTS WHO RECEIVED DIFFERENT MODELLING INSTRUCTIONS

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With the promotion of mathematical modelling in the teaching and learning of mathematics in China, an increasing attention has been paid to which teaching approaches are appropriate to promote students' modelling competencies.

In this study, we measured two groups of grade eight students' modelling competencies who received two different ways of modelling instructions. Group A received a kind of modelling instruction which was consistent with the traditional approach of mathematical teaching that was teacher-centered and demonstration-oriented; and the instruction was led by an experienced teacher. Group B received the modelling instruction that organized in the way of group cooperative learning, with five to six students working together on modelling tasks; and there was one pre-service teachers provided interventions for each student group. Group A, which involved 45 students, received modelling training in June 2022; while Group B included 32 students was involved into the training in May 2023. According to their class teachers, the two groups performed equally on the learning of school mathematics. After receiving the modelling instructions, the two groups of students were immediately asked to individually complete a modelling test which includes three modelling tasks.

In this paper, we presented the measurement of the two groups' performances on the two same modelling tasks in the modelling test they received after the modelling instructions. A hybrid approach (Cevikbas et al., 2021) was employed to assess the students' modelling competencies through their performances on the two tasks, which includes a holistic approach which concentrated on students' full-scale modelling process and an atomistic approach which focused on the phases of simplifying situations and making assumptions. The research results will reveal the differences in the students' performances on the modelling tasks in terms of holistic and atomistic aspects of modelling competencies, and further imply the influences of different modelling instructions on students' modelling competencies.

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VALIDATION IN SIXTH-GRADE DATA-DRIVEN MODELLING

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Validation in mathematical and statistical modelling is crucial for all citizens to understand and critique mathematical and statistical models in a data-driven society (e.g., Czocher et al., 2018; Gal & Geiger, 2022). However, the nature of validation and modification attempted by young learners remains unclear. To address this research gap, our study focused on validation with sixth-grade students during the modelling of a data-rich situation, that is, *data-driven modelling* (DDM). DDM refers to activities involving the generation, validation, and modification of models (mathematical and statistical models), based on data/context to make better predictions and estimates (Kawakami, 2023). In the framework, mathematical and statistical models reflect the subject's deterministic and stochastic interpretation of the data/context, respectively. The framework can conceptualise data-cantered validation, which compares models for prediction and estimation with the actual results, and subsequent modification related to models and data.

We addressed the following question: *What attempts do sixth-grade students make when comparing models for prediction and estimation with the actual results during DDM?* For this purpose, we designed and implemented a DDM teaching experiment consisting of 7 lessons for 35 sixth-grade students. The students worked in groups on an authentic *ring-decorating task,* developing models for estimating the number of origami sheets needed to decorate the gym with ring-decorations for a thank-you party they were organising. They repeatedly made assumptions, and generated, measured, structured, and represented the necessary data. After the lessons, they constructed the decorations, completed a task-based questionnaire in which they watched a video of the hanging of the decorations (because of COVID-19 school closures), and compared the models' results with the actual decorations, and modified the models if necessary. We analysed the data from the questionnaire in terms of what they focused on in their models, what they relied on for validation, and how they tried to improve their models.

The preliminary results show that the students validated the context in which the decorations were made, the way the data were collected and measured, and the assumptions of the models. Students who noticed measurement errors critically rethought a mathematical model that was considered deterministic and that assumed proportional relationships, and mentioned the need to reinterpret it stochastically and to modify it to a statistical model, taking into account errors and uncertainty. These findings suggest that validation can play an age-appropriate and innovative role in learning to model data-filled situations.

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STRENGTHENING TEACHERS' CAPABILITIES WITH BIG DATA

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We live in a time of disruption – not just digital disruption but also long-standing international challenges such as those associated with health, the environment, and food and energy security (see UN General Assembly, 2015). Responses to disruptions invariably rely on the use of mathematics and statistics (M&S). M&S are also used to update the public about the state of disruptions, make predictions about their progress, and to justify decisions made by authorities about responses – all communicated in public forums (e.g., mass and social media). This means that citizens must be able to understand, interpret and evaluate M&S information (Gal & Geiger 2022) to remain informed and empowered to contribute to a prosperous, just and equitable society (Maass et al., 2019).

The COVID-19 pandemic highlighted how M&S can be used as *evidence*, by government agencies, politicians, commercial entities, experts, and non-experts, to support arguments about how to respond to the crisis. These arguments often involved claims that were contradictory, inconclusive, and misleading. For citizens to understand how evidence is used in decision making and assess the veracity of claims made during debates, they must possess the capability to *critically* evaluate M&S *evidence* (Gal & Geiger, 2022). This requires processes described in science education as *evidentiary practices* (e.g., Duncan et al., 2018) – a foundation for prudent decision-making.

In this presentation, we describe a current study that aims to develop students' capabilities to evaluate claims and arguments in the media through the modelling of large data sets related to sustainability issues. The session will provide background to the study and opportunity to discuss the challenges associated with the design of tasks appropriate for students in Years 9/10 and relevant data collection instruments. Tasks are focused on issues such as levels of atmospheric CO_2, and international levels of poverty, among others. Each task is based on two media items that provide contradictory information about a disruption, aligned with a selection of at least three databases. Data collection instruments include observation, computer screen capture, surveys, and student interviews.

The outcomes of the research are focused on: 1. The processes used to identify misleading ot contradictory information within media items; 2. Students' choice of data base in response to a task; 3. Student modelling of selected data in respect to the issue of concern; and 4. Students' decision-making processes in forming judgements about the veracity of claims and arguments in a media item.

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IMPROVING UNDERGRADUATE STATISTICS EDUCATION: EDUCATIONAL LESSONS FROM PEDAGOGICAL EXPERIMENTS

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The main goal of this research is to fill in some gaps in the research literature on the teaching of statistics. This research includes two independent studies (experiments). The first study examined the instructional effects of physical versus virtual manipulatives on learning outcomes in introductory statistics, whereas the second study investigated the impact of different styles in teaching statistics (inverted classroom versus traditional classroom) on learning outcomes in introductory statistics. In general, this research attempted to join many other reform efforts to explore instructional ways that engage students in reasoning and modeling statistically. To combat the abstract nature of probability and statistics, the use of manipulatives may represent one of the most effective strategies in the statistics classroom. There are fundamental reasons to inherently value the inverted classroom’s emphasis on activity-based learning and increased responsibility of the students to become active participants in their own learning.

The results of the first study revealed that, in terms of GPA one year (as the outcome), there were no statistically significant interaction effects between types of manipulatives (the business-as-usual group received traditional concrete manipulatives and the experimental group received online virtual manipulatives) and high school ACT mathematics scores, informing the literature that ability levels neither intensify nor weaken the effects of types of manipulatives. The results of the study did not show a statistically significant difference in GPA one year later between the experimental group and the business-as-usual group.

The results of the second study revealed some significant differences between the business-as-usual group who received traditional lecture and the experimental group who received inverted instruction. We compared seven outcomes for the two groups: projects average, tests average, classwork, midterm attendance average, class final attendance average, midterm grade, and class final grade. After controlling for student individual background (age, gender, and ethnicity), students in the traditional classroom did better than students in the inverted classroom in projects average, overall classwork, and midterm grade. After controlling for high school background (high school GPA and ACT mathematics scores), students in the traditional classroom did better than students in the inverted classroom in projects average, overall classwork, and midterm grade. After controlling for university program background (university cumulative GPA and student major), students in the traditional classroom performed similarly to students in the inverted classroom in all seven outcomes. Finally, when controlling for all (i.e., student individual background, high school background, and university program background), students in the traditional classroom did better than students in the inverted classroom in midterm grade only.

PRE-SERVICE TEACHERS' NOTICING OF MATHEMATICAL MODELLING PROCESSES

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For teaching mathematical modelling in a classroom setting broad knowledge about many aspects of mathematical modelling, such as modelling tasks, modelling cycles, multiple solutions and potential difficulties (as described in the 4-dimensional model by Borromeo Ferry and Blum (2010)) is needed. Moreover, it is essential that teachers are able to react spontaneously to solution approaches, which were not anticipated, and students' learning obstacles, which can sometimes not be foreseen. In more detail, teachers need to selectively perceive noteworthy aspects in a situation with an overwhelming amount of information, interpret the perceived incident by relying on their knowledge and weigh different options to react. This competence of *noticing* is important not only for teaching-and-learning-processes in general, but for mathematical modelling teaching-and-learning-processes in particular. It also differs among novice and expert teachers (Cai et al. 2021).

The development of noticing competencies should therefore be already supported in teacher education, especially as it links theoretical knowledge offered at university courses and practical activities in school. To bridge this gap, link theory and practice and prepare pre-service teachers for their future work at school, a seminar about mathematical modelling was offered, which included theoretical concepts on the one hand and the analysis of practice-oriented artifacts, such as students' written solutions, text vignettes and videos on the other hand in order to foster pre-service teachers' noticing competencies. Thus, the following research questions are posed:

To what extent were pre-service teachers able to notice important events within modeling processes? How did their ability to notice differ when analyzing classroom sequences displayed by videos from their ability to notice when analyzing text vignettes?

To answer the research questions pre-service teachers noticing competencies were evaluated with a video-based test and text-vignettes, which included analogous situations, e.g., the same difficulty occurs although the modelling problem differed. The instruments were used before and after several master seminars with 55 pre-service teachers. The answers of the participants were qualitatively analyzed. It can be hypothesized that video-based formats capture perception more precisely, while especially interpretation and decision-making can also be assessed sufficiently by text-based formats.

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COMBINING BRUTE FORCE AND IT TO SOLVE A CLASSICAL SHOPPING PROBLEM

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Two classic word problems with a similar background are mathematically modeled and a solution is provided. The first problem, the puzzle of the "Dutchmen's Wives", can be found as early as 1739 in a women's magazine (May (1739)**.** The Ladies Diary). A similar problem ("Find Ada's Surname", Workman (1906)) was formulated at the beginning of the last century. The mathematical model leads to a Diophantine quadratic equation, which can be solved using the third binomial formula and a prime factor analysis. First, a "stupid trial and error method" (brute force) is used to search for a solution. The CASIO-calculator/emulator ClassPad II will be used.

The first word problem: the puzzle of the "Dutchmen's Wives" "There came three Dutchmen of my acquaintance to see me, being lately married. The men's names were Hendrick, Claas, and Cornelius; the women's, Geertruii, Catriin, and Anna. But I forgot the name of each man's wife. They told me they had been at market to buy hogs. Each person bought as many hogs as they gave shillings for each hog. Hendrick bought 23 hogs more than Catriin, and Claas bought 11 more than Geertruii; likewise, each man laid out three guineas more than his wife. I desire to know the name of each man's wife." May (1739), Dudeney (1917), Hemme (2019).

The second word problem: the puzzle "Find Ada's Surname" Dudeney (1917), Workman(1906,1918)

We want to sensitize the pupils to MINT (**m**athematics, computer science (**i**nformatics), **n**atural sciences and **t**echnology) and to link interest with personal experiences. We show the pupils that MINT is much more than the cliché of mathematics and computer programming. For this purpose, the inhibition threshold for dealing with MINT should be lowered in the school project on the basis of playful, project-oriented didactics and at the same time interest in the topic should be encouraged. Various projects are available. "In conclusions, we point to the fact that there are several ways to demonstrate interrelations between traditional Maths and IT." Hvorecký, Korenova, Barot (2022).

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A STUDY ON MATHEMATICAL MODELING UTILIZING APOLLONIUS' CIRCLE

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The purpose of this paper is to present a comparison of student reality in a paper-drawn and GeoGebra-drawn mathematical modelling and problem-solving class in the field of graphics for third-year junior high school students, using the everyday and easily interesting subject ”Where can I see Tokyo Tower and Sky Tree at the same height?".

As a result of 2015, it became clear that the learners used logical thinking in various situations in the process of mathematical modelling problem solving in this problem. (2015).

The study suggested that it is possible to realize what Ikeda calls "the flow from intuitive, analogical, and inductive understanding to deductive understanding in a curriculum sequence, and to teach in such a way that students can experience the deepening of logic".

2022, school education in Japan is changing drastically with the deployment of a tablet computer for each student in junior high schools. Therefore, it is now possible for students themselves to open their tablets and use GeoGebra to draw Apollonius' circle on a map of Tokyo, including Tokyo Tower and Sky Tree.

A post-class survey showed that students were more interested in Apollonius circles when using GeoGebra, and were also more interested in the use of mathematical modelling. Furthermore, these two results will be used to discuss the difference between the thinking of drafting regarding mathematical modelling with GeoGebra and the thinking of drafting with a compass and ruler.

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EXPLORATORY STUDY ON STUDENTS' UNDERSTANDING OF SIMULATIONS - HOW MATHEMATICAL MODELLING COMPETENCY NEEDS TO BE COMPLEMENTED TO PROMOTE UNDERSTANDING OF SIMULATIONS

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Computer simulations have been a central method in scientific findings for a long time. Scientists use simulations to test their hypotheses, generate new ones and make predictions. In addition, simulations are used for communicating scientific results. Experts exchange information among themselves or communicate their knowledge to politicians and other stake holders. Therefore, simulations serve as basis for decisions of social relevance. To maintain credibility the decision-making process of persons in charge is explained by sharing (simplified) simulations results through the media. This is particularly important when political decisions require actions from the citizens as for example in the case of the COVID-19 pandemic. Consequently, computer simulations are no longer just the matter of scientists, they are part of our everyday life and the informed citizen and consumer need to be able to reflect simulation results critically. Hence, it is the responsibility of schools to impart competencies for understanding simulations (OECD, 2018, p. 26).

This exploitative study aims to come up with a theoretical construct of understanding simulations. What does it take to understand simulations? Which kind of competencies and skills are needed to be able to access all information provided by the simulation and put it into the right context?

After analysing various definitions of the term computer simulation, two main characteristics emerge: Computer simulations are based on mathematical models and the results are the consequences of the system dynamics (e.g. OECD, 2018, p.18). Therefore, we propose that modelling competency is a part of understanding simulations, as well as understanding mathematical models themselves and system thinking.

To discuss these hypotheses critically and identify more partial competencies and skills, semi-structured interviews with experts on computer simulation are being conducted. By engaging experts from various fields such as education, philosophy, engineering, systems theory and computer science, we aim to explore understanding simulations from multiple perspectives and gain comprehensive insights. Outcomes of this interview study will be presented at the conference.

This interview study is part of a broader investigation with the overall aim of developing learning sequences on computer simulations.

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MATHEMATICAL MODELING PROJECTS IN ADVANCED ORDINARY DIFFERENTIAL EQUATIONS COURSES FOR MATHEMATICS AND ENGINEERING STUDENTS

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Differential equations (DEs) are used for modelling a wide spectrum of phenomena in engineering, physics, natural and social sciences and are taught in most STEM programs. Traditional courses concentrate attention mainly on different techniques for solving DEs, whereas contemporary approaches to teaching and learning of the subject tend to emphasize qualitative, graphical, and numerical methods. Despite the centrality of DEs in the undergraduate curricula, educational research is scarce and focuses primarily on the understanding of the concept of DE, solutions to first order DEs and systems of DEs whereas "mathematical modeling and the role that technology can play in the modeling process are missing" (Rasmussen & Wawro, 2017, p. 555).

Recent research addressed the use of mathematical modelling (MM) as a vehicle for more authentic learning of DEs (Czocher et al., 2021), how the emphasis on MM principles can be beneficial for students in standard DEs courses (Czocher, 2017) and how modelling tasks motivate the development of new explorative mathematical routines contributing to the improvement of students' mathematical and programming skills (Rogovchenko, 2021).

In this paper, we discuss how MM projects were integrated in two DEs courses at the University of Agder, Mathematics for Mechatronics for senior engineering students and Nonlinear Differential Equations and Dynamical Systems for graduate mathematics students. In the former course, MM projects were included in summative assessments during the semester and included student small group work, submission of written individual reports and presentation of the projects to the class. In the latter course, students could choose to work on theory based or MM individual projects, and submission of individual written reports and presentation of the projects were compulsory for the admission to the final oral exam. In both cases MM projects included experimental parts. We analyse and compare students' work on MM projects and projects' impact on students' learning in courses where there was little room available in the syllabus for addressing MM in more detail.

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MODELLING TASKS FOR NON-STEM UNIVERSITY STUDENTS AND THE CHARACTERISTICS OF THEIR MODELLING PROCESS

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In Japan, the teaching of *mathematical modelling* (MM) for all university students, including non-STEM students, is attracting more attention depending on the spread of data sciences in our society. However, actually, most Japanese university students, especially non-STEM students, do not have the opportunity to experience MM in their educational history, due to the lack of MM courses in school and university. Here are our research questions: *What tasks can prompt non-STEM students' MM activities? How is their MM process, and how does it differ from STEM students?*

Regarding the questions, we developed a one-semester MM course with the following topics: (1) the Japanese government's project to halve the number of deer and wild boar in Japan (cf. Wild animal population control, 2014), (2) the effectiveness of a new AI-based cancer detection (cf. Klein et al, 2021), and (3) changes in the market share of smartphone operating systems (cf. Statcounter, 2023). The MM task for each topic can be approached using a recurrence formula of a sequence, Bayesian estimation, and Markov chains, respectively. The course was conducted in the spring and the autumn semesters of the 2022 academic year with mixed but unbalanced classes of STEM and non-STEM students: non-STEM students dominated 78% of the spring class (non-STEM class); STEM students dominated 89% of the autumn class (STEM class). Students worked in groups of 3-4. Activity logs and reports on each task produced by the groups were collected and qualitatively analysed.

The results are as follows. In the non-STEM class, students' awareness of the mathematics used in MM activities was weaker and the models were less diverse than in the STEM class: the recurrence formula in Task (1) appeared only after the teacher's suggestion to express models in mathematical terms; only the Markov chain model was used in Task (3). Despite these differences, rich MM activities, making assumptions, simplifying the real situation, and evaluating, modifying the models, were observed in both classes for Task (1), (3); the differences seem to be eliminable with the cooperation of both students and with the help of the teacher. On the other hand, such activities were poor in both classes for Task (2). This was probably because the mathematisation from the situation models was the main activity for Task (1), (3), whereas, in the case of Task (2), it was to understand the terminology and formulas used in the medical field that is based on Bayes' theorem, and then to interpret the real situation within them. That seemed to be difficult for the students of both types.

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COMPARING AND CONTRASTING STATISTICAL AND MATHEMATICAL MODELLING – A STUDY OF THE ROLE OF DATA

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Data is depicted as a central part of the modelling cycle as "the epistemological basis for the different sub-processes" (Blomhøj & Kjeldsen, 2006, p. 166) and as a crucial role as a driver for the inquiry process and a unique component of the media to be studied (Chevallard, 2019). To question and study data are vital for statistical inquiries leading to statistical modelling and the development of statistical literacy and reasoning. Similar aims can be said for mathematical modelling where modelling can drive the learning of mathematics. According to Chevallard (2019) can data both be quantitative and qualitative in nature (possibly gathered as part of the inquiry or modelling process).

We compare and contrast existing research regarding statistical and mathematical modelling with an emphasis put on the role of data and how data may shape the inquiry and modelling processes conducted by students. Our main focus is two Danish Study and Research Paths (SRP; Chevallard, 2006) studying the real-world problems of 'health and physically activity' with grade five students (Østergaard & Larsen, submitted) and 'facial recognition system' with upper secondary school students (Jessen, 2022). We found, how the dynamic data visualisation tool, TinkerPlots, and the dynamic geometry software, GeoGebra, drew students' learning processes and were crucial milieus; to generate first hypotheses, to question and explore students' collected data and answers found in media and, hence, as a condition for the questioning and media-milieu dialectics.

We further elaborates on the development and implementation of SRP in classrooms, and we question specific views of interaction between teachers and researchers and investigate the hypothesis that research could work as an integrated part of paradidactic infrastructure (Miyakawa & Winsløw, 2013), such as in the format of lesson study and other forms of teacher–researcher collaborations.

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ASPECTS OF DATA MODELLING AMONG JAPANESE JUNIOR HIGH SCHOOL STUDENTS: FOCUSING ON CRITICAL CONSIDERATION

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In recent years, research linking mathematical and statistical modelling has been actively promoted (e.g. Ärlebäck & Kawakami, 2023; Kawakami & Mineno, 2021). In Japan's new Courses of Study, published in 2017, there is an urgent need to enhance statistics education and develop students' abilities to critically examine data through lessons. Thus, practical research must be conducted. The purpose of this study is to describe the actual mathematical activities demonstrated by students in the second grade of a Japanese junior high school through data modelling (Pfannkuch et al., 2018), an activity type of statistical modelling. The study also aims to clarify the level of criticality demonstrated by the students.

The lessons were based on the cycle of PPDAC (Wild & Pfannkuch, 1999), and a 16-h unit plan was developed and implemented. In this unit, we asked, "How can we improve our peripheral vision skills?" To identify the characteristics of students with faster times in the "Touch the Numbers" action game application, which requires quick number pressing, we conducted an analysis to determine the factors contributing to their speed. The questionnaire (Q1: grade; Q2: time spent using smartphones and games; Q3: favourite subject; Q4: club activities; Q5: frequency of reading) was cross-tabulated with time and the data were analysed based on statistics (number of data points, minimum, maximum, mean, and median), histograms, frequency tables, and box-and-whisker diagrams.

The following aspects of the students were observed in class situations, in which they exchanged analysis results through group activities: 1. When summarising the findings, students tended to incorporate the conclusions of other groups' analyses into their own; 2. In data analysis, it was difficult for students to identify "no trend"; 3. Difficulties were observed in critically interpreting information beyond the data. These findings can serve as guidelines for improving future data modelling classes. In addition, this study revealed that students constructed new mathematical models from the given statistical models when problem-solving. These observations provide suggestive evidence of the relationship between mathematical and statistical modelling.

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WHAT ARE THE POTENTIAL RELATIONS BETWEEN MATHEMATICAL MODELLING AND COMPUTING EDUCATION – THE CASE OF SIMULATIONS

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Data science and computing education in upper secondary mathematics has sparked debate among scholars about the traditional view of math being a gateway to STEM (Boaler & Lewitt, 2019; Conrad & Mazzeo, 2022). Mathematical modelling has emerged as a way to connect STEM and math, but further research is needed to understand technology's role in this integration (Niss & Blum, 2020; Siller et al, 2022).

This paper explores the use of technology in mathematical modelling through simulations, building on previous case studies in this area (Greefrath & Siller, 2017). Drawing on data science education elements (Heinemann et al., 2018), we investigate the role of technology in a Danish upper secondary math case study where students work on "Are boys better at mathematics than girls?"

Our study analyses audio and screen recordings to assess how students use technology focusing on question-answer dynamics and media-milieu interplay (Jessen, 2022). We ask: What are the potential relations between mathematical modelling and data science when teaching simulations in upper secondary school? Can task design that facilitates explicit media-milieu dialectics support students learning of both.

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DIAGNOSTIC COMPETENCE OF PRESERVICE TEACHERS IN THE CONTEXT OF OPEN MODELLING TASKS

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Open problems take on an important role in mathematics education. One particular type of open tasks are open modelling tasks, which are characterized by their relation to reality and require demanding translation processes between reality and mathematics (Niss & Blum, 2020). Researchers repeatedly underlined the necessity of teaching students how to solve open modelling tasks in the classroom (Krawitz et al., 2018). For the teaching of modelling tasks, (preservice) teachers should be able to assess students' performance on open modelling tasks (Greefrath et al., 2022). Indeed diagnostic competence is a competency facet in the models of teachers' professional competence in modelling (Greefrath et al., 2022) and should address also the openness of the modelling tasks. These tasks require students to (1) notice missing information and (2) make realistic assumptions for this missing information (Krawitz et al., 2018). However, empirical considerations on the assessment of student solutions to open modelling tasks have been lacking. To address this research gap, we conducted an interview study in the framework of the DOModA project (Diagnose von Schülerlösungen zu Offenen Modellierungsaufgaben, Diagnostic Modelling Competencies of Student Solutions on Open Modelling Problems). The aim of this study is to gain insights into the diagnostic competencies of (preservice) teachers in the context of open modelling tasks.

Ten preservice teachers at the University of Münster were first asked to solve four open modelling tasks and then to rank various student solutions to these open modelling tasks from good to worse and justify their judgements. The interviews take part in a pair of two preservice teachers. The student solutions were designed following the model by Krawitz et al. (2018). The student solutions differed regarding (1) whether an assumption was made for a missing data, (2) whether the assumption was realistic, (3) whether the openness was identified in the answer.

One important finding of the study is that the preservice teachers do not make assumptions, when they solve modelling problems. Furthermore, preservice teachers do not perceive the necessity for assumptions and do not expect those from students. These findings contribute to a better understanding of how preservice teachers judges student solutions regarding open modelling tasks.

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DIMENSIONS OF THE PEDAGOGICAL CONTENT KNOWLEDGE IN A FIRST MATHEMATICAL MODELLING PRACTICE: THE CASE OF LUCY

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Implementing modelling practices in the classroom is still challenging for many teachers (Niss & Blum, 2020). To overcome these challenges, we understand that teacher training in Mathematical Modelling, in addition to contemplating theoretical knowledge, needs to articulate knowledge associated with modelling and its use in classroom practices.

Our research focused on teacher knowledge necessary for teaching, specifically the Pedagogical Content Knowledge (PCK) proposed by Shulman (1986). Borromeo Ferri and Blum (2010) showed four dimensions of teachers' PCK for modelling: a theoretical dimension, a task dimension, an instructional dimension, and a diagnostic dimension. In general, each of these dimensions encompasses specific knowledge that the teaching of mathematics, mediated by modelling, and the teaching of modelling itself require from the modeller teacher.

In this article, we seek to reflect on the following questions: what dimensions of the PCK manifest in planning and implementing a first practice with modelling from an in-service teacher? How are these dimensions articulated in the planning and implementation of the practice? To address these questions, we conducted a case study (Yin, 2002) – the case of Lucy. In 2021, Lucy attended a course on Mathematical Modelling from the Teaching Perspective, where the collaborative planning of practice with modelling and its implementation occurred.

The qualitative analysis, structured via Trees of Association of Ideas (Spink, 2013), was supported by data from audio and video recordings of the classes of the subject in which the planning was carried out and the implementation of the practice by Lucy with her 8th-grade students. (13 years old). The results revealed that the PCK dimensions were articulated through teachers' interventions during collaborative planning and the report of the implemented practice, as well as in Lucy's actions during classroom management. These results reinforce the need to improve new designs for training courses that promote the articulation of the teachers’ PCK dimensions.

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AN ETHNOMODELLING PERSPECTIVE OF EMIC, ETIC, AND DIALOGIC ANALYSIS OF ETHNOMODELS IN THE AFRO-DESCENDANT CARIBBEAN DANCE *PALO DE MAYO* OF COSTA RICA

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This study is the result of a master's research on the traditional dances of Costa Rica, which was held at the Universidade Federal Ouro Preto, in Minas Gerais, Brazil. Its intent was to highlight local knowledge and ethnomathematical practices that are present in these dances. Thus, this presentation analyses ethnomodels that can be elaborated in relation to the Afro-descendant dance of *Palo Mayo*, which can contribute to the development of pedagogical actions in the perspective of ethnomodelling.

This is qualitative research in which some of the results are related to the elaboration of ethnomodels: a) emic (local) based on observations made with the dancers of the traditional dance of *Palo de Mayo*, b) etic (global) based on the perceptions of a mathematics teacher and the researcher, and c) dialogic (glocal) related to dialogue that happened between the Palo de Mayo dancers and the researcher, who has both emic and etic views of mathematical knowledge, as he is a member of the dance culture and he is also a mathematics teacher.

The connections between ethnomathematics and the sociocultural perspective of mathematical modelling provide for the development of pedagogical actions in classrooms, which are directed towards to the awareness of the relevance of the social and cultural aspects of mathematics (Orey & Rosa, 2021).

Figure 1 shows the elaboration of a dialogic ethnomodel that represents the mathematical knowledge of *Palo de Mayo* dancers in relation to the school/academic mathematical knowledge that is implicit during the development of this dance. This ethnomodel generates an appreciation and respect for both the emic (local) and etic (global) knowledge through a cultural dynamism (dialogic/glocal).

Figure 1: A dialogic ethnomodel. Source: Personal file of the authors.

	This ethnomodel shows the representation of the choreography proposed by the choreographer and analyze some translations in Geogebra, can evidence the precise translation of the figures.	This QR code shows a video that highlights the dynamics of the choreography, which represents the dialogic ethnomodel.

The partial results of this study, which are related to data collected in participant observation, show the development of a dialogue between emic (local)mathematical (cultural dynamism) present in the sociocultural traditional *Palo de Mayo* dance with school/academic mathematical knowledge, which values and respects this dynamic of the encounter of distinct cultures.

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A SUGGESTION OF A FRAMEWORK TO CAPTURE THE STRUCTURE OF REGIONAL PROBLEMS AND ITS META PROBLEM

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Mathematical modeling problems for students to solve in lessons have been developed. For example, the International Mathematical Modeling Challenge (IM^2C) in Australia is releasing problems. "Waste not, want not" is one of problems developed by the IM^2C. Attempting to solve the problem is beneficial for students. However, in many cases, mathematical modeling problems tend to be localized to original situations faced by specific areas and residents. Therefore, teachers must modify the variables in the original problems to suit the local situation in which students live. For example, the original "waste not, want not" problem has variables such as initial value for population in Australia and amount of waste per person per year in Australia. Certainly, teachers in Japan must review the variables for students in Japan. However, the modeling process will be fixed between specific situations.

To overcome this problem, Misono and Takanashi (2022) suggested a framework to capture the structure of regional problems and its meta problem. Figure 1 shows its translation to English.

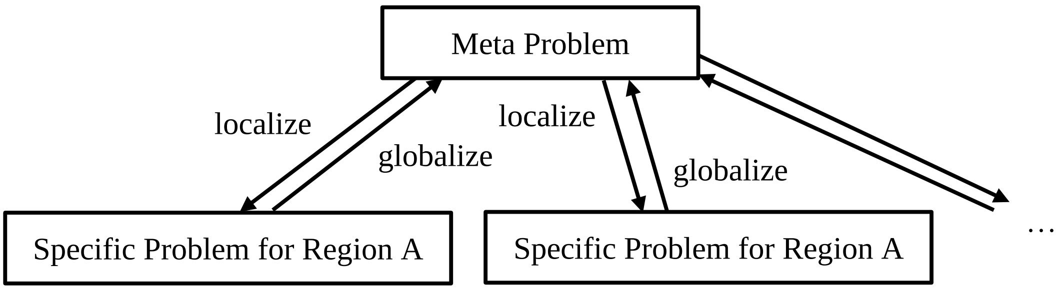

Figure 1. A framework to capture the structure of regional problems and its meta problem

In our previous study, we only refer to problems and situations limited to Japan. However, in this study, we apply this framework to worldwide situations.

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Poster Presentation

EXAMINING EARLY-GRADE CHILDREN'S MATHEMATICAL MODELS IN PROPORTIONAL SITUATIONS: A GLIMPSE FROM TASK-BASED INTERVIEWS

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The purpose of this paper is to explore aspects of mathematical models related to proportional reasoning in early-grade primary school children. This paper bases "Models and Modeling Perspectives" (MMP, Lesh & Lehrer, 2003) and examines children's interpretations and descriptions of proportional situations in task-based interviews.

Proportionality is a key idea and model used in mathematics and science. However, it has been shown that students who have studied proportionality apply it without examining the problem situation (e.g., De Bock, et al., 2002). In reality, they do not necessarily master the powerful model of proportionality. By capturing the case of early-grade children from the MMP, it is expected to reveal factors that have not been fully recognized so far and to provide useful suggestions on how proportionality can be learned. For example, one of the concerns in previous studies of young children is how they overcome the overreliance to whole-number information and how a progress from a simple sensory perception to an attention to numerical relations of proportionality is facilitated (e.g., Hurst & Cordes, 2018).

The research question addressed is "What models do children exhibit to find unknown quantities, and what are their own ways of perceiving and hypothesizing quantity relationships?" In this paper, we offer cases of children's situated problem-solving from our task-based interviews with 18 children, aged 5-8, by using everyday situations. In the interview, several proportional situations were presented in which two quantities are related at a certain rate, e.g., "We exchange 2 chocolate bars and 3 marbles. How many marbles can you get if you have 6 chocolate bars?"

From the results of the interviews, it became apparent that the children's models of the relationship between the quantities varied, compared with the ones intended by the interviewer. They attended broader aspects of the quantities and relationship by reflecting on their everyday experiences and values. It is also observed that the children revised their interpretations by trying to resolve their perceived unfitness. By reflecting on these cases, we discuss what implications children's models offer for the teaching/learning of the basis of proportional reasoning in early-grade primary school.

References

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HOW DO STUDENTS CRITICALLY INTERPRET PLURAL WAYS TO DETERMINE THE MOST POPULAR ONIGIRI? -ATOMISTIC APPROACH FOCUSED ON INTERPRETATION, VALIDATION AND MODIFICATION-

Minami Asahara[1], Takashi Sumioka[1], Makoto Chonan[1] and Kazuki Koike[1]

[1]*Yokohama National University (Master's student), Japan*

In a society where online information and tools, such as Chat GPT, are becoming increasingly prevalent, it is essential to develop competencies that enable individuals to identify and analyze hidden or explicit assumptions in the given information and make decisions based on that analysis. This activity is strongly concerned with prescriptive modelling, which aims to pave the way for taking action based on decisions resulting from a certain type of mathematical consideration (Niss, 2015).

This study aimed to foster sub-competencies in modelling among sixth-grade students by applying an atomistic approach (Kaiser & Brand, 2015), focusing on interpretation, validation, and modification. This study sought to determine how students interpret, compare, and validate the three voting models in response to the question, "What is the best ingredient for an onigiri?"

The teaching experiment involved teachers presenting six different onigiri ingredients to 28 students and three voting models to determine the best ingredients. The three voting models were (a) a single-entry method, (b) a deciding vote method, and (c) a point-distribution method. The teachers gave the students the opportunity to vote using these three models and then analyzed the results.

This study analyzed the transformation of students' thinking, dividing the analysis into three stages: (1) personal thinking after understanding the problem, (2) personal thinking after voting in class, and (3) personal thinking after group discussions. This study also asked about the significance of this type of problem in the future.

The results showed that more than 60% of the students' thinking was transformed after the three stages, and about 85% of them had a positive opinion on the effectiveness of this kind of problem solving. During the teaching experiment, students were able to point out the advantages and disadvantages of the voting method as their ideas differed from each other. For example, an advantage of the deciding vote method is that it results in fewer dead votes. The advantage of the point-distribution method is that it accurately reflects feelings, whereas the disadvantage is that it results in an artificially extreme score. At the end of the class, some students suggested changing the model according to their purpose.

While the teaching experiment focused on the atomistic approach in this material, which is one of the teaching methods used to encourage students' interpretation and validation, the study found that students were unable to modify the given voting models or create new ones. Therefore, encouraging students to pursue better models remains a challenge.

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HOW TO USE ELEMENTARY MATH TEXTBOOKS TO PROMOTE MATHEMATIZATION

Hatsuda Hiroki

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Teacher education in the teaching of mathematical modelling has become a worldwide topic (Blum, 2015). In Japan, methods to facilitate training for teachers in using modelling at schools are also being considered (Saeki et al.,2023). In this study, we will focus on methods for arranging problems in textbooks that teachers use daily to allow non-specialist teachers of mathematics to gradually incorporate the teaching of modelling at the elementary school level. The purpose of this study is to consider how word problems can be arranged to promote mathematization and to clarify their effectiveness and challenges through practical instruction. The following methods are used to arrange the textbooks: (1) inserting sentences that serve the purpose of the text, (2) using actual objects to make realistic situations in the text questions, and (3) asking questions to shake up the students' view of mathematics according to their actual situation.

In the actual class, we select the task from the textbook, "Divide 72 sheets of colored paper among three students in equal numbers. How many sheets does one person get?" The first step is to state "folding paper airplanes" as the objective for solving the problem. The reason is that the way to answer the question will change depending on the objective. The second step is to use colored paper when thinking about the 72 sheets of paper, to remind the students of the implicit assumption that the sheets are to be divided regardless of color. The problem we arrange is: "Three students make paper airplanes. 72 sheets of colored paper are divided equally among three students. " As a third step, after solving the problem and getting the answer from 24 sheets, the teacher asks "One of the three students says that the bundle of papers over there is better. Why do you think so?"

The practical class is given to 32 fourth-grade elementary school students for two hours (45 minutes x 2). In the introductory stage, three students are asked to divide 72 sheets of origami paper equally in front of everyone. After the students work out the division equation and give the answer of 24 sheets per person, we make evocative questioning by shuffling the paper. Based on the children's murmurs such as "We should not use gold or silver because the colors are too bright" or "After sorting the sheets of origami paper by color, we should divide them into three students," a development leads to the consideration and revision of the mathematization of setting new assumptions.

From the above study, we find that the implicit assumption becomes apparent by including a purpose in the text, making the problem situation realistic, and making evocative questioning according to the student's actual situation. However, further study is needed to find out how to modify the model based on the implicit assumption.

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FOSTERING STUDENTS' ABILITY TO COMPARE AND CRITICALLY EXAMINE PLURAL MODELS TO SOLVE REAL WORLD PROBLEMS -TEACHING EXPERIMENT BY USING A TENT PROBLEM-

Shota Kita[1], Masashi Ikemura[1], Kodai Aratani[1] and Toshikazu Ikeda[2]

[1]*Yokohama National University(Master's student),* [2]*Yokoaham National University, Japan*

This article explored how changes in society, such as the proliferation of Internet information and chatbots, make everyday life's modelling processes more diverse. People are expected to consider at least two models and make judgments based on a comparison of their advantages and disadvantages. This kind of activity is related to the atomistic approach to modelling, which focuses on sub-competencies such as interpretation, validation, and modification (Kaiser & Brand, 2015). This approach can be applied to real-world problem-solving. This study aimed to identify the effectiveness and challenges of the atomistic approach for 50 twelfth-grade students by analyzing their descriptions during and after the teaching experiment.

The modelling task in the teaching experiment was to make the tent interior comfortable. The teacher posed the question of how to determine the angle between the two poles set in the tent. Two options were presented: 1) maximizing the volume inside the tent and 2) maximizing the length of the radius of a semicircle tangent to the triangle composed of two poles. Two questionnaires were administered at different stages of the experiment. The first question asked which of the two models was more suitable for the following three stages: individual thoughts before mathematical analysis, individual thoughts after mathematical analysis, and individual thoughts after group discussions. The second question, "Is it meaningful in a society to consider a more suitable model by comparing and validating plural models like this tent problem?" was asked after the teaching experiment using the Likert scale method. In addition, students were asked to describe their reasons for selecting their preferred model.

In the experimental teaching, students pointed out through group discussions the advantages of having a high space within the tent, which was derived from Model 2), for greater freedom of movement, and the usefulness of having a large floor, which was derived from Model 1), for greater comfort while sleeping. Furthermore, some have pointed out the importance of adapting tent designs according to their purposes. The results showed that over 70% of the students' thoughts were transformed through the three stages of analysis in the first question. Approximately 95% of students selected "totally agree" and "agree" in the Likert scale questionnaires regarding the second question. Furthermore, 70% of students described the usefulness of considering at least two models and making a judgment by comparing and validating multiple models. However, 30% of the students did not appreciate its usefulness.

Overall, this article highlighted the importance of fostering students' abilities to compare and critically examine multiple models for solving real-world problems. The study results suggested that the atomistic approach can be an effective way to achieve this, although further discussion is needed regarding the teacher's role in providing the two models for comparison.

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WHAT METAPHORS DO CHILDREN UTILIZE TO MAKE CONNECTIONS BETWEEN MATHEMATICS AND THE REAL WORLD?

Kensuke Koizumi[1], Ryuta Tani[2] and Ryo Hanzawa[3]

[1]Gunma University, Japan; [2]Tanaka Gakuen Ritsumeikan Keisyo Primary School, Japan; [3]Seya Primary School, Japan

We learn to see time as money from the expression "time is money”. Thus, metaphors serve as a kind of filter through which we try to better understand certain things. The filter expresses a perspective that reflects the person's own experience. Carreira (2001) focused on the relationship between the concept of mathematical models and the concept of metaphors, noting that mathematics and the real world influence each other to create meaning. This study examines the connection between mathematics and the real world from the perspective of metaphor. The meaning of “metaphor” in this study relies on the discussion of Lakoff & Johnson (2003) and Sfard (1997).

This study was conducted as a case study on the semantic generation of mathematical symbols such as □ and △, and was intended to identify the following two research questions.

RQ1. How does the connection between mathematics and the real world work in making sense?

RQ2. What kind of effects can be expected by eliciting the generation of meaning by children (especially through classroom interaction)?

The findings of this case study of 23 children (fifth grade elementary school) revealed the follows.

In an activity that attempted to explain the distinction between a) variables and b) unknowns, the meaning of □ and △ were associated with the real world as follows. (The above is related to RQ1)

a) chameleons, days of the week, first and last name relationships, community and town relationships, attendees and absentees relationships, etc.

b) Omikuji (a paper fortune), capsule toy, lucky bag, etc.

When such metaphors are incorporated into lesson planning, the following effects are expected. First, understanding with conviction related to concrete experiences. Second, understanding of mathematical concepts with multifaceted meanings. The activity of interpreting multiple metaphorical expressions that extracted different elements, could constitute an opportunity to organize the multifaceted meanings of mathematical concepts, including meanings that were implicit until then. Third, understanding through the diversity of base selection. Furthermore, in some cases, as a perspective on the development of learning, it can be a trigger for other learning content (in this case, for example, “community and town relationships”). (The above is related to RQ2)

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HOW DOES FIRST-YEAR SCITECH EXPERIENCE OF PROFESSIONAL DEVELOPMENT EFFECT MATHEMATIC EDUCATION?

Satoru Fujitani[1] and Motoko Fujitani[2]

[1]Tamagawa University, Japan; [2]Joetsu University of Education, Japan

This paper reports professional development practices of first-year college education based on the perspective of science, technology, engineering, and mathematics (STEM) education. Meaningful science for young children builds on the emotional underpinnings of their curiosity and concerns about the everyday world, and their pleasure in exploring it (Harlan & Rivkin, 2003). Through the implementation of the course named 'Introductory Study on Natural Environment and Science in Childhood Education', Fujitani (2022) has engaged in activities for first-year teacher-education students to cultivate STEM interests and to build a foundation in mathematics for primary education.

We describe an example of mathematical modelling in practice in this course as follows.

During a visit to the garden inside the shrine (Figure 1), students worked on the estimation of the natural regeneration of forests after learning forester's research practice at the time of afforestation for the shrine. The students attempted to find the time period of regeneration toward the climax stage of forests.

Figure 1: Visit for nature observation

Students also undertook an activity to build an antenna for digital television broadcasting (Figure 2). The size of the loop antenna they made was defined by the frequency band of the radio waves they planned to receive. Following the instruction, the students cut out aluminium foil according to the wavelength of the radio waves to produce their antennas.

Figure 2: Making TV antennas

In addition, the students also have engaged in the experiences of mammalian and aquatic animal rearing and cultivation of vegetables and flowers through this course, for instance. By introducing students to these scientific experiences early on, the students, who will be teachers, are able to build the foundation for developing scientific thinking and inquiry skills that would be the scaffold for the spark of the student's interest and excitement in STEM fields. The course makes teacher-training students foster a love and curiosity for science, and encourages them to pursue further STEM learning opportunities, and it can work as an access of mathematical modelling for them. It can also affect the children who are taught by the students as teachers in future likewise.

References

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Conference Information

Climate

Awaji has a Humid subtropical climate (Köppen Cfa) characterized by warm summers. The average annual temperature in Awaji is 16.3 °C / 61.3 °F. The average annual rainfall is 1,600 mm, with September as the wettest month. The temperatures are highest on average in August, at around 26.6 °C, and especially deadly heat in this summer. The temperatures in September are forecasted hot more than usual.

VISA

Information for foreign nationals about visas, including the application procedures and other related FAQs. Please access to the following web site of Ministry of Foreign Affairs of Japan.
https://www.mofa.go.jp/j_info/visit/visa/index.html

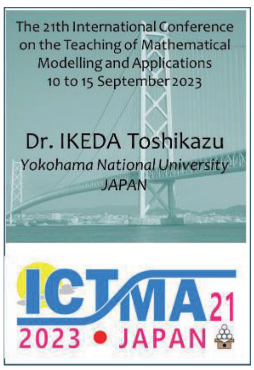
Electricity in Japan

- In Japan the power plug sockets are of type A and B. The standard voltage is 100 V and the frequency is 50 / 60 Hz.
- In Japan the power plug sockets are mainly type A. Check out the following pictures; Type A: mainly used in North and Central America, China and Japan.
- The power plug sockets of Awaji Yumebutai International Conference Center are only Type A.

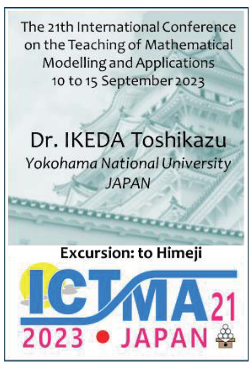
Registration

- Please check your name, affiliation, country, and lunch tickets.
- Have a name holder on a neck strap during the conference.
- Excursion is option, and check your choice (your choice is printed on name holder).

Opt. tour: Kyoto



Opt. tour: None



Opt. tour: Himeji

- Please fill in the form 'Personal Information Handling Agreement'.

Time table of the bus stops

- Cash or transportation IC card (ICOCA, SUICA, PASMO) are available.
- Please check the following web site if you will use transporatation IC card.
  [ICOCA] https://www.westjr.co.jp/global/en/howto/icoca/
  [Suica] https://www.jreast.co.jp/multi/en/welcomesuica/welcomesuica.html
  [PASMO] https://www.pasmo.co.jp/visitors/en/about

新神戸・三宮・高速舞子～淡路夢舞台　　高速バス時刻表

TIME TABLE FROM SHINKOBE TO (VIA SANNOMIYA STATION･MAIKO STATION) **YUMEBUTAI BUS STOP**

◆高速バスのりばのご案内

★[新幹線 新神戸駅] 高速バスのりば

新幹線 新神戸駅 コンコース2F → エスカレーターで1階へ → JRバスのりば（東浦BT行）

★ [JR三宮駅] 高速バスのりば

JR三宮駅 東口(南 → 左方向へ 歩いてすぐ → 三宮バスターミナル 4番のりば(東浦BT行) ※11:30発は5番のりば

★[高速舞子] 高速バスのりば

JR舞子駅 / 山陽電鉄 舞子公園駅 → 連絡通路 → 淡路島行き高速バスターミナルビル(5階) 明石海峡大橋路側帯バス停 2番のりば :東浦バスターミナル行(一部の便において津名港行)

◆所要時間、運賃

新神戸ー夢舞台 (60分) ¥1,050
三ノ宮 ー夢舞台 (45分) ¥1,050
高速舞子ー夢舞台 (15分) ¥590

運行日	行先	新神戸 SHINKOBE	三ノ宮 SANNOMIYA	高速舞子 MAIKO	淡路夢舞台 YUMEBUTAI
平日のみ	東浦BT	–	–	6:30	6:45
土休日のみ	東浦BT	6:05	6:20	6:50	7:05
平日のみ	東浦BT	6:45	7:00	7:30	7:48
	東浦BT	7:15	7:30	8:00	8:18
	東浦BT	7:45	8:00	8:30	8:45
	東浦BT	8:15	8:30	9:00	9:18
	東浦BT	8:40	8:55	9:30	9:45
	東浦BT	9:15	9:30	10:00	10:18
土休日のみ	東浦BT	9:45	10:00	10:30	10:45
	東浦BT	10:15	10:30	11:00	11:15
土休日のみ	東浦BT	10:45	11:00	11:30	11:45
	東浦BT	11:15	11:30	12:00	12:15
	東浦BT	12:15	12:30	13:00	13:15
	東浦BT	13:15	13:30	14:00	14:18
	東浦BT	14:15	14:30	15:00	15:15
	東浦BT	–	–	15:45	16:03
	東浦BT	15:15	15:30	16:00	16:15
	東浦BT	15:45	16:00	16:30	16:48
	東浦BT	16:05	16:20	16:50	17:05
	東浦BT	16:45	17:00	17:30	17:45
	東浦BT	17:15	17:30	18:00	18:18
土休日のみ	東浦BT	17:35	17:50	18:20	18:35
	東浦BT	17:45	18:00	18:30	18:45
	東浦BT	18:15	18:30	19:00	19:15
	東浦BT	18:45	19:00	19:30	19:45

運行日	行先	新神戸 SHINKOBE	三ノ宮 SANNOMIYA	高速舞子 MAIKO	※国道夢舞台 KOKUDOU YUMEBUTAI
	東浦BT	19:15	19:30	20:00	20:15
	東浦BT	19:45	20:00	20:30	20:45
	東浦BT	20:15	20:30	21:00	21:15
	東浦BT	20:45	21:00	21:30	21:45
	東浦BT	21:35	21:50	22:20	22:35
	東浦BT	22:45	23:00	23:30	23:45

運行日	行先	淡路夢舞台 YUMEBUTAI	高速舞子 MAIKO	三ノ宮 SANNOMIYA	新神戸 SHINKOBE
土休日のみ	新神戸	8:15	8:31	8:58	9:08
平日のみ	新神戸	8:20	8:36	9:03	9:13
	新神戸	8:55	9:15	9:42	9:52
	新神戸	9:15	9:31	9:58	10:08
土休日のみ	新神戸	9:30	9:46	10:13	10:23
平日のみ	新神戸	9:35	9:51	10:18	10:28
土休日のみ	新神戸	9:45	10:01	10:28	10:38
平日のみ	新神戸	9:55	10:11	10:38	10:48
	新神戸	10:15	10:35	11:02	11:12
土休日のみ	新神戸	10:35	10:51	11:18	11:28
平日のみ	新神戸	10:45	11:01	11:28	11:38
土休日のみ	新神戸	10:55	11:11	11:38	11:48
	新神戸	11:15	11:31	11:58	12:08
	新神戸	11:45	12:01	12:28	12:38
	新神戸	12:50	13:06	13:33	13:43
	新神戸	13:45	14:01	14:28	14:38
	新神戸	14:20	14:40	15:07	15:17
	新神戸	14:45	15:01	15:28	15:38
	新神戸	15:15	15:35	16:02	16:12
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	新神戸	16:15	16:35	17:02	17:12
	新神戸	16:45	17:01	17:28	17:38
土休日のみ	新神戸	17:15	17:31	17:58	18:08
平日のみ	新神戸	17:35	17:55	18:22	18:32
土休日のみ	新神戸	17:45	18:05	18:32	18:42
	新神戸	18:15	18:35	19:02	19:12
平日のみ	高速舞子	18:45	19:05	–	–
	新神戸	19:15	19:35	20:02	20:12
	新神戸	20:15	20:31	20:58	21:08
	新神戸	21:15	21:31	21:58	22:08
	新神戸	21:55	22:11	22:38	22:48
	新神戸	22:35	22:51	23:18	23:28

2023年4月1日改正 As of 2023/4/1

※全席自由席

on weekdays only
on weekends & holidays only

西日本JRバス WEST JAPAN RAILWAY CO. 06-6371-0121
(平日9:00～17:45)
本四海峡バス HONSHIKAIKYO BUS CO. 0799-74-0600

(参考) 新神戸駅から淡路夢舞台まで中型タクシー(5人乗り)を使用した場合の料金(高速料金含む)は、約17,000(所用時間50分)となります。

Taxi (from Shin-Kobe Station to Yumebutai) :approx. 50 min. ￥17,000 (Expressway toll and Akashi Kaikyo Bridge toll are included.)

https://www.yumebutai.org/pdf/time_sin_san_mai.pdf

Wi-Fi

- Free Wi-Fi is available in each room. Please check SSID & Password in each room.
- Free Wi-Fi is also available outside the rooms and in the corridor (SSID is floor name and no password).

Welcome Reception (9th Sunday)

- Welcome reception is held at the lobby (B1F). It's free of charge.
- Light meals, snacks, and drinks are available.

Coffee Break (It's included in the registration fee.)

- Coffee, tea, and some drinks are available on 11th (Monday), 12th (Tuesday), and 14th (Thursday). The place is at the lobby (B1F) in the morning on the 11th (Monday), and the 3rd and 4th floor break area in the rest.
- In the afternoon on the 10th (Monday) and on the 15th (Friday), coffee break is available at the lobby (B1F).

Happy Hour (10th Monday & 11th Tuesday)

- Happy hours are held at the lobby (B1F). It's free of charge.
- On the 10th (Monday), the time is 18:30 to 19:30.
- On the 11th (Tuesday), the time is 17:00 to 18:30, and concurrently with poster session.
- Light meals, snacks, and drinks are available.

Lunch (It's included in the registration fee.)

- Lunch is available every day from 11th (Monday) to the 14th (Thursday).
- Coffee, tea, and some drinks are also available except on the 13th (Wednesday).
- Please exchange lunch ticket for lunch at break area except the 13th (Wednesday).
- On the 13th (Wednesday), exchange lunch ticket for lunch box at the lobby (B1F).
- Don't forget take away lunch box before participating the excursion (optional tour) please.
- Lunch menu will NOT change from your registration request (None, Vegetarian, or Other). Please check the lunch tickets.
- It is possible to take lunch into the building and outside, except to the public floor of Grand Nikko Awaji (2F).

Conference Dinner (14th Thursday) (It's included in the registration fee.)

- Conference dinner is held at the Reception Hall B (2F).
- To enter the Reception Hall B, you have to place a name holder on a neck strap.

Corporate Exhibition

Exhibitions by supported companies (Casio Institute for Educational Development, Gakkotosho, and Kyoiku-shuppan) is held at 404.

Smoking

No smoking on the premises. Smoking is only permitted outside in designated areas (4F courtyard).

Other inquires

- If you have questions, please contact LOC staffs and/or students' staffs who have a yellow-colored name holder on a neck strap.
- Rooms 401 and 402 are the LOC office rooms. Please come the rooms if you need anything.

Excursion (13th Wednesday) *Option

After the plenary lecture 2, please meet at a designated place in front of the building. Please don't forget to take away lunch box.

Hospitals

If you have illness or poor physical condition etc…, please inform the staff. Our staff will introduce the following hospital.

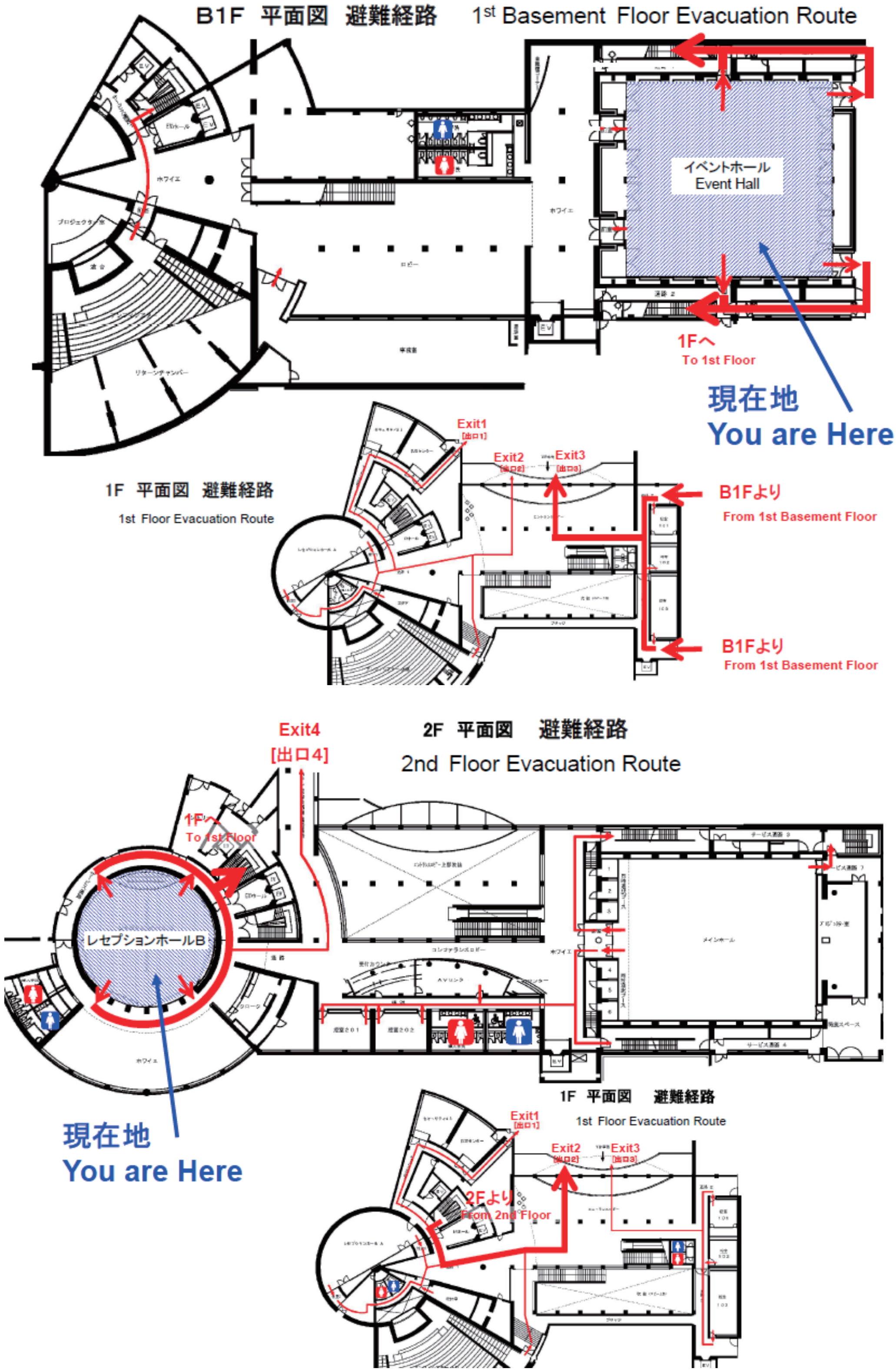
- **Seirei Awaji Hospital**

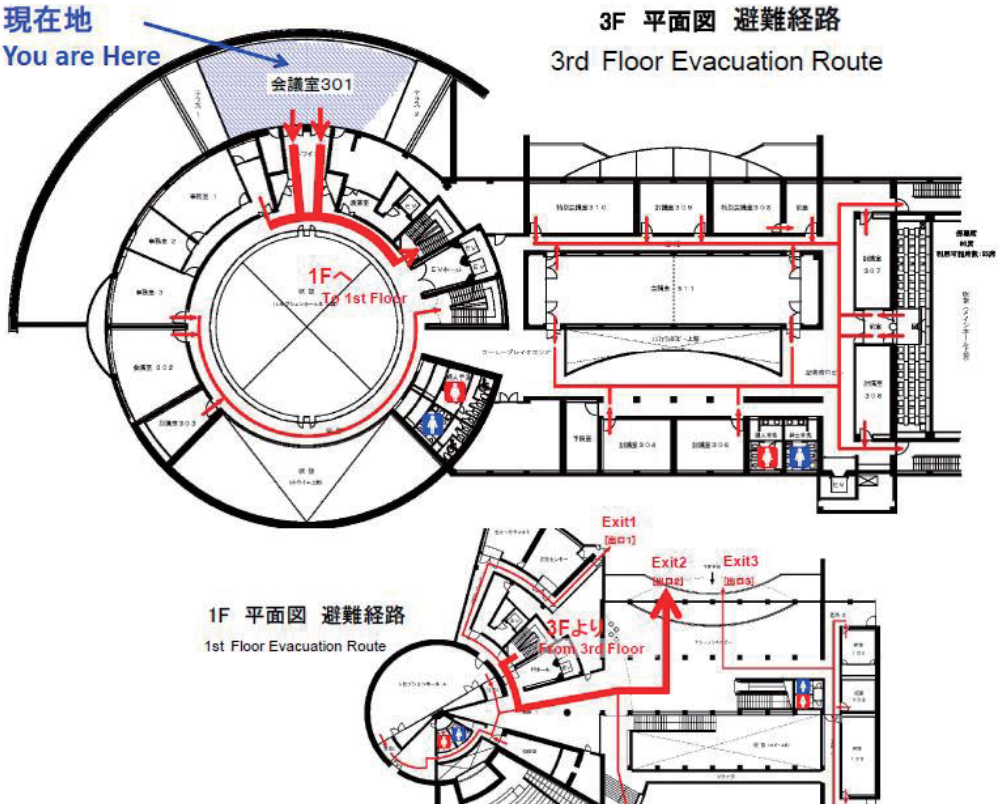
[URL] https://www.seirei.or.jp/awaji/index.html
Desk: 8:30-11:30, 13:30-16:30 *10th Sunday is closed.
Address: Yumebutai 1-1, Awaji-City, Hyogo
Tel: (+81) 799-72-3636

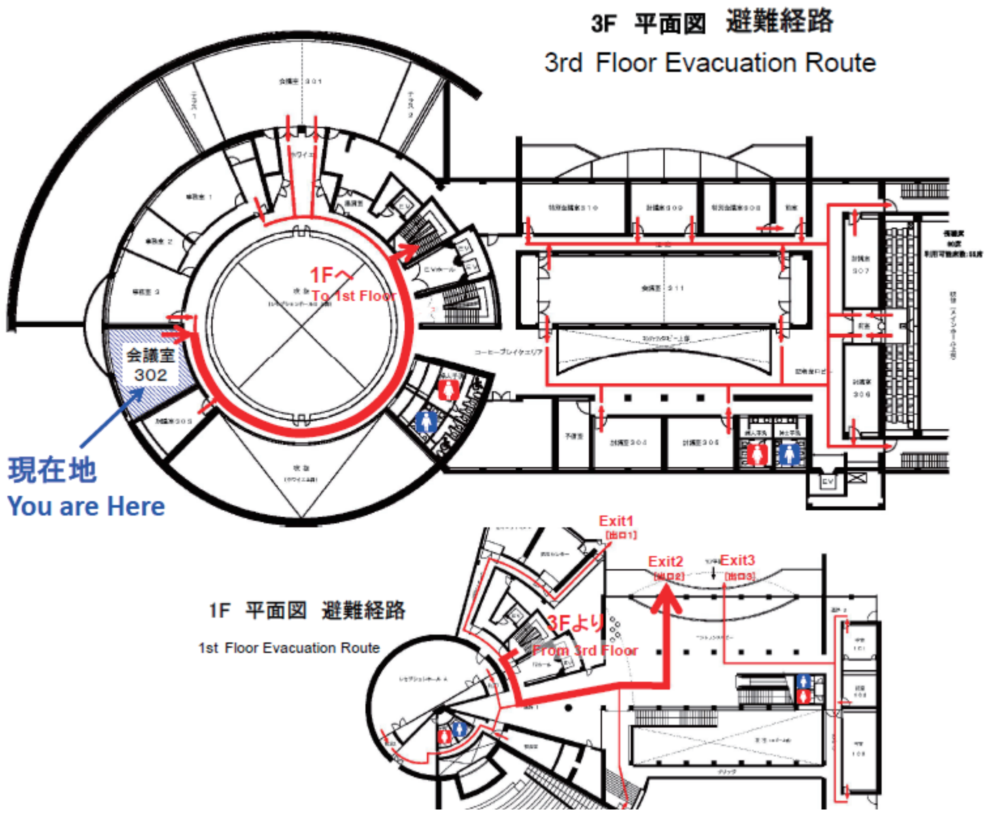
- **Higashiura Heisei Hospital**

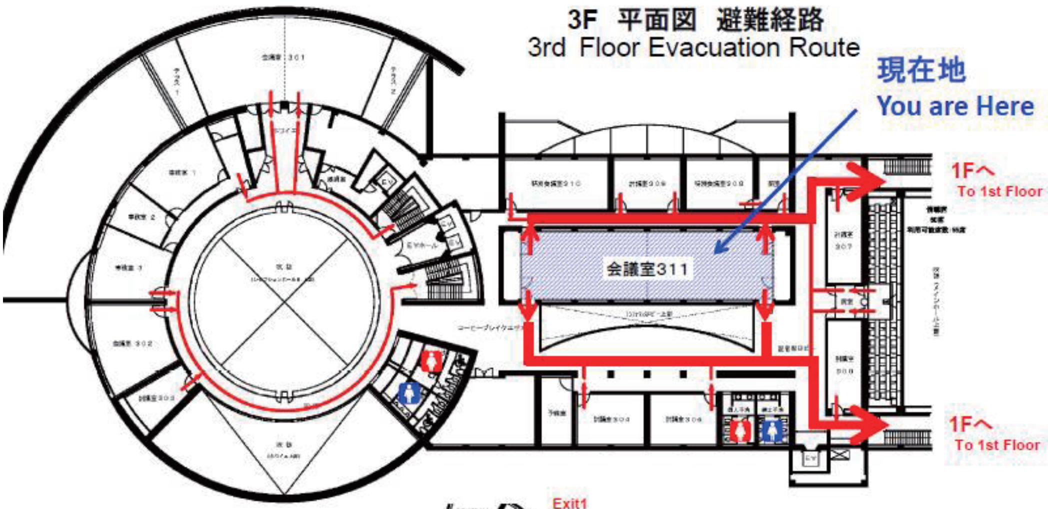
[URL] https://higashihp.jp/news/id_3278
Consultation time: 9:00-12:00, 15:00-17:00 *10th Sunday is closed.
Address: Kuruma 1867, Awaji City, Hyogo
Tel: (+81) 799-74-0503

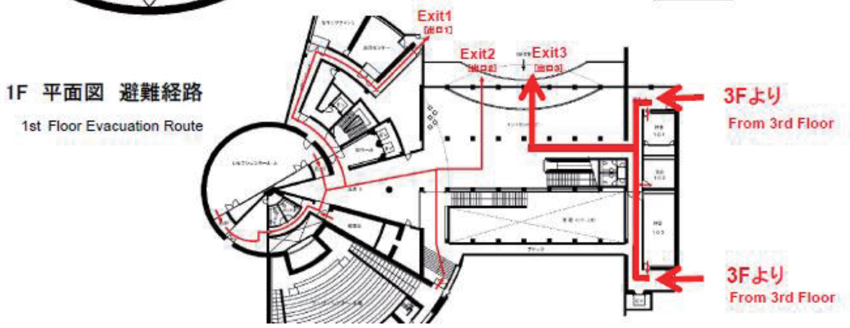
Evacuation routes

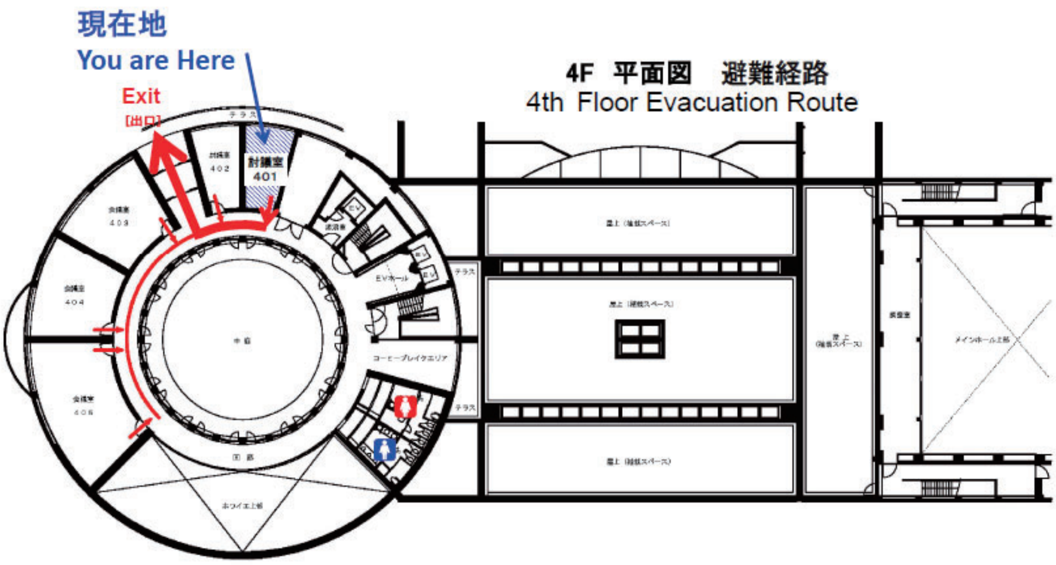


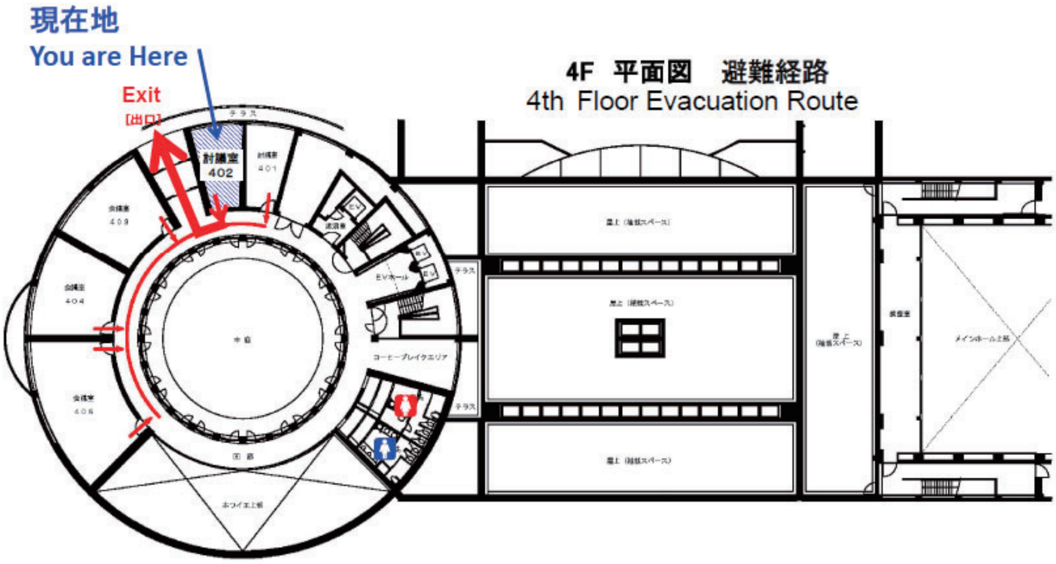

現在地
You are Here
会議室301
3F 平面図 避難経路
3rd Floor Evacuation Route
1Fへ
To 1st Floor
Exit1
[出口1]
Exit2
[出口2]
Exit3
[出口3]
1F 平面図 避難経路
1st Floor Evacuation Route
3Fより
From 3rd Floor


3F 平面図 避難経路
3rd Floor Evacuation Route
1Fへ
To 1st Floor
会議室
302
現在地
You are Here
Exit1
[出口1]
Exit2
[出口2]
Exit3
[出口3]
1F 平面図 避難経路
1st Floor Evacuation Route
3Fより
From 3rd Floor

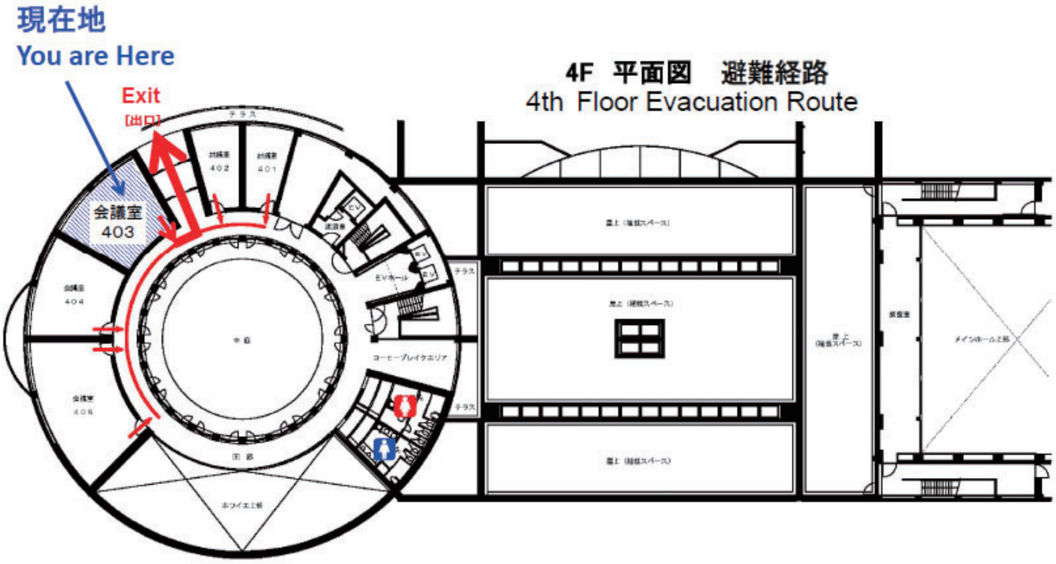

3F 平面図 避難経路
3rd Floor Evacuation Route
現在地
You are Here
会議室311
1F へ
To 1st Floor
1F へ
To 1st Floor


1F 平面図 避難経路
1st Floor Evacuation Route
Exit1
Exit2
Exit3
3Fより
From 3rd Floor
3Fより
From 3rd Floor

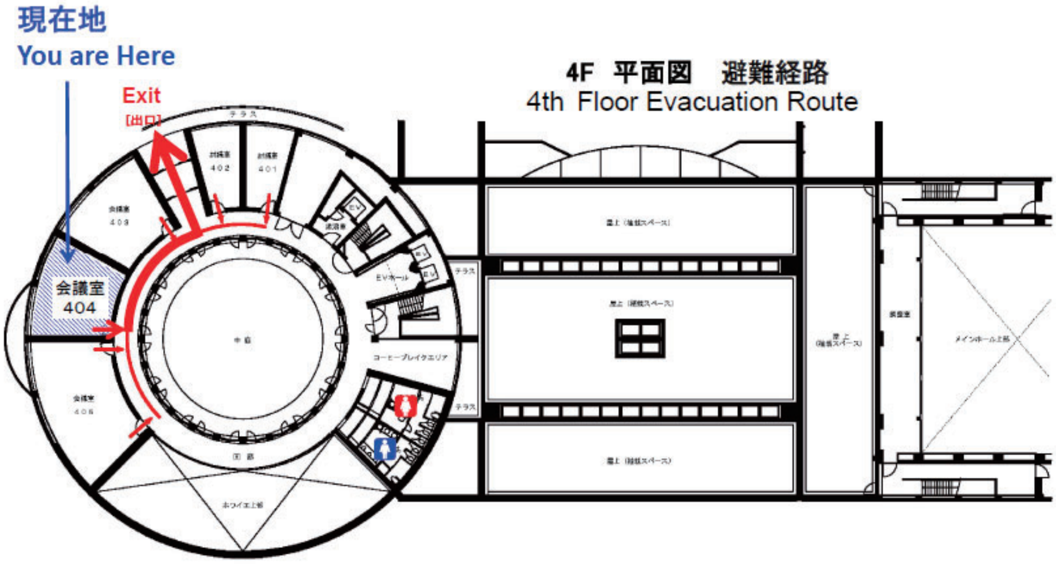

現在地
You are Here
Exit
[出口]
討議室
401
4F 平面図 避難経路
4th Floor Evacuation Route

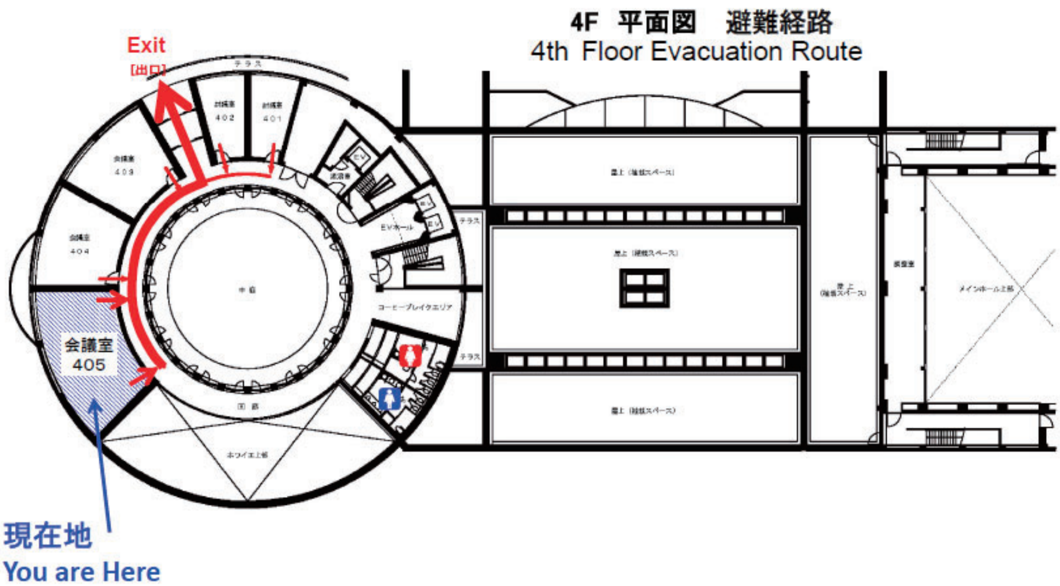


現在地
You are Here
Exit
[出口]
4F 平面図 避難経路
4th Floor Evacuation Route
討議室
402



現在地
You are Here
Exit
[出口]
4F 平面図 避難経路
4th Floor Evacuation Route
会議室
403


現在地
You are Here
4F 平面図 避難経路
4th Floor Evacuation Route
Exit
[出口]
会議室
404


4F 平面図 避難経路
4th Floor Evacuation Route
Exit
[出口]
会議室
405
現在地
You are Here

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Japan Society of Mathematical Education

一般社団法人 数学教育学会
Mathematics Education Society of Japan

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Ugsonkid Songchai

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Uozumi Shohei

Urayama Daiki

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Watanabe Yuki

Wedelin Dag

Wiehe Maria Katharina

Wienecke Lisa-Marie

Wirth Laura

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Yamanaka Hitoshi

Yan Yuan

Yanagimoto Akira

Yang Kai-Lin

Yang Xinrong

Yata Kosuke

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