Toward an Approach to the Hierarchy Problem via Flux Compactification

(フラックスコンパクト化による階層性問題のアプローチに向けて)

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Abstract

この博士論文では、背景磁場を含むコンパクト化（フラックスコンパクト化）された高次元理論において、高次元ゲージ場の余剰成分スカラー場（ウィルソンライン (WL) スカラー場) のゼロモードの質量に関する量子補正が有限でかつ消えない状況を実現する可能性について注目する。可換ゲージ理論から非可換ゲージ理論へと拡張し、さらに非可換ゲージ理論にはある高次元演算子を加えた理論も考える。これらの理論で WL スカラー場の質量に関する量子補正が相殺していることを示す。フラックスコンパクト化における有限かつ消えない WL スカラー場の質量を実現するため、質量に関する 1 ループレベルの量子補正の中に現れる一般化されたループ積分を解析する。さらに、その解析から有限な量子補正が得られる 4 点相互作用項と 3 点相互作用項を推測、分類する。これらの相互作用項のうち、ある単純な形の相互作用項に着目し、6 次元スカラー量子電磁気学の枠組で WL スカラー場の質量に関する量子補正が実際に有限に得られることを例証する。最後に、以上の議論の応用として、フラックスコンパクト化された理論における新しいインフレーション理論を提唱し、Planck 2018 の観測データと我々の結果を比較する。

Toward an Approach to the Hierarchy Problem via Flux Compactification

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Abstract

In this thesis, we focus on the possibilities to realize a nonvanishing finite quantum correction to the mass of zero-mode of the Wilson-Line (WL) scalar in flux compactification. We extend Abelian gauge theories to non-Abelian gauge theories and add some higher dimensional operators, and then we show that the quantum corrections to WL scalar mass are canceled. To realize a nonvanishing finite WL scalar mass in flux compactification, we analyze the generalized loop integrals in the quantum correction to WL scalar mass at one-loop. We further guess and classify the four-point and three-point interaction terms generating the finite quantum correction to WL scalar mass at one-loop level. Of these interaction terms, we focus on a simplest interaction term and illustrate the finite quantum correction to the WL scalar mass in a six-dimensional scalar QED. Finally, we propose a new inflation scenario in flux compactification as an application of the above discussion and compare our results to Planck 2018 data.

Contents

Chapter 1

Introduction

A theory containing elementary particles (quarks, leptons, gauge bosons and Higgs boson) is called the Standard Model (SM) of particle physics. From the point of view of gauge theory, the SM is the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge theory. $SU(3)_C$ group implies quantum chromodynamics and $SU(2)_L \times U(1)_Y$ group means electroweak theory. After Higgs boson was discovered at LHC experiment in 2012 [1, 2], the SM has been established as a theory explaining real phenomena.

However, the SM is not a final destination and has many phenomena which cannot be explained. Since the SM is just a low-energy effective theory with electroweak scale as a cutoff scale, the SM cannot explain some ultraviolet (UV) physics. As a guiding principle of search for the physics beyond the Standard Model (BSM), the hierarchy problem has been considered [3, 4]. In the SM, the quantum correction to the mass of Higgs field is sensitive to the square of the UV cutoff scale of the theory (for example, Planck scale or the scale of grand unified theory). Since the cutoff scale is much larger than an experimental value of the Higgs mass (125 GeV), the solution of the hierarchy problem requires an unnatural fine-tuning of parameters or exploring a new physics beyond the SM at order of TeV scale.

Historically, the latter approaches have been mainly studied so far. The origin of the hierarchy problem is that there are no symmetries forbidding the mass of the scalar field. As an example of the solution of the hierarchy problem, supersymmetry has been considered [5,6]. Supersymmetry is the symmetry exchanging boson and fermion. If we impose supersymmetry on a theory, the quantum corrections from boson loops and fermion loops are canceled at all order. Although supersymmetry predicts some superpartners, no signature of them has been found at TeV scale. As another example, a higher dimensional field theory has been considered [7–10]. In particular, gauge-Higgs unification has been paid attention in order to solve the hierarchy problem [10–14]. Gauge-Higgs unification is the theory that zero-mode of the scalar field induced from extra components of higher dimensional gauge field (called as Wilson-line (WL) scalar field) is identified with Higgs boson. In the gauge-Higgs unification, the finite Higgs mass is generated by the quantum corrections at one-loop. Higher dimensional field theory also predicts Kaluza-Klein fields and compact space. Both of them however has not been found at TeV scale.

Toward the approaches to the hierarchy problem, we consider a higher dimensional theory with magnetic flux compactification. Magnetic flux compactification has been originally studied in string theory [15, 16]. Even in the field theories, flux compactification has many attractive properties: attempt to explain the number of the generations of the SM fermion [17–19], computation of Yukawa coupling [20–22], and spontaneously supersymmetry breaking [23]. Recently, it has been considered that the quantum corrections to the masses of zero-mode of the WL scalar are canceled [24–29] and are finite [30]. The physical reason of the cancellation is that the shift symmetry from translation in extra spaces forbids the mass term of WL scalar field. In that situation, the zero-mode of the WL scalar field can be identified with Nambu-Goldstone (NG) boson (or with pseudo-NG boson in [30]) of spontaneously broken translational symmetry. It is not possible for results in [24–29] to apply to the hierarchy problem as it stands since the WL scalar field is also massless at quantum level. However, even if the new physics scale (or compactification scale) is much higher than the electroweak scale and the KK fields are very massive, the hierarchy problem may be solved in the framework of flux compactification [30].

In this thesis, we focus on a six-dimensional field theory with flux compactification and mainly investigate the quantum corrections to WL scalar mass. At first, we review Abelian gauge theories in six dimensions without or with flux and discuss the difference between the quantum corrections without and with flux [24, 25]. Next, we extend to non-Abelian gauge theories and also calculate the quantum corrections to WL scalar mass [27]. Moreover, we add higher dimensional operators and compute the quantum corrections to WL scalar mass [29]. We show that the quantum corrections are canceled in these theories. To obtain the finite quantum correction, we investigate the loop integral in the quantum correction to WL scalar mass at one-loop [30]. Then, the conditions for the loop integral and mode sum to be finite are derived. We further guess and classify the four-point and three-point interaction terms generating the finite quantum correction to WL scalar mass at one-loop level. Of these interaction terms, we focus on a simplest interaction term and illustrate the finite quantum correction to the WL scalar mass in a six-dimensional scalar QED. Finally, we apply the theory with the finite quantum correction to inflationary theory [31]. From the effective potential, we can calculate inflationary parameters. We compare our results to Planck 2018 data [32].

This thesis is organized as follows. We explain the basis of flux compactification in chapter 2. The idea of flux compactification is based on quantum mechanics in magnetic field. Thus, after introducing the quantum mechanics in magnetic field, we consider a six-dimensional field theory with flux compactification. In chapter 3, we review Abelian gauge theories in six dimensions without or with flux. The quantum corrections to WL scalar mass in a theory without flux are finite. On the other hand, the quantum corrections to WL scalar mass in a theory with flux vanish. We discuss this difference. We also see that the physical reason of the cancellation is that the shift symmetry from translation in extra spaces forbids the mass term of WL scalar field. We extend Abelian gauge theories to non-Abelian gauge theories and also calculate the quantum corrections to WL scalar mass in chapter 4. We also discuss higher dimensional operators and compute the quantum corrections to WL scalar mass. In chapter 5, we study possibilities to realize a nonvanishing finite WL scalar mass in flux compactification by analyzing the generalized loop integrals in the quantum correction to WL scalar mass at one-loop. In chapter 6, we propose an inflation scenario in flux compactification. We calculate inflationary parameters and compare our results to Planck 2018 data. Finally, we devote our conclusion in this thesis. In appendix A, we review Poisson resummation formula. In appendices B and C, we summarize the calculations of $2\mathrm{Tr}[D_L F_{MN} D^L F^{MN}]$ and $\mathrm{Tr}[F^4]$ in chapter 4, respectively. The properties of Hurwitz zeta function are summarized in appendix D.

Chapter 2

Flux Compactification

Magnetic flux compactification is a compactification with nontrivial magnetic background. Originally, flux compactification has been studied in string theory and related with D-brane (see [16]). Higher dimensional theory with flux compactification has also many attractive properties: attempt to explain the number of the generations of the standard fermion [17–19], realization of four-dimensional chiral fermion zero-mode and computation of four-dimensional Yukawa coupling from higher dimensional theory [20–22]. In this chapter, we give a basic idea for magnetic flux compactification.

2.1 Quantum mechanics in magnetic field

Before considering flux compactification, let's remind us of quantum mechanics in magnetic field [33]. We consider that a charged particle with a charge e and a mass m moves in a uniform magnetic field B. The two-dimensional Hamiltonian is given by

$$\begin{aligned} H &= \frac{1}{2m}\left\{(p_x - eA_x(x,y))^2 + (p_y - eA_y(x,y))^2\right\} \\ &= \frac{1}{2m}\left\{(iD_x)^2 + (iD_y)^2\right\}, \end{aligned} \tag{2.1}$$

where $p_x = -i\partial_x$, $p_y = -i\partial_y$ are momenta and $D_i = \partial_i - ieA_i (i = x, y)$ are the covariant derivatives. Eq.(2.1) is similar to the Hamiltonian of harmonic oscillators. Computing the commutation relation between iD_x and iD_y, we obtain

$$[iD_x, iD_y] = ie(\partial_x A_y - \partial_y A_x) = ieB, \tag{2.2}$$

where we use a magnetic field $B = \partial_x A_y - \partial_y A_x$. When eq.(2.2) is normalized by eB, the commutation relation is rewritten by

$$[Q, P] = i, \quad Q \equiv \frac{iD_x}{\sqrt{eB}}, \quad P \equiv \frac{iD_y}{\sqrt{eB}}. \tag{2.3}$$

Once creation and annihilation operators are defined by

$$a \equiv \frac{Q + iP}{\sqrt{2}}, \quad a^\dagger \equiv \frac{Q - iP}{\sqrt{2}}, \tag{2.4}$$

the Hamiltonian (2.1) is expressed by

$$H = \omega \left(a^\dagger a + \frac{1}{2} \right), \tag{2.5}$$

where $\omega = eB/m$. From eq.(2.5), the energy level is discrete and is called as Landau level.

Next, we focus on eigenvalues of momentum. For simplicity, we choose the gauge fields (called Landau gauge) as follows

$$A_x = 0, \quad A_y = Bx. \tag{2.6}$$

Eq.(2.1) is expressed by

$$\begin{aligned} H &= \frac{1}{2m} \left\{ p_x^2 + (k_y - eBx)^2 \right\} \\ &= \frac{1}{2m} \left\{ p_x^2 + e^2 B^2 \left(x - \frac{k_y}{eB} \right)^2 \right\}, \end{aligned} \tag{2.7}$$

where p_y is replaced by k_y, which is the eigenvalue of p_y. Note that an eigenvalue k_y is arbitrary and has no relation to energy eigenvalue n. To deal with arbitrary k_y, we impose periodic boundary conditions in the x direction with length L_x and in the y direction with length L_y. This is the same as periodic boundary conditions of torus. From the periodic boundary condition of y direction, k_y is discretized as follows

$$k_y = \frac{2\pi}{L_y} l \quad (l \in \mathbb{Z}). \tag{2.8}$$

On the other hand, we find that the center of wavefunction in the x direction is k_y/eB from eq.(2.7). To locate the center of this wavefunction between 0 and L_x, it needs satisfying the inequality

$$0 \leq l \leq \frac{eBL_xL_y}{2\pi}.$$

Since the eigenvalue k_y has no relation to the energy eigenvalue, the quantity

$$N \equiv \frac{eB}{2\pi} L_x L_y \tag{2.9}$$

means the number of the degeneracy.

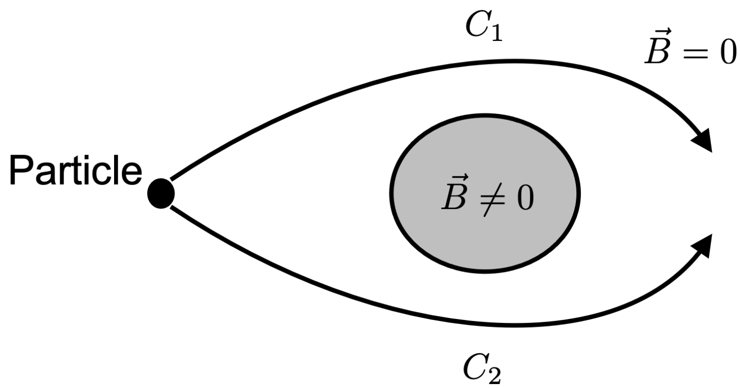

Figure 2.1: The setup for Aharonov-Bohm effect. Magnetic field only exists in the shadow region.

To discretize the degeneracy N, we consider the gauge transformation for wavefunction $\psi(x)$ (Aharonov-Bohm effect):

$$\psi'(x) = e^{ie\chi(x)}\psi(x), \tag{2.10}$$

where $\chi(x)$ is a degree of freedom of gauge transformation $A'_\mu = A_\mu - \partial_\mu \chi$. Figure 2.1 is the setup for the Aharonov-Bohm effect. A particle can pass through two paths C_1 and C_2. Wavefunction has a phase difference $\exp[ie(\chi_{C_1} - \chi_{C_2})]$ between path C_1 and path C_2. By using a gauge field $\vec{A}$, $e(\chi_{C_1} - \chi_{C_2})$ is represented by

$$\begin{aligned} \theta_{AB} &\equiv e(\chi_{C_1} - \chi_{C_2}) \\ &= e\left[\int_{C_1} A(s)\cdot ds - \int_{C_2} A(s)\cdot ds\right] = e\oint A(s)\cdot ds \\ &= e\int B\cdot dS = e\Phi = eBL_xL_y, \end{aligned} \tag{2.11}$$

where we use the Stoke's theorem and Φ is a magnetic flux in this section. θ_{AB} is called Aharonov-Bohm phase. Since the wavefunction must be single-valued, we get

$$eBL_xL_y = 2\pi N \quad (N \in \mathbb{Z}). \tag{2.12}$$

Thus, the degeneracy N is discretized.

2.2 Flux compactification in six-dimensional theory

We consider a higher dimensional field theory with flux compactification. In general, magnetic flux can be introduced into a compact space. In superstring theory which implies ten-dimensional spacetime with six-dimensional compact space, there are many ways to introduce the magnetic flux [16]. In higher dimensional field theories, the magnetic flux can be introduced into a torus T^2 [20,24] or a sphere S^2 [34,35]. Although the flux compactification on S^2 is interesting, we consider the flux compactification on T^2 hereafter.

We assume the six-dimensional spacetime M^6 is a product of four-dimensional Minkowski spacetime M^4 and two-dimensional torus T^2: $M^6 = M^4 \times T^2$. In this thesis, the six-dimensional spacetime index is given by $M, N = 0, 1, 2, 3, 5, 6$ and the Minkowski spacetime M^4 index is $\mu, \nu = 0, 1, 2, 3$ and compact space T^2 index is $m, n = 5, 6$. We follow the metric convention as $\eta_{\mu\nu} = (-1, +1, \cdots, +1)$. In general, a torus can be defined on a complex plane $\mathbb{C}$ (or two dimensional real plane $\mathbb{R}^2$) by modding out a Λ_2, which is two-dimensional lattice generated by two vectors $\{\vec{e}_1, \vec{e}_2\}$. The size of T^2 is parametrized by the length L_1, L_2 and the shape is $\tau \in \mathbb{C}$. For simplicity, we set $L = L_1 = L_2 = 1$ and $\tau = i$ (square torus).

We introduce the magnetic flux. The magnetic flux is given by the nontrivial background (or vacuum expectation value (VEV)) of the fifth and the sixth component of the gauge fields $A_{5,6}$. We choose the background of $A_{5,6}$ as

$$\langle A_5 \rangle = -\frac{1}{2} f x_6, \quad \langle A_6 \rangle = \frac{1}{2} f x_5, \tag{2.13}$$

which is called symmetric gauge. This background introduces a constant magnetic flux density $\langle F_{56} \rangle = f$ with a real number f. Note that this solution breaks an extra-dimensional translational invariance spontaneously. The degeneracy is obtained from $\langle F_{56} \rangle$ integrating over T^2 as follows

$$\frac{g}{2\pi} \int_{T^2} dx_5 dx_6 \, \langle F_{56} \rangle = \frac{g}{2\pi} L^2 f = N \in \mathbb{Z}, \tag{2.14}$$

where g is a gauge coupling. Eq.(2.14) is the same as eq.(2.9) and means that the magnetic flux is quantized.

We take a kinetic term for six-dimensional charged scalar field Φ as an example:

$$S_6 = \int d^6x\Big(- (D_M\Phi)^* D^M\Phi\Big), \tag{2.15}$$

where $D_M = \partial_M - igA_M$ is the covariant derivative. Decomposing this into the part of Minkowski spacetime and that of compact space, we have

$$\begin{aligned} S_6 &= \int d^6x\Big(- \eta^{\mu\nu}(D_\mu\Phi)^* D_\nu\Phi - (D_5\Phi)^* D_5\Phi - (D_6\Phi)^* D_6\Phi\Big) \\ &\simeq \int d^6x\Big(- \eta^{\mu\nu}(D_\mu\Phi)^* D_\nu\Phi - \Phi^*(-D_5^2 - D_6^2)\Phi\Big), \end{aligned} \tag{2.16}$$

where we drop the surface terms in performing an integration by parts in the second line. The second term in eq.(2.16) will be a mass term in terms of four-dimensional effective theory. As in the previous section, we recall the commutation relation, which was defined in eq.(2.4). Replacing D_x, D_y, e, B by D_5, D_6, g, f respectively, creation and annihilation operators in the present case are given by

$$a = \sqrt{\frac{1}{2gf}}(iD_5 - D_6), \quad a^\dagger = \sqrt{\frac{1}{2gf}}(iD_5 + D_6), \quad [a, a^\dagger] = 1. \tag{2.17}$$

By using these creation and annihilation operators, the mass term can be rewritten by

$$-D_5^2 - D_6^2 = 2gf\left(a^\dagger a + \frac{1}{2}\right). \tag{2.18}$$

This mass spectrum becomes a Landau level.

We denote Landau level by n ($n = 0, 1, 2, \cdots$) and the degeneracy by j ($j = 0, 1, \cdots, N-1$). If the zero mode function in compact space is expressed by $\xi_{0,j}$, the zero mode function is determined by [20]

$$a\xi_{0,j} = 0, \quad a^\dagger\bar{\xi}_{0,j} = 0. \tag{2.19}$$

By using creation and annihilation operators, the higher mode function $\xi_{n,j}$ can be obtained by [36]

$$\xi_{n,j} = \frac{1}{\sqrt{n!}}(a^\dagger)^n\xi_{0,j}, \quad \bar{\xi}_{n,j} = \frac{1}{\sqrt{n!}}(a)^n\bar{\xi}_{0,j}. \tag{2.20}$$

The higher mode function $\xi_{n,j}$ also satisfies an orthonormality condition

$$\int_{T^2} d^2x\bar{\xi}_{n',j'}\xi_{n,j} = \delta_{n,n'}\delta_{j,j'}. \tag{2.21}$$

To derive a four-dimensional effective Lagrangian by Kaluza-Klein reduction, we need to expand Φ in terms of mode functions $\xi_{n,j}$ (Kaluza-Klein expansion, KK expansion):

$$\Phi = \sum_{n,j} \Phi_{n,j}(x_\mu)\xi_{n,j}(x_m) = \sum_{n,j} \Phi_{n,j}(x_\mu)\frac{1}{\sqrt{n!}}(a^\dagger)^n \xi_{0,j}(x_m), \tag{2.22}$$

$$\Phi^* = \sum_{n,j} \Phi^*_{n,j}(x_\mu)\bar{\xi}_{n,j}(x_m) = \sum_{n,j} \Phi^*_{n,j}(x_\mu)\frac{1}{\sqrt{n!}}(a)^n \bar{\xi}_{0,j}(x_m). \tag{2.23}$$

By using this KK expansion and the orthonormality condition, the four-dimensional effective action is obtained as

$$\begin{aligned} S_4 &= \int d^4x \left(\int_{T^2} d^2x \Big(-\eta^{\mu\nu}(D_\mu\Phi)^* D_\nu\Phi - \Phi^*(-D_5^2 - D_6^2)\Phi \Big) \right) \\ &= \int d^4x \sum_{n,j} \left(-(D_\mu\Phi_{n,j})^* D^\mu\Phi_{n,j} - (2gf)\left(n + \frac{1}{2}\right)\Phi^*_{n,j}\Phi_{n,j} \right). \end{aligned} \tag{2.24}$$

Chapter 3

Abelian Gauge Theory Analysis in Six Dimensions

In this chapter, we review an Abelian gauge theory in six dimensions. In particular, we see a quantum electrodynamics (QED) in six-dimensional theory [24–26]. First, we will calculate a four-dimensional effective Lagrangian from the six-dimensional Lagrangian. Then, we will calculate the quantum corrections to Wilson-line scalar mass without or with magnetic flux, and discuss their properties.

3.1 Six-dimensional action

3.1.1 Gauge field

Before considering a six-dimensional action for gauge field, it is useful to define ∂, z and ϕ as

$$\phi = \frac{1}{\sqrt{2}}(A_6 + iA_5), \quad z = \frac{1}{2}(x_5 + ix_6), \quad \partial = \partial_5 - i\partial_6. \tag{3.1}$$

Since VEV is given by eq.(2.13), $\langle\phi\rangle = f\bar{z}/\sqrt{2}$ is obtained, and then we expand ϕ around the flux background $\langle\phi\rangle$ as

$$\phi = \langle\phi\rangle + \varphi = \frac{f}{\sqrt{2}}\bar{z} + \varphi. \tag{3.2}$$

To distinguish φ from a bulk scalar Φ, which we will introduce later, we call φ Wilson line (WL) scalar field.

Six-dimensional action for gauge field is expressed by

$$\begin{aligned} S_{6g} &= \int d^6x \left(-\frac{1}{4}F^{MN}F_{MN}\right) \\ &= \int d^6x \left(-\frac{1}{4}\right)\Big\{F^{\mu\nu}F_{\mu\nu} + 2(F^{\mu 5}F_{\mu 5} + F^{\mu 6}F_{\mu 6} + F^{56}F_{56})\Big\}, \end{aligned} \tag{3.3}$$

where $F_{MN} = \partial_M A_N - \partial_N A_M$ is the field strength. Terms from the second term to the fourth term in eq.(3.3) are expressed in terms of φ and the result is

$$\begin{aligned} S_{6g} = \int d^6x \Big(&-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \partial^\mu\varphi^*\partial_\mu\varphi - \frac{1}{4}(\partial\varphi^* + \bar{\partial}\varphi)^2 - \frac{1}{2}f^2 \\ &- \frac{1}{2}\bar{\partial}A^\mu\partial A_\mu - \frac{i}{\sqrt{2}}\partial_\mu A^\mu(\partial\varphi^* - \bar{\partial}\varphi)\Big). \end{aligned} \tag{3.4}$$

Note that eq.(3.4) only contains quadratic terms for gauge and WL scalar fields, but eq.(3.4) does not contain the interaction terms.

3.1.2 Scalar field

Six-dimensional action for the scalar field is the same as eq.(2.15) in the section 2.2:

$$\begin{aligned} S_{6s} &= \int d^6x \Big(-(D_M\Phi)^* D^M\Phi\Big) \\ &= \int d^6x \Big(-(D_\mu\Phi)^* D^\mu\Phi - (D_m\Phi)^* D^m\Phi\Big). \end{aligned} \tag{3.5}$$

The second term in eq.(3.5) involves the mass term and the interaction terms between WL scalar φ and bulk scalar Φ. The second term in eq.(3.5) is calculated as

$$\begin{aligned} (D_m\Phi)^* D^m\Phi &= (D_5\Phi)^* D^5\Phi + (D_6\Phi)^* D^6\Phi = \frac{1}{2}D^*\Phi^* D\Phi + \frac{1}{2}\bar{D}^*\Phi^*\bar{D}\Phi \\ &\simeq -\frac{1}{2}\Phi^*(\mathcal{D}\bar{\mathcal{D}} + \bar{\mathcal{D}}\mathcal{D})\Phi - \sqrt{2}g\varphi^*\Phi^*\mathcal{D}\Phi + \sqrt{2}g\varphi\Phi^*\bar{\mathcal{D}}\Phi + 2g^2\varphi^*\varphi\Phi^*\Phi \end{aligned} \tag{3.6}$$

where the covariant derivatives $D, \bar{D}$ in the complex coordinates are defined by

$$D = D_5 - iD_6 = \partial - \sqrt{2}g\phi = \mathcal{D} - \sqrt{2}g\varphi, \tag{3.7}$$

$$\bar{D} = \bar{D}_5 + i\bar{D}_6 = \bar{\partial} + \sqrt{2}g\phi^* = \bar{\mathcal{D}} + \sqrt{2}g\varphi^*, \tag{3.8}$$

$$\mathcal{D} = \mathcal{D}_5 - i\mathcal{D}_6 = \partial - \sqrt{2}g\left\langle\phi\right\rangle, \tag{3.9}$$

$$\bar{\mathcal{D}} = \bar{\mathcal{D}}_5 + i\bar{\mathcal{D}}_6 = \bar{\partial} + \sqrt{2}g\left\langle\phi^*\right\rangle. \tag{3.10}$$

Note that $\mathcal{D}_m$ means the covariant derivatives with VEV $\langle A_m \rangle$, and $D^*(\bar{D}^*)$ is not $\bar{D}(D)$ since $D^* = \bar{\partial} - \sqrt{2}g\phi^*$ and we drop the surface terms in performing integration by parts in the second line. Thus, eq.(3.5) is rewritten by

$$S_{6s} = \int d^6x \Big(- (D_\mu \Phi)^* D^\mu \Phi - \frac{1}{2}\Phi^* \Big[- (\mathcal{D}\bar{\mathcal{D}} + \bar{\mathcal{D}}\mathcal{D}) \Big] \Phi$$
$$+ \sqrt{2} g \varphi^* \Phi^* \mathcal{D}\Phi - \sqrt{2} g \varphi \Phi^* \bar{\mathcal{D}}\Phi - 2g^2 \varphi^* \varphi \Phi^* \Phi \Big). \tag{3.11}$$

3.1.3 Fermion field

Before considering a six-dimensional action for fermion fields, we introduce our convention of gamma matices [5,37]. First, σ^μ is defined as

$$\sigma^0 = -1_{2\times 2} = \bar{\sigma}^0, \quad \sigma^i = -\bar{\sigma}^i, \tag{3.12}$$

where σ^i are Pauli mattices. Gamma matrices in four dimensions are given by

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}, \quad \gamma^5 = -i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{3.13}$$

When ψ_L and ψ_R is expressed by

$$\psi_L = \begin{pmatrix} \psi \\ 0 \end{pmatrix}, \quad \psi_R = \begin{pmatrix} 0 \\ \bar{\chi} \end{pmatrix}, \tag{3.14}$$

ψ_L and ψ_R are satisfied with $\gamma^5\psi_L = -\psi_L$ and $\gamma^5\psi_R = \psi_R$ as the eigenfunction of γ^5. Note that the Weyl fermion ψ and χ have charges $-g$ and $+g$ respectively. Gamma matrices in six dimensions are given by

$$\Gamma^\mu = \begin{pmatrix} \gamma^\mu & 0 \\ 0 & \gamma^\mu \end{pmatrix}, \quad \Gamma^5 = \begin{pmatrix} 0 & i\gamma^5 \\ i\gamma^5 & 0 \end{pmatrix}, \quad \Gamma^6 = \begin{pmatrix} 0 & -\gamma^5 \\ \gamma^5 & 0 \end{pmatrix}. \tag{3.15}$$

Thus, a six-dimensional Weyl fermion Ψ is defined as

$$\Psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}, \quad \Gamma^7 = -\Gamma^0\Gamma^1\Gamma^2\Gamma^3\Gamma^5\Gamma^6 = \begin{pmatrix} \gamma^5 & 0 \\ 0 & -\gamma^5 \end{pmatrix}. \tag{3.16}$$

In this convention, $\Gamma^7\Psi = -\Psi$ is satisfied.

Six-dimensional action for fermion is given by

$$S_{6f} = \int d^6x \; i\bar{\Psi}\Gamma^M D_M \Psi$$
$$= \int d^6x \; i(\bar{\Psi}\Gamma^\mu D_\mu \Psi + \bar{\Psi}\Gamma^5 D_5 \Psi + \bar{\Psi}\Gamma^6 D_6 \Psi). \tag{3.17}$$

Calculating the second and third terms in eq.(3.17), one has

$$\begin{aligned}i(\bar{\Psi}\Gamma^5 D_5\Psi + \bar{\Psi}\Gamma^6 D_6\Psi) &= -\chi D_5\psi + \bar{\psi}D_5\bar{\chi} + i(\chi D_6\psi + \bar{\psi}D_6\bar{\chi})\\ &\simeq -\chi(\partial - gf\bar{z} - \sqrt{2}g\varphi)\psi - \bar{\chi}(\bar{\partial} - gfz - \sqrt{2}g\varphi^*)\bar{\psi}. \quad (3.18)\end{aligned}$$

Thus, eq.(3.17) is expressed as

$$\begin{aligned}S_{6f} = \int d^6x \Big(&- i\psi\sigma^\mu D_\mu^*\bar{\psi} - i\chi\sigma^\mu D_\mu\bar{\chi}\\ &- \chi(\partial - gf\bar{z} - \sqrt{2}g\varphi)\psi - \bar{\chi}(\bar{\partial} - gfz - \sqrt{2}g\varphi^*)\bar{\psi}\Big). \quad (3.19)\end{aligned}$$

3.2 Quantum correction: Without flux

It is meaningful to compare quantum correction to WL scalar mass with flux to quantum correction without flux. First, we consider the quantum correction to WL scalar mass without flux. In the case without flux, KK expansion of scalar field Φ and fermion fields ψ, χ are given by

$$\Phi = \sum_{n,m} \Phi_{n,m}(x_\mu)\lambda_{n,m}(x_m), \quad (3.20)$$

$$\psi = \sum_{n,m} \psi_{n,m}(x_\mu)\lambda_{n,m}(x_m), \quad (3.21)$$

$$\chi = \sum_{n,m} \chi_{n,m}(x_\mu)\bar{\lambda}_{n,m}(x_m) \quad (3.22)$$

where $n, m \in \mathbb{Z}$. The mode functions of compact space $\lambda_{n,m}(x_m)$ are determined as

$$\lambda_{n,m}(x_m) = \frac{1}{L}\exp\left[\frac{2\pi i}{L}(nx_5 + mx_6)\right], \quad (3.23)$$

from the periodic boundary condition of torus. The mode function $\lambda_{n,m}$ also satisfies an orthonormality condition

$$\int_{T^2} d^2x \bar{\lambda}_{n',m'}\lambda_{n,m} = \delta_{n,n'}\delta_{m,m'}. \quad (3.24)$$

3.2.1 Scalar QED

Noting that the covariant derivatives $\mathcal{D}$ and $\bar{\mathcal{D}}$ become the normal partial derivatives ∂ and $\bar{\partial}$ without flux respectively, eq.(3.4) and eq.(3.11) have

$$S_{sQED} = \int d^4x\Big(- \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \partial_\mu\varphi^*\partial^\mu\varphi + \sum_{n,m}\Big(- (D_\mu\Phi_{n,m})^* D^\mu\Phi_{n,m} - |M_{n,m}|^2\Phi^*_{n,m}\Phi_{n,m} + \sqrt{2}gM^*_{n,m}\varphi\Phi^*_{n,m}\Phi_{n,m} + \sqrt{2}gM_{n,m}\varphi^*\Phi^*_{n,m}\Phi_{n,m} - 2g^2\varphi^*\varphi\Phi^*_{n,m}\Phi_{n,m}\Big)\Big), \tag{3.25}$$

by using eq.(3.20) and eq.(3.23). In eq.(3.25), we omit KK gauge fields. KK modes for A_m (or φ, φ^*) are absorbed into the longitudinal part of KK gauge fields. Here, $M_{n,m} = 2\pi(m + in)/L$ is the KK mass spectrum and we ignore the constant term. In eq.(3.25), we deal with WL scalar φ as zero-mode.

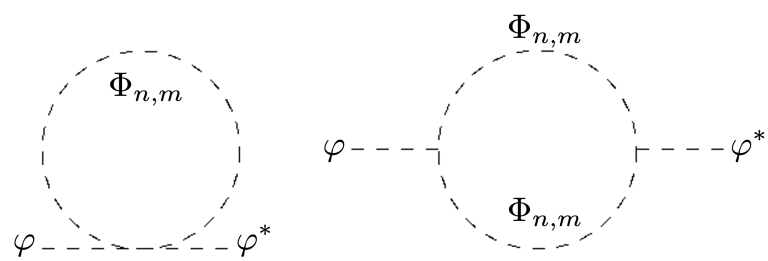

Figure 3.1: Scalar loop correction

We are ready to calculate the quantum correction to WL scalar mass from eq.(3.25). Two Feynman diagrams from the scalar field $\Phi_{n,m}$ loop contributions are depicted in figure 3.1. Denoting I_{b4pt} and I_{b3pt} as the contributions from the four-point interaction $\varphi^*\varphi\Phi^*_{n,m}\Phi_{n,m}$ and the three-point interaction $\varphi\Phi^*_{n,m}\Phi_{n,m}$+ h.c. respectively, I_{b4pt} and I_{b3pt} are obtained as

$$I_{b4pt} = -i2g^2 \sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}\int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2 + |M_{n,m}|^2}, \tag{3.26}$$

$$I_{b3pt} = +i2g^2 \sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}\int \frac{d^4k}{(2\pi)^4}\frac{|M_{n,m}|^2}{(k^2 + |M_{n,m}|^2)^2}, \tag{3.27}$$

where Wick rotation is applied in momentum integrals. Thus, the quantum correction to WL scalar mass has

$$\begin{aligned}\delta m_b^2 &= i(I_{b4pt} + I_{b3pt}) \\ &= 2g^2 \sum_{n,m} \int \frac{d^4k}{(2\pi)^4} \left(\frac{1}{k^2 + |M_{n,m}|^2} - \frac{|M_{n,m}|^2}{(k^2 + |M_{n,m}|^2)^2} \right) \\ &= 2g^2 \sum_{n,m} \int \frac{d^4k}{(2\pi)^4} \frac{k^2}{(k^2 + |M_{n,m}|^2)^2}. \end{aligned} \tag{3.28}$$

Since the contribution to quantum correction from scalar loop (3.28) is similar to the contribution from fermion loop, which we will calculate in the next subsection, the momentum integral and summation for KK mode m, n will be performed in the next subsection.

3.2.2 QED

Eq.(3.4) and eq.(3.19) have

$$\begin{aligned} S_{QED} = \int d^4x \Big(&-\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \partial^\mu \varphi^* \partial_\mu \varphi \\ &+ \sum_{n,m} \Big(- i\psi_{n,m} \sigma^\mu D_\mu^* \bar{\psi}_{n,m} - i\chi_{n,m} \sigma^\mu D_\mu \bar{\chi}_{n,m} \\ &- (M_{n,m} - \sqrt{2} g\varphi) \chi_{n,m} \psi_{n,m} - (M_{n,m}^* - \sqrt{2} g\varphi^*) \bar{\chi}_{n,m} \bar{\psi}_{n,m} \Big) \Big), \end{aligned} \tag{3.29}$$

by using eq.(3.21), eq.(3.22) and eq.(3.23) without flux. We also omit KK gauge fields in eq.(3.29).

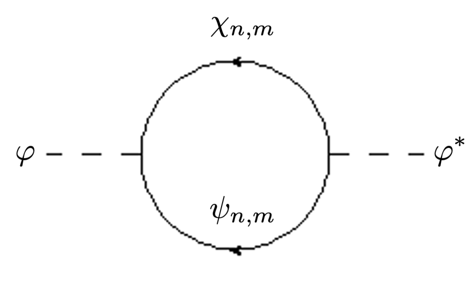

Figure 3.2: Fermion loop correction

As in the previous subsection, we calculate the quantum correction to WL scalar mass from the action (3.29). A Feynman diagram from the fermion field loop contribution is depicted in figure 3.2. Denoting I_f by the contribution from the three-point interaction $\varphi\chi_{n,m}\psi_{n,m}$, I_f is obtained as

$$\begin{aligned} I_f &= (-1) \times 2g^2 \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \int \frac{d^4k}{(2\pi)^4} \frac{\mathrm{Tr}[\sigma^\mu \bar{\sigma}^\nu] k_\mu k_\nu}{(k^2 + |M_{n,m}|^2)^2} \\ &= +i4g^2 \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \int \frac{d^4k}{(2\pi)^4} \frac{k^2}{(k^2 + |M_{n,m}|^2)^2}, \end{aligned} \tag{3.30}$$

where we employed the relation $\mathrm{Tr}[\sigma^\mu \bar{\sigma}^\nu] = -2\eta^{\mu\nu}$. Note that this quantum correction is applied by two-component spinor techniques [38]. Thus, the quantum correction to WL scalar mass has

$$\delta m_f^2 = iI_f = -4g^2 \sum_{n,m} \int \frac{d^4k}{(2\pi)^4} \frac{k^2}{(k^2 + |M_{n,m}|^2)^2}. \tag{3.31}$$

Note that eq.(3.31) has a relation[1] $\delta m_f^2 = -2\delta m_b^2$.

We continue to compute eq.(3.31):

$$\begin{aligned} \delta m_f^2 &= -4g^2 \sum_{n,m} \int_0^\infty dtt \int \frac{d^4k}{(2\pi)^4} k^2 e^{-t(k^2 + |M_{n,m}|^2)} = -\frac{g^2}{2\pi^2} \sum_{n,m} \int_0^\infty \frac{dt}{t^2} e^{-|M_{n,m}|^2 t} \\ &= -\frac{g^2}{2\pi^2} \sum_{n,m} \int_0^\infty \frac{dt}{t^2} \exp\left[-\frac{4\pi^2 t}{L^2}(m^2 + n^2)\right]. \end{aligned} \tag{3.32}$$

where we used Schwinger representation

$$\frac{\Gamma(s)}{A^s} = \int_0^\infty e^{-At} t^{s-1} dt. \tag{3.33}$$

The summation for KK mode m, n in eq.(3.32) are performed by using Poisson resummation (see appendix A):

$$\sum_{n=-\infty}^{\infty} \exp\left[-\frac{(n+a)^2}{R^2 l}\right] = R\sqrt{\pi l} \sum_{m=-\infty}^{\infty} e^{2\pi i m a} e^{-\pi^2 l m^2 R^2}. \tag{3.34}$$

Replacing R and l for $L/2\pi$ and $1/t$ respectively and set $a = 0$, the summation part in eq.(3.32) is rewritten as

$$\sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \exp\left[-\frac{4\pi^2 t}{L^2}(m^2 + n^2)\right] = \left(\frac{L}{2\pi}\right)^2 \frac{\pi}{t} \sum_{r=-\infty}^{\infty} \sum_{s=-\infty}^{\infty} \exp\left[-\frac{L^2}{4t}(r^2 + s^2)\right]. \tag{3.35}$$

[1] If we impose supersymmetry, the quantum correction to WL scalar mass from bosonic loop and fermionic loop are canceled (see [24]).

Thus, δm_f^2 is calculated as

$$
\begin{aligned}
\delta m_f^2 &= -\frac{g^2 L^2}{8\pi^3} \sum_{r,s} \int_0^\infty \frac{dt}{t^3} \exp\left[-\frac{L^2}{4t}(r^2+s^2)\right] \\
&= -\frac{2g^2}{\pi^3 L^2} \sum_{(r,s)\neq(0,0)} \frac{1}{(r^2+s^2)^2}.
\end{aligned} \tag{3.36}
$$

In the second line of eq.(3.36), we have performed the integral for t. Note that we subtracted a zero-mode $(r,s)=(0,0)$ from the summation for winding mode r,s since δm_f^2 is diverged at $(r,s)=(0,0)$. Since the zero mode part is just the constant part from the point of view of the one-loop effective potential, it is possible to subtract the zero-mode part. Computing eq.(3.36) numerically, one has

$$
\delta m_f^2 \approx -0.39 \times \frac{g^2}{L^2}, \tag{3.37}
$$

and we find δm_f^2 has a finite value. In [24], δm_f^2 (or δm_b^2) is calculated by using Jacobi theta function and is also derived from one-loop effective potential.

3.3 Quantum correction: With flux

3.3.1 Scalar QED

In the case with flux, we regard the covariant derivatives $\mathcal{D}$ and $\bar{\mathcal{D}}$ as creation and annihilation operators by

$$
a = \frac{1}{\sqrt{2gf}} i\bar{\mathcal{D}}, \quad a^\dagger = \frac{1}{\sqrt{2gf}} i\mathcal{D}, \tag{3.38}
$$

which satisfy the commutation relation $[a, a^\dagger] = 1$. In this thesis, we denote $\alpha = 2gf$.

By using eq.(3.38), eq.(3.11) is rewritten as

$$
\begin{aligned}
S_{6s} = \int d^6x \Big(&- (D_\mu \Phi)^* D^\mu \Phi - \alpha \left(n + \frac{1}{2}\right) \Phi^* \Phi \\
&- \sqrt{2} i g \sqrt{\alpha} \varphi^* \Phi^* a^\dagger \Phi + \sqrt{2} i g \sqrt{\alpha} \varphi \Phi^* a \Phi - 2g^2 \varphi^* \varphi \Phi^* \Phi \Big).
\end{aligned} \tag{3.39}
$$

As we have seen in quantum mechanics, creation and annihilation operators $a^\dagger, a$ act on mode functions $\xi_{n,j}$ as

$$
a\xi_{n,j} = \sqrt{n}\,\xi_{n-1,j}, \quad a^\dagger \xi_{n,j} = \sqrt{n+1}\,\xi_{n+1,j}. \tag{3.40}
$$

The third and fourth terms in eq.(3.39) can be calculated by using eqs.(2.22), (2.23) and (3.40)

$$-\sqrt{2}ig\sqrt{\alpha}\varphi^*\Phi^* a^\dagger \Phi = -\sqrt{2}ig\varphi^* \sum_{n,j}\sum_{n',j'} \sqrt{\alpha(n+1)}\Phi^*_{n',j'}\Phi_{n,j}\bar{\xi}_{n',j'}\xi_{n+1,j}, \tag{3.41}$$

$$\sqrt{2}ig\sqrt{\alpha}\varphi\Phi^* a\Phi = \sqrt{2}ig\varphi \sum_{n,j}\sum_{n',j'} \sqrt{\alpha n}\Phi^*_{n',j'}\Phi_{n,j}\bar{\xi}_{n',j'}\xi_{n-1,j}. \tag{3.42}$$

Thus, eq.(3.4) and eq.(3.39) have

$$\begin{aligned} S_{sQED} = \int d^4x \Big(&-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \partial_\mu\varphi^*\partial^\mu\varphi \\ &+ \sum_{n,j}\Big(-(D_\mu\Phi_{n,j})^* D^\mu\Phi_{n,j} - \alpha\left(n+\frac{1}{2}\right)\Phi^*_{n,j}\Phi_{n,j} \\ &\quad - \sqrt{2}ig\sqrt{\alpha(n+1)}\varphi^*\Phi^*_{n+1,j}\Phi_{n,j} + \sqrt{2}ig\sqrt{\alpha(n+1)}\varphi\Phi^*_{n,j}\Phi_{n+1,j} \\ &\quad - 2g^2\varphi^*\varphi\Phi^*_{n,j}\Phi_{n,j}\Big)\Big) \end{aligned} \tag{3.43}$$

by using eq.(2.21).

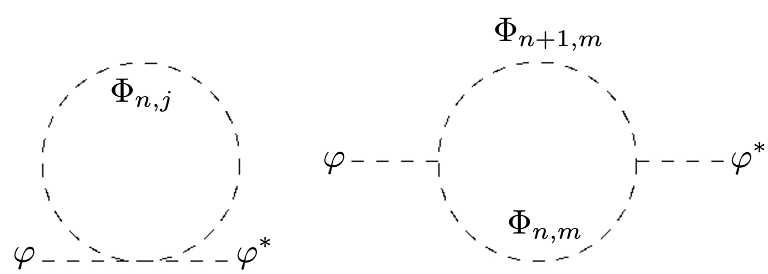

Figure 3.3: Scalar loop correction with flux

The quantum correction to WL scalar mass can be calculated from eq.(3.43). Two Feynman diagrams from the scalar field loop contributions are depicted in figure 3.3. As in the subsection 3.2.1, I_{b4pt} and I_{b3pt}, which are denoted by the contributions from the four-point interaction and three-point interaction respectively, are obtained as

$$I_{b4pt} = -i2g^2 \sum_{n,j} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 + \alpha\left(n+\frac{1}{2}\right)}, \tag{3.44}$$

$$I_{b3pt} = +i2g^2 \sum_{n,j} \int \frac{d^4k}{(2\pi)^4} \frac{\alpha(n+1)}{\left(k^2 + \alpha\left(n+\frac{1}{2}\right)\right)\left(k^2 + \alpha\left(n+\frac{3}{2}\right)\right)}. \tag{3.45}$$

Thus, the quantum correction to WL scalar mass has

$$
\begin{aligned}
\delta m_b^2 &= i(I_{b4pt} + I_{b3pt}) \\
&= 2g^2 \sum_{n,j} \int \frac{d^4k}{(2\pi)^4} \left(\frac{1}{k^2 + \alpha\left(n + \frac{1}{2}\right)} - \frac{\alpha(n+1)}{\left(k^2 + \alpha\left(n + \frac{1}{2}\right)\right)\left(k^2 + \alpha\left(n + \frac{3}{2}\right)\right)} \right) \\
&= 2g^2|N| \sum_{n=0}^{\infty} \int \frac{d^4k}{(2\pi)^4} \left(\frac{1}{k^2 + \alpha\left(n + \frac{1}{2}\right)} - (n+1) \left(\frac{1}{k^2 + \alpha\left(n + \frac{1}{2}\right)} - \frac{1}{k^2 + \alpha\left(n + \frac{3}{2}\right)} \right) \right) \\
&= 2g^2|N| \sum_{n=0}^{\infty} \int \frac{d^4k}{(2\pi)^4} \left(\frac{n+1}{k^2 + \alpha\left(n + \frac{3}{2}\right)} - \frac{n}{k^2 + \alpha\left(n + \frac{1}{2}\right)} \right). \qquad (3.46)
\end{aligned}
$$

By the shift $n \to n+1$ in the second term of eq.(3.46), the quantum correction vanishes:

$$\delta m_b^2 = 0. \qquad (3.47)$$

3.3.2 QED

Eq.(3.4) and eq.(3.19) have

$$
\begin{aligned}
S_{QED} = \int d^6x \Big(&- \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \partial^\mu \varphi^* \partial_\mu \varphi - i\psi\sigma^\mu D_\mu^* \bar{\psi} - i\chi\sigma^\mu D_\mu \bar{\chi} \\
&- \chi(\partial - gf\bar{z} - \sqrt{2}g\varphi)\psi - \bar{\chi}(\bar{\partial} - gfz - \sqrt{2}g\varphi^*)\bar{\psi} \Big). \qquad (3.48)
\end{aligned}
$$

As in the previous subsection, we regard the covariant derivatives in the complex coordinates as creation and annihilation operators in the case of fermion. To derive the mass-squared operators, we find Dirac equation for ψ or $\bar{\chi}$ from eq.(3.48):

$$i\bar{\sigma}^\mu \partial_\mu \psi + (\bar{\partial} + gfz)\bar{\chi} = 0, \qquad (3.49)$$

$$i\sigma^\mu \partial_\mu \bar{\chi} + (\partial - gf\bar{z})\psi = 0, \qquad (3.50)$$

where we ignore the interaction terms since we focus on the mass-squared operators. Acting $i\sigma^\mu\partial_\mu$ or $i\bar{\sigma}^\mu\partial_\mu$ on eq.(3.49) or (3.50) and using the relation $\sigma^\mu\bar{\sigma}^\nu + \sigma^\nu\bar{\sigma}^\mu = -2\eta^{\mu\nu}$, Klein-Gordon equations

$$\Box\psi - (\bar{\partial} + gfz)(\partial - gf\bar{z})\psi = 0, \qquad (3.51)$$

$$\Box\chi - (\bar{\partial} - gfz)(\partial + gf\bar{z})\chi = 0, \qquad (3.52)$$

can be obtained. If creation and annihilation operators are defined as

$$a_- = \frac{i}{\sqrt{\alpha}}(\partial - gf\bar{z}), \quad a_-^\dagger = \frac{i}{\sqrt{\alpha}}(\bar{\partial} + gfz), \tag{3.53}$$

$$a_+ = \frac{i}{\sqrt{\alpha}}(\bar{\partial} - gfz), \quad a_+^\dagger = \frac{i}{\sqrt{\alpha}}(\partial + gf\bar{z}), \tag{3.54}$$

we can read the mass-squared operators $\mathcal{M}_-^2 = \alpha a_-^\dagger a_-$ from eq.(3.51) or $\mathcal{M}_+^2 = \alpha(a_+^\dagger a_+ + 1)$ from eq.(3.52). The difference between $\mathcal{M}_-^2$ and $\mathcal{M}_+^2$ is a feature of flux compactification, which means that a zero-mode of chiral fermion in four dimensions can be obtained from ψ. Denoting the mode functions as $\xi_{n,j}$ and $\bar{\xi}_{n,j}$, the mode functions on the ground state satisfy $a_-\xi_{0,j} = 0$ and $a_+\bar{\xi}_{0,j} = 0$. As we have seen eq.(2.20), the mode functions are expressed as

$$\xi_{n,j} = \frac{i^n}{\sqrt{n!}}(a_-^\dagger)^n \xi_{0,j}, \quad \bar{\xi}_{n,j} = \frac{i^n}{\sqrt{n!}}(a_+^\dagger)^n \bar{\xi}_{0,j}, \tag{3.55}$$

where i^n is convention. Acting creation and annihilation operators on the mode functions, one has

$$a_-\xi_{n,j} = i\sqrt{n}\xi_{n-1,j}, \quad a_-^\dagger \xi_{n,j} = -i\sqrt{n+1}\xi_{n+1.j}, \tag{3.56}$$

$$a_+\bar{\xi}_{n,j} = i\sqrt{n}\bar{\xi}_{n-1,j}, \quad a_+^\dagger \bar{\xi}_{n,j} = -i\sqrt{n+1}\bar{\xi}_{n+1.j}, \tag{3.57}$$

like eq.(3.40). KK expansion for ψ and χ are expressed as

$$\psi = \sum_{n,j} \psi_{n,j}(x^\mu)\xi_{n,j}(x^m), \tag{3.58}$$

$$\chi = \sum_{n,j} \chi_{n,j}(x^\mu)\bar{\xi}_{n,j}(x^m). \tag{3.59}$$

Using KK expansion (3.58), (3.59) and the orthonormality condition (2.21), eq.(3.48) has

$$\begin{aligned} S_{QED} = \int d^6x \Big(&- \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \partial^\mu\varphi^*\partial_\mu\varphi - i\psi\sigma^\mu D_\mu^*\bar{\psi} - i\chi\sigma^\mu D_\mu\bar{\chi} \\ &- \chi(-i\sqrt{\alpha}a_- - \sqrt{2}g\varphi)\psi - \bar{\chi}(-i\sqrt{\alpha}a_+ - \sqrt{2}g\varphi^*)\bar{\psi}\Big) \\ = \int d^4x \Big(&- \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \partial^\mu\varphi^*\partial_\mu\varphi + \sum_{n,j}\Big(- i\psi_{n,j}\sigma^\mu D_\mu^*\bar{\psi}_{n,j} - i\chi_{n,j}\sigma^\mu D_\mu\bar{\chi}_{n,j} \\ &- \sqrt{\alpha(n+1)}\chi_{n,j}\psi_{n+1,j} + \sqrt{2}g\varphi\chi_{n,j}\psi_{n,j} + \text{h.c.}\Big)\Big). \end{aligned} \tag{3.60}$$

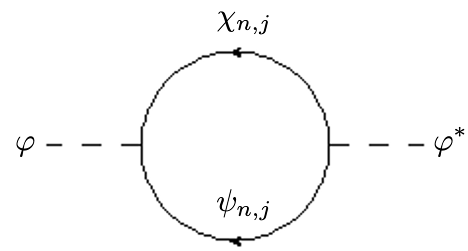

Figure 3.4: Fermion loop correction with flux

As in the previous subsection, we compute the quantum correction to WL scalar mass from eq.(3.60). A Feynman diagram from fermion field loop contribution is depicted in figure 3.4. As in the subsection 3.2.2, I_f is obtained as

$$\begin{aligned} I_f &= +i4g^2|N| \sum_n \int \frac{d^4k}{(2\pi)^4} \frac{k^2}{(k^2+\alpha n)(k^2+\alpha(n+1))} \\ &= +i4g^2|N| \sum_{n=0}^{\infty} \int \frac{d^4k}{(2\pi)^4} \left(\frac{n+1}{k^2+\alpha(n+1)} - \frac{n}{k^2+\alpha n} \right). \end{aligned} \tag{3.61}$$

As in the subsection 3.3.1, the quantum correction δm_f^2 vanishes by the shift $n \to n+1$ in the second term of eq.(3.61):

$$\delta m_f^2 = 0. \tag{3.62}$$

Thus, the cancellation of the quantum correction to WL scalar mass at one-loop level is shown [2]. These results (3.47), (3.62) are also shown by dimensional regularization [26].

We compare the result (3.37) without flux to the result (3.62) with flux. In the case without flux, the inverse of the compactification radius L^{-1} plays a role of cutoff scale. This means that the result (3.37) is finite if the compactification radius L has a finite length. Finiteness of the quantum correction has been seen in higher dimensional gauge theory, in particular in gauge-Higgs unification ([10, 11, 14]). On the other hand, the quantum correction to WL scalar mass with flux (3.62) is canceled at one-loop level[3]. The physical reason of this cancellation is that the shift symmetry from translation in compact spaces forbids the mass term of WL scalar field. In that situation, the

[2] It is shown that the quantum correction to WL scalar mass is canceled at two-loop level [28].
[3] It will be interesting that this cancellation mechanism is similar to the quantum correction in S^2 compactification without flux [11].

zero-mode of WL scalar field φ can be identified with Nambu-Goldstone (NG) boson of spontaneously broken translational symmetry. This issue will be seen in next section.

We comment on the results (3.47) and (3.62). These results imply that the quantum corrections to WL scalar mass from bosonic contribution (3.47) and fermionic contribution (3.62) are separately canceled even if a theory involves scalar fields and fermion fields. These results are not changed if supersymmetry is imposed [24]. The cancellation of the quantum correction by introducing magnetic flux is a new attractive feature and this feature might be a hint of the alternative solution of the hierarchy problem.

3.4 WL scalar as a Nambu-Goldstone boson

Six-dimensional actions (3.39) or (3.48) are invariant under the translation on torus $\delta_T = \epsilon\partial + \bar{\epsilon}\bar{\partial}$, where ϵ and $\bar{\epsilon}$ are infinitesimal parameters. This translation acts on WL scalar $\phi = \langle\phi\rangle + \varphi$ as

$$\begin{aligned} \delta_T\phi &= (\epsilon\partial + \bar{\epsilon}\bar{\partial})\phi \\ &= \frac{\bar{\epsilon}}{\sqrt{2}}f + (\epsilon\partial + \bar{\epsilon}\bar{\partial})\varphi. \end{aligned} \tag{3.63}$$

Since we regard φ as zero-mode of WL scalar, $\partial\varphi = 0, \bar{\partial}\varphi = 0$ are satisfied, and then

$$\delta_T\phi = \frac{\bar{\epsilon}}{\sqrt{2}}f. \tag{3.64}$$

Eq.(3.64) means that $\delta_T\phi$ is a constant shift. We find that the symmetry of translation on torus is spontaneously broken because of the constant shift (3.64). Eq.(3.64) is understood as follows. In order for four-dimensional effective Lagrangian to be invariant under the shift transformation, the Lagrangian must involve only the derivative terms of WL scalar, and the mass term of WL scalar has to be forbidden. Therefore, WL scalar φ behaves as a NG boson under shift symmetry of the translation on torus.

Chapter 4

Non-Abelian Gauge Theory Analysis in Six Dimensions

To realize more realistic model, we extend an Abelian gauge group to a non-Abelian gauge group, and calculate the quantum correction to WL scalar mass [27]. As a non-Abelian gauge group, we choose an SU(2) group. Extending to non-Abelian gauge group, self-interactions of non-Abelian gauge fields are included, and an analysis of the quantum corrections in the case of non-Abelian gauge group is non-trivial compared to the analysis in the case of Abelian gauge group. Moreover, we refer to [29] for an analysis of quantum corrections in a theory with higher dimensional operators.

4.1 Yang-Mills theory

We consider a six-dimensional SU(2) Yang-Mills theory with a constant magnetic flux, and the Lagrangian is

$$\begin{aligned}\mathcal{L}_{YM} &= -\frac{1}{4}F^a_{MN}F^{aMN} \\ &= -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - \frac{1}{2}F^a_{\mu 5}F^{a\mu 5} - \frac{1}{2}F^a_{\mu 6}F^{a\mu 6} - \frac{1}{2}F^a_{56}F^{a56},\end{aligned} \tag{4.1}$$

where the field strength and the covariant derivative are defined by

$$F^a_{MN} = \partial_M A^a_N - \partial_N A^a_M - ig[A_M, A_N]^a, \tag{4.2}$$

$$\begin{aligned}D_M A^a_N &= \partial_M A^a_N + g\varepsilon^{abc}A^b_M A^c_N \\ &= \partial_M A^a_N - ig[A_M, A_N]^a,\end{aligned} \tag{4.3}$$

and $a,b,c=1,2,3$ are gauge indices.

Following the way to introduce a constant magnetic flux in the Abelian theory (2.13), we introduce a constant magnetic flux as

$$\langle A_5^1\rangle = -\frac{1}{2}fx_6, \qquad \langle A_6^1\rangle = \frac{1}{2}fx_5, \qquad \langle A_5^{2,3}\rangle = \langle A_6^{2,3}\rangle = 0. \tag{4.4}$$

Note that this background satisfies classical equation of motion $D^m\langle F_{mn}\rangle = 0$ since the background of the field strength becomes $\langle F_{56}^a\rangle = f\delta^{a1}$. The degeneracy can be obtained as eq.(2.14) in the direction of gauge index $a=1$.

In this chapter, we also use the notation (3.1) except for the gauge index of WL scalar: ϕ^a. In analogy to eq.(3.2), we expand ϕ^a around the flux background $\langle\phi^a\rangle$ as

$$\phi^a = \langle\phi^a\rangle + \varphi^a = \frac{f}{\sqrt{2}}\bar{z}\delta^{a1} + \varphi^a. \tag{4.5}$$

Eq.(4.1) is expressed in terms of WL scalar φ:

$$\begin{aligned}
\mathcal{L}_{YM} = &-\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} - \partial_\mu\varphi^{a*}\partial^\mu\varphi^a - \frac{1}{2}\mathcal{D}A_\mu^a\bar{\mathcal{D}}A^{a\mu} + g^2[A_\mu,\varphi]^a[A^\mu,\varphi^*]^a \\
&-\frac{i}{\sqrt{2}}(\partial_\mu\varphi^a\bar{\mathcal{D}}A^{a\mu} - \partial_\mu\varphi^{a*}\mathcal{D}A^{a\mu}) \\
&+ig\Big\{\partial_\mu\varphi^a[A^\mu,\varphi^*]^a + \partial^\mu\varphi^{a*}[A_\mu,\varphi]^a\Big\} \\
&-\frac{g}{\sqrt{2}}\Big\{-\mathcal{D}A_\mu^a[A^\mu,\varphi^*]^a + \bar{\mathcal{D}}A^{a\mu}[A_\mu,\varphi]^a\Big\} \\
&-\frac{1}{4}\Big(\mathcal{D}\varphi^{a*} + \bar{\mathcal{D}}\varphi^a - \sqrt{2}g[\varphi,\varphi^*]^a + \sqrt{2}f\delta^{a1}\Big)^2,
\end{aligned} \tag{4.6}$$

where $\mathcal{D}$ and $\bar{\mathcal{D}}$ are the covariant derivatives in the complex coordinates and are defined as

$$DX^a = (D_5 - iD_6)X^a = \partial X^a - \sqrt{2}g[\phi,X]^a = \mathcal{D}X^a - \sqrt{2}g[\varphi,X]^a, \tag{4.7}$$

$$\bar{D}X^a = (D_5 + iD_6)X^a = \bar{\partial}X^a + \sqrt{2}g[\phi^*,X]^a = \bar{\mathcal{D}}X^a + \sqrt{2}g[\varphi^*,X]^a, \tag{4.8}$$

$$\mathcal{D}X^a = (\mathcal{D}_5 - i\mathcal{D}_6)X^a = \partial X^a - \sqrt{2}g[\langle\phi\rangle, X]^a, \tag{4.9}$$

$$\bar{\mathcal{D}}X^a = (\bar{\mathcal{D}}_5 + i\bar{\mathcal{D}}_6)X^a = \bar{\partial}X^a + \sqrt{2}g[\langle\phi^*\rangle, X]^a. \tag{4.10}$$

The second line in eq.(4.6) is removed by the following gauge-fixing terms:

$$\begin{aligned}
\mathcal{L}_{gf} &= -\frac{1}{2\xi}(D_\mu A^{a\mu} + \xi\mathcal{D}_m A^{am})^2 \\
&= -\frac{1}{2\xi}D_\mu A^{a\mu}D_\nu A^{a\nu} + \frac{\xi}{4}(\mathcal{D}\varphi^{a*} - \bar{\mathcal{D}}\varphi^a)^2 + \frac{i}{\sqrt{2}}(\partial_\mu\varphi^a\bar{\mathcal{D}}A^{a\mu} - \partial_\mu\varphi^{a*}\mathcal{D}A^{a\mu}),
\end{aligned} \tag{4.11}$$

where ξ is called a gauge parameter.

Once we have gauge-fixed, we need to introduce the ghost fields by following Faddeev-Popov procedure to quantize gauge fields. The ghost Lagrangian reads

$$\mathcal{L}_{ghost} = -c^{a*}(D_\mu D^\mu + \xi D_m \mathcal{D}^m)c^a. \tag{4.12}$$

Then, the total Lagrangian is

$$\begin{aligned}\mathcal{L}_{total} =& -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - \frac{1}{2\xi}D_\mu A^{a\mu}D_\nu A^{a\nu} - \partial_\mu\varphi^{a*}\partial^\mu\varphi^a \\ & -\frac{1}{2}\mathcal{D}A^a_\mu\bar{\mathcal{D}}A^{a\mu} + g^2[A_\mu,\varphi]^a[A^\mu,\varphi^*]^a - \frac{g}{\sqrt{2}}\Big\{-\mathcal{D}A^a_\mu[A^\mu,\varphi^*]^a + \bar{\mathcal{D}}A^{a\mu}[A_\mu,\varphi]^a\Big\} \\ & + ig\Big\{\partial_\mu\varphi^a[A^\mu,\varphi^*]^a + \partial^\mu\varphi^{a*}[A_\mu,\varphi]^a\Big\} \\ & -\frac{1}{4}\Big(\mathcal{D}\varphi^{a*} + \bar{\mathcal{D}}\varphi^a - \sqrt{2}g[\varphi,\varphi^*]^a + \sqrt{2}f\delta^{a1}\Big)^2 + \frac{\xi}{4}(\mathcal{D}\varphi^{a*} - \bar{\mathcal{D}}\varphi^a)^2 \\ & - c^{a*}(D_\mu D^\mu + \xi D_m\mathcal{D}^m)c^a.\end{aligned} \tag{4.13}$$

4.2 Mass spectrum

In this section, we find mass eigenstates and eigenvalues of the fields A^a_μ, φ^a, c^a.

4.2.1 Gauge field

First, we find mass eigenvalue and eigenstate of the non-Abelian gauge field. The mass term of gauge field corresponds to the first term in the second line in eq.(4.13) as

$$\mathcal{L}_{mass} = -\frac{1}{2}\mathcal{D}A^a_\mu\bar{\mathcal{D}}A^{a\mu} = -\frac{1}{2}A^a_\mu[-\mathcal{D}\bar{\mathcal{D}}]A^{a\mu}. \tag{4.14}$$

In the section 2.2, we have seen that $-\mathcal{D}\bar{\mathcal{D}}$ corresponds to the Hamiltonian of the harmonic oscillator. We regard the covariant derivatives $\mathcal{D}$ and $\bar{\mathcal{D}}$ as creation and annihilation operators like the subsection 3.3.1. Expressing them in a matrix form as

$$\mathcal{D}^{ac} = \begin{pmatrix} \partial & 0 & 0 \\ 0 & \partial & -\sqrt{2}i\varepsilon^{213}g\langle\phi^1\rangle \\ 0 & -\sqrt{2}i\varepsilon^{312}g\langle\phi^1\rangle & \partial \end{pmatrix} = \begin{pmatrix} \partial & 0 & 0 \\ 0 & \partial & igf\bar{z} \\ 0 & -igf\bar{z} & \partial \end{pmatrix}, \tag{4.15}$$

$$\bar{\mathcal{D}}^{ac} = \begin{pmatrix} \bar\partial & 0 & 0 \\ 0 & \bar\partial & \sqrt{2}i\varepsilon^{213}g\langle\phi^{1*}\rangle \\ 0 & \sqrt{2}i\varepsilon^{312}g\langle\phi^{1*}\rangle & \bar\partial \end{pmatrix} = \begin{pmatrix} \bar\partial & 0 & 0 \\ 0 & \bar\partial & -igfz \\ 0 & igfz & \bar\partial \end{pmatrix}, \tag{4.16}$$

their commutation relation can be calculated as

$$[i\bar{\mathcal{D}}, i\mathcal{D}]^{ac} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -2igf \\ 0 & 2igf & 0 \end{pmatrix} = 2igf\varepsilon^{a1c}. \tag{4.17}$$

Thus, the creation and annihilation operators can be defined as

$$a = \frac{1}{\sqrt{\alpha}} i\bar{\mathcal{D}}, \quad a^\dagger = \frac{1}{\sqrt{\alpha}} i\mathcal{D}, \tag{4.18}$$

and the commutation relation can be rewritten as $[a, a^\dagger]^{ac} = i\varepsilon^{a1c}$.

Since these expressions are non-diagonal, we diagonalize $\mathcal{D}$ and $\bar{\mathcal{D}}$ as

$$\mathcal{D}_{diag} = U^{-1}\mathcal{D}U = \begin{pmatrix} \partial & 0 & 0 \\ 0 & \partial - gf\bar{z} & 0 \\ 0 & 0 & \partial + gf\bar{z} \end{pmatrix}, \tag{4.19}$$

$$\bar{\mathcal{D}}_{diag} = U^{-1}\bar{\mathcal{D}}U = \begin{pmatrix} \bar{\partial} & 0 & 0 \\ 0 & \bar{\partial} + gfz & 0 \\ 0 & 0 & \bar{\partial} - gfz \end{pmatrix}, \tag{4.20}$$

with a unitary matrix

$$U = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} & 0 & 0 \\ 0 & 1 & i \\ 0 & i & 1 \end{pmatrix}. \tag{4.21}$$

From diagonalization of $\mathcal{D}$ and $\bar{\mathcal{D}}$, the mass eigenstates of gauge fields are defined by

$$\widetilde{A}^a_\mu = A^a_\mu U, \quad \widetilde{A}^{a\mu} = U^{-1} A^{a\mu} \tag{4.22}$$

The commutation relation is also diagonalized:

$$[a, a^\dagger] = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \tag{4.23}$$

Each component of creation and annihilation operators are summarized as follows.

$$\begin{cases} a_1 \equiv \frac{1}{\sqrt{\alpha}} i\bar{\partial} \\ a_2 \equiv \frac{1}{\sqrt{\alpha}} i(\bar{\partial} + gfz) \\ a_3 \equiv \frac{1}{\sqrt{\alpha}} i(\bar{\partial} - gfz) \end{cases}, \begin{cases} a_1^\dagger \equiv \frac{1}{\sqrt{\alpha}} i\partial \\ a_2^\dagger \equiv \frac{1}{\sqrt{\alpha}} i(\partial - gf\bar{z}) \\ a_3^\dagger \equiv \frac{1}{\sqrt{\alpha}} i(\partial + gf\bar{z}) \end{cases}. \tag{4.24}$$

We note that a_1 and $a_1^\dagger$ play no role of annihilation and creation operators. Although a_2 and $a_2^\dagger$ are ordinary annihilation and creation operators, the role of annihilation and creation operators for a_3 and $a_3^\dagger$ are inverted because of $[a_3, a_3^\dagger] = -1$. The mode functions on the ground state are determined by $\partial\lambda_{0,0} = 0(\bar{\partial}\lambda_{0,0} = 0)$, $a_2\psi^2_{0,j} = 0$, $a_3^\dagger\psi^3_{0,j} = 0$, where $j = 0, \cdots, |N| - 1$ labels the degeneracy of the ground state. Higher mode functions are constructed as

$$\psi^1_{l,m} = \begin{pmatrix} \lambda_{l,m} \\ 0 \\ 0 \end{pmatrix}, \quad \psi^2_{n_2,j} = \begin{pmatrix} 0 \\ \xi_{n_2,j} \\ 0 \end{pmatrix}, \quad \psi^3_{n_3,j} = \begin{pmatrix} 0 \\ 0 \\ \bar{\xi}_{n_3,j} \end{pmatrix}, \tag{4.25}$$

where $\lambda_{l,m}$ and $\xi_{n,j}$, $\bar{\xi}_{n,j}$ are given by eq.(3.23) and eq.(2.20) respectively. These mode functions satisfy a orthonormality condition

$$\int_{T^2} dx^2 (\psi^1_{l',m'})^\dagger \psi^1_{l,m} = \delta_{ll'}\delta_{mm'}, \quad \int_{T^2} dx^2 (\psi^{a'}_{n'_a,j'})^\dagger \psi^a_{n_a,j} = \delta^{a'a}\delta_{n'_a n_a}\delta_{j'j}, \tag{4.26}$$

where a, a' in the right equation of eq.(4.26) mean $a = 2, 3$. When creation operators $a_2^\dagger$ and a_3 or annihilation operators a_2 and $a_3^\dagger$ act on the mode function, the relation can be obtained as

$$\begin{cases} a_2\psi^2_{n_2,j} = \sqrt{n_2}\psi^2_{n_2-1,j} \\ a_3\psi^3_{n_3,j} = \sqrt{n_3+1}\psi^3_{n_3+1,j} \end{cases}, \quad \begin{cases} a_2^\dagger\psi^2_{n_2,j} = \sqrt{n_2+1}\psi^2_{n_2+1,j} \\ a_3^\dagger\psi^3_{n_3,j} = \sqrt{n_3}\psi^3_{n_3-1,j} \end{cases}. \tag{4.27}$$

The mass-squared operator for gauge field is diagonalized as

$$m^2_{YM} \equiv -\mathcal{D}_{diag}\bar{\mathcal{D}}_{diag} = \begin{pmatrix} \beta(l^2+m^2) & 0 & 0 \\ 0 & \alpha n_2 & 0 \\ 0 & 0 & \alpha(n_3+1) \end{pmatrix}, \tag{4.28}$$

where $l, m \in \mathbb{Z}$ and $n_{2,3} = 0, 1, 2\cdots$ are Landau level and $\beta = (2\pi/L)^2$.

It seems that there are two massless gauge bosons in eq.(4.28). If the commutation relation of adjoint representation for gauge boson is calculated, we understand that a massless gauge boson is appeared. Concretely, we can check the commutation relation of the representation $(t^1)_{ab} = \epsilon^{a1b}$ for gauge field $A^1_{5,6}$ and the representation $(t^c)_{ab} = \epsilon^{acb}$ for gauge field A^c_μ:

$$[t^1, t^1] = 0 \text{ for } c = 1, \quad [t^1, t^c] \neq 0 \text{ for } c = 2, 3. \tag{4.29}$$

Thus, the (1,1) component of eq.(4.28) is a massless gauge boson. On the other hand, the (2,2) component of eq.(4.28) is a fictitious massless gauge boson. This component is removed by shift symmetry as we will see in the subsection 4.4.1 or the section 4.8. We conclude that the SU(2) gauge symmetry is broken to U(1) by the flux background.

4.2.2 WL scalar field

Next, we find mass eigenvalues of WL scalar fields. Extracting quadratic terms for φ^a from eq.(4.13), we obtain

$$\begin{aligned}\mathcal{L}_{\varphi\varphi} = &-\frac{1}{4}\Big(\mathcal{D}\varphi^{a*}\mathcal{D}\varphi^{a*} + \mathcal{D}\varphi^{a*}\bar{\mathcal{D}}\varphi^{a} + \bar{\mathcal{D}}\varphi^{a}\mathcal{D}\varphi^{a*} + \bar{\mathcal{D}}\varphi^{a}\bar{\mathcal{D}}\varphi^{a} - 4gf[\varphi,\varphi^*]^1\Big)\\ &+\frac{\xi}{4}\Big(\mathcal{D}\varphi^{a*}\mathcal{D}\varphi^{a*} - \mathcal{D}\varphi^{a*}\bar{\mathcal{D}}\varphi^{a} - \bar{\mathcal{D}}\varphi^{a}\mathcal{D}\varphi^{a*} + \bar{\mathcal{D}}\varphi^{a}\bar{\mathcal{D}}\varphi^{a}\Big). \end{aligned} \tag{4.30}$$

As the discussion in the previous subsection, we need to diagonalize them. In order to justify that WL scalar masses can be simultaneously diagonalized by the same unitary rotation

$$\widetilde{\varphi}^a = U^{-1}\varphi^a, \quad \widetilde{\varphi}^{a*} = \varphi^{a*}U, \tag{4.31}$$

as that of non-Abelian gauge field, we give some arguments below. Because of $\mathcal{D}\varphi^{a*}\bar{\mathcal{D}}\varphi^a = -\varphi^{a*}\mathcal{D}\bar{\mathcal{D}}\varphi^a$, the second and the third terms in the first line of eq.(4.30) can be diagonalized by the unitary matrix U.

Next, we focus on the first term in the first line of eq.(4.30)

$$\begin{aligned}\mathcal{D}\varphi^{a*}\mathcal{D}\varphi^{a*} &= -\varphi^{a*}\mathcal{D}\mathcal{D}\varphi^{a*}\\ &= \alpha\Big(\widetilde{\varphi}^{1*}(a_1)^2\widetilde{\varphi}^{1*} - i\widetilde{\varphi}^{2*}(a_2)^2\widetilde{\varphi}^{3*} - i\widetilde{\varphi}^{3*}(a_3)^2\widetilde{\varphi}^{2*}\Big).\end{aligned} \tag{4.32}$$

Integrating these forms out on the square torus, the second and third terms in eq.(4.32) vanish thanks to the orthogonality of the mode functions. The first term in eq.(4.32) also vanishes since we will consider the zero mode of $\widetilde{\varphi}^{1*}$ independent of $z, \bar{z}$. This argument can be also applied to $\bar{\mathcal{D}}\varphi^a\bar{\mathcal{D}}\varphi^a$. The last term in the first line of eq.(4.30) can be also diagonalized by the unitary matrix U

$$-4gf[\varphi,\varphi^*]^1 = 2\alpha\varphi^{a*}(i\varepsilon^{a1b})\varphi^b = 2\times\alpha\widetilde{\varphi}^{a*}\begin{pmatrix}0&0&0\\0&1&0\\0&0&-1\end{pmatrix}\widetilde{\varphi}^a. \tag{4.33}$$

Note that the complex conjugate of adjoint representation $(t^a)^*$ becomes $(t^a)^* = -t^a$ since the structure constants are real and totally antisymmetric. Applying the same argument to WL scalar mass terms from the gauge fixing terms in the second line of eq.(4.30), the mass eigenvalues of WL scalar can be finally obtained as

$$m_{WL}^2 = gf \begin{pmatrix} (1+\xi)\beta(l^2+m^2) & 0 & 0 \\ 0 & \frac{\alpha}{2}((1+\xi)n_2+1) & 0 \\ 0 & 0 & \frac{\alpha}{2}((1+\xi)n_3+\xi) \end{pmatrix}. \tag{4.34}$$

4.2.3 Ghost field

Finally, we find mass eigenvalues of the ghost field. Extracting the quadratic terms for c^a, one has

$$\mathcal{L}_{cc} = -c^{a*}\xi \mathcal{D}_m \mathcal{D}^m c^a \tag{4.35}$$

Rewriting the differential operator $\mathcal{D}_m\mathcal{D}^m$ in terms of creation and annihilation operators, we obtain

$$\begin{aligned} (\mathcal{D}_m\mathcal{D}^m)^{ab} &= (\mathcal{D}_5^2 + \mathcal{D}_6^2)^{ab} \\ &= -[(i\mathcal{D})(i\bar{\mathcal{D}})]^{ab} - \frac{1}{2}[\mathcal{D}, \bar{\mathcal{D}}]^{ab} \\ &= -\alpha \left[(a^\dagger a)^{ab} + \frac{1}{2} i\varepsilon^{a1b} \right], \end{aligned} \tag{4.36}$$

where we used $[\mathcal{D}_5, \mathcal{D}_6] = [\mathcal{D}, \bar{\mathcal{D}}]/2i$. Note that eq.(4.36) is an non-Abelian extension of eq.(2.18). Thus, the ghost mass matrix is diagonalized as

$$m_{ghost}^2 = \xi \begin{pmatrix} \beta(l^2+m^2) & 0 & 0 \\ 0 & \alpha\left(n_2+\frac{1}{2}\right) & 0 \\ 0 & 0 & \alpha\left(n_3+\frac{1}{2}\right) \end{pmatrix}. \tag{4.37}$$

Mass eigenstate of the ghost field is defined as

$$\widetilde{c}^{a*} = c^{a*}U, \quad \widetilde{c}^a = U^{-1}c^a. \tag{4.38}$$

4.3 Effective Lagrangian

To derive the effective Lagrangian in four dimensions from the Lagrangian in six dimensions by KK reduction, we expand A_μ^a, φ^a, c^a in terms of KK mode except for φ^1

because we are interested in the quantum corrections to the mass for zero-mode of φ^1. In other words, the components of KK modes running in the loop are $a = 2, 3$ in the quantum corrections.

$$\widetilde{A}^{1\mu} = \sum_{l,m} \widetilde{A}^{1\mu}_{l,m} \psi^1_{l,m}, \quad \widetilde{A}^{a\mu} = \sum_{n_a,j} \widetilde{A}^{a\mu}_{n_a,j} \psi^a_{n_a,j} \ (a = 2, 3), \tag{4.39}$$

$$\widetilde{A}^1_\mu = \sum_{l,m} \widetilde{A}^1_{\mu,l,m} \psi^{1\dagger}_{l,m}, \quad \widetilde{A}^a_\mu = \sum_{n_a,j} \widetilde{A}^a_{\mu,n_a,j} \psi^{a\dagger}_{n_a,j} \ (a = 2, 3), \tag{4.40}$$

$$\widetilde{\varphi}^a = \sum_{n_a,j} \widetilde{\varphi}^a_{n_a,j} \psi^a_{n_a,j}, \quad \widetilde{\varphi}^{a*} = \sum_{n_a,j} \widetilde{\varphi}^{a*}_{n_a,j} \psi^{a\dagger}_{n_a,j} \ (a = 2, 3), \tag{4.41}$$

$$\widetilde{c}^1 = \sum_{l,m} \widetilde{c}^1_{l,m} \psi^1_{l,m}, \quad \widetilde{c}^a = \sum_{n_a,j} \widetilde{c}^a_{n_a,j} \psi^a_{n_a,j} \ (a = 2, 3), \tag{4.42}$$

$$\widetilde{c}^{1*} = \sum_{l,m} \widetilde{c}^{1*}_{l,m} \psi^1_{l,m}, \quad \widetilde{c}^{a*} = \sum_{n_a,j} \widetilde{c}^{a*}_{n_a,j} \psi^{a\dagger}_{n_a,j} \ (a = 2, 3). \tag{4.43}$$

Notice that eq.(4.39), the first equations of eq.(4.41) and eq.(4.42) are regarded as column vector and eq.(4.40), the second equations of eq.(4.41) and eq.(4.43) are regarded as row vector. Using the KK expansion from eqs.(4.39) to (4.43), the total Lagrangian (4.13) is given as

$$\begin{aligned}
\mathcal{L}_{total} =& -\frac{1}{4}\widetilde{F}^a_{\mu\nu}\widetilde{F}^{a\mu\nu} - \partial_\mu \widetilde{\varphi}^{a*}\partial^\mu \widetilde{\varphi}^a - \widetilde{c}^{a*} D_\mu D^\mu \widetilde{c}^a \\
& - \frac{1}{2}\widetilde{A}^a_\mu m^2_{YM} \widetilde{A}^{a\mu} - \widetilde{\varphi}^{a*} m^2_{WL} \widetilde{\varphi}^a - \widetilde{c}^{a*} m^2_{ghost} \widetilde{c}^a \\
& + ig\Big\{\partial_\mu \varphi^a [A^\mu, \varphi^*]^a + \partial^\mu \varphi^{a*}[A_\mu, \varphi]^a\Big\} + g^2 [A_\mu, \varphi]^a [A^\mu, \varphi^*]^a \\
& - \frac{g}{\sqrt{2}}\Big\{ - \mathcal{D}A^a_\mu [A^\mu, \varphi^*]^a + \bar{\mathcal{D}} A^{a\mu}[A_\mu, \varphi]^a \Big\} \\
& + \frac{g}{\sqrt{2}}(\mathcal{D}\varphi^* + \bar{\mathcal{D}}\varphi)^a [\varphi, \varphi^*]^a - \frac{1}{2} g^2 [\varphi, \varphi^*]^a [\varphi, \varphi^*]^a \\
& - \frac{g\xi}{\sqrt{2}}\Big([\varphi, c^*]^a \bar{\mathcal{D}} c^a - [\varphi^*, c^*]^a \mathcal{D} c^a \Big).
\end{aligned} \tag{4.44}$$

In eq.(4.44), only the quadratic terms in the first and second lines are written in terms of mass eigenstate. To read vertices for Feynman diagram calculations, we must rewrite the remaining interaction terms in terms of the corresponding mass eigenstate, which will be done below.

4.3.1 Gauge field

First, we consider the interaction terms including the non-Abelian gauge fields. It is easy to expand the quartic term,

$$g^2[A_\mu, \varphi]^a[A^\mu, \varphi^*]^a = -g^2\varepsilon^{ab1}\varepsilon^{ab'1}\sum_{n_b,j}\sum_{n'_b,j'} A^b_{\mu,n_b,j}A^{b'\mu}_{n'_b,j'}\varphi^1\varphi^{1*}\psi^b_{n_b,j}\psi^{b'}_{n'_b,j'}. \tag{4.45}$$

Then, the orthonormality condition for the mode functions leads to

$$\mathcal{L}_{\varphi\varphi AA} = -g^2\eta^{\mu\nu}\sum_{n_b,j}\widetilde{A}^b_{\mu,n_b,j}\widetilde{A}^b_{\nu,n_b,j}\varphi^1\varphi^{1*}, \tag{4.46}$$

where b means $b = 2, 3$ in the above expression. Next, we calculate the cubic term of φAA in a mass eigenstate. Expanding $\mathcal{D}A^a_\mu[A^\mu, \varphi^*]^a$ by KK modes, we have

$$\begin{aligned}\mathcal{D}A^a_\mu[A^\mu, \varphi^*]^a &\supset A^a_\mu U\overleftarrow{\mathcal{D}}_{diag}U^{-1}(i\varepsilon_{ab1})UU^{-1}A^{b\mu}\varphi^{1*} \\ &= \mathcal{D}_{diag}\widetilde{A}^a_\mu U^{-1}(-i\varepsilon_{a1b})U\widetilde{A}^{b\mu}\varphi^{1*} \\ &= -\frac{\sqrt{\alpha}}{i}a^\dagger_2\widetilde{A}^2_\mu\widetilde{A}^{2\mu}\varphi^{1*} + \frac{\sqrt{\alpha}}{i}a^\dagger_3\widetilde{A}^3_\mu\widetilde{A}^{3\mu}\varphi^{1*}\end{aligned} \tag{4.47}$$

where a symbol $\supset$ in the first line means that only the non-vanishing terms by the orthonormality condition are left and $\overleftarrow{\mathcal{D}}_{diag}$ means it acts on A^a_μ not $A^{b\mu}$. Using the relation eq.(4.27) and the orthonormality condition for mode functions, we obtain

$$\begin{aligned}\mathcal{L}_{\varphi AA} = &+\sum_{n_2,j}\frac{g\sqrt{\alpha(n_2+1)}}{\sqrt{2}i}\widetilde{A}^2_{\mu,n_2+1,j}\widetilde{A}^{2\mu}_{n_2,j}\varphi^{1*} - \sum_{n_3,j}\frac{g\sqrt{\alpha(n_3+1)}}{\sqrt{2}i}\widetilde{A}^3_{\mu,n_3,j}\widetilde{A}^{3\mu}_{n_3+1,j}\varphi^{1*} \\ &-\sum_{n_2,j}\frac{g\sqrt{\alpha(n_2+1)}}{\sqrt{2}i}\widetilde{A}^2_{\mu,n_2,j}\widetilde{A}^{2\mu}_{n_2+1,j}\varphi^{1} + \sum_{n_3,j}\frac{g\sqrt{\alpha(n_3+1)}}{\sqrt{2}i}\widetilde{A}^3_{\mu,n_3+1,j}\widetilde{A}^{3\mu}_{n_3,j}\varphi^{1}.\end{aligned} \tag{4.48}$$

As for the cubic terms $\partial_\mu\varphi^a[A^\mu, \varphi^*]^a$ or $\partial^\mu\varphi^{*a}[A_\mu, \varphi]^a$, these terms turn out to be vanished thanks to the orthogonality condition for mode functions. Thus, there is no contribution to the cubic terms in the third line of eq.(4.44) in four-dimensional effective Lagrangian.

4.3.2 WL scalar field

Next, we calculate the cubic and quartic terms for the WL scalar field. It is also easy to compute the quartic term.

$$
\begin{aligned}
-\frac{1}{2}g^2[\varphi,\varphi^*]^a[\varphi,\varphi^*]^a &= \frac{1}{2}g^2\varepsilon^{abc}\varepsilon^{ab'c'}\varphi^b\varphi^{c*}\varphi^{b'}\varphi^{c'*} \\
&= 2\times\frac{1}{2}g^2\varepsilon^{abc}\varepsilon^{ab'c'}\sum_{n_b,j}\sum_{n_c,j'}\varphi^b_{n_b,j}\varphi^{c'*}_{n_{c'},j'}\varphi^{b'}\varphi^{c*}\psi^b_{n_b,j}\psi^{c'*}_{n_{c'},j'}.
\end{aligned} \tag{4.49}
$$

The reason why a factor 2 appears is that there are two ways to choose a pair of KK expansions: $\varphi^b\varphi^{*c'}$ or $\varphi^{b'}\varphi^{*c}$ since one of the two $\varphi(\varphi^*)$ is taken to be $\varphi^1(\varphi^{*1})$. Thus, we obtain

$$
\mathcal{L}_{\varphi\varphi\varphi\varphi} = g^2\varepsilon^{abc}\varepsilon^{ab'c'}\delta^{bc'}\sum_{n_b,j}\widetilde{\varphi}^b_{n_b,j}\widetilde{\varphi}^{c'*}_{n_b,j}\varphi^{b'}\varphi^{c*}. \tag{4.50}
$$

Next, we calculate the cubic term of φ in a mass eigenstate. Expanding $\mathcal{D}\varphi^{a*}[\varphi,\varphi^*]^a$ by KK modes, we have

$$
\begin{aligned}
\mathcal{D}\varphi^{a*}[\varphi,\varphi^*]^a &= i\varepsilon_{abc}\mathcal{D}\varphi^{a*}\varphi^b\varphi^{c*} \\
&\supset \mathcal{D}\varphi^{a*}(i\varepsilon_{a1b})(\varphi^{b*})^T\varphi^1 - \mathcal{D}\varphi^{a*}(i\varepsilon_{a1b})\varphi^b\varphi^{1*} \\
&= \mathcal{D}_{diag}\widetilde{\varphi}^{a*}U^{-1}(i\varepsilon_{a1b})(U^{-1})^T(\varphi^{b*})^T\varphi^1 + \mathcal{D}_{diag}\widetilde{\varphi}^{a*}U^{-1}(-i\varepsilon_{a1b})U\widetilde{\varphi}^b\varphi^{1*},
\end{aligned}
$$

where we add the transpose T to φ^{b*} to compute the first term. $U^{-1}(i\varepsilon_{a1b})(U^{-1})^T$ has no diagonalized component, thus one has

$$
\mathcal{D}\varphi^{a*}[\varphi,\varphi^*]^a \supset -\frac{\sqrt{\alpha}}{i}a_2^\dagger\widetilde{\varphi}^{2*}\widetilde{\varphi}^2\varphi^{1*} + \frac{\sqrt{\alpha}}{i}a_3^\dagger\widetilde{\varphi}^{3*}\widetilde{\varphi}^3\varphi^{1*}. \tag{4.51}
$$

As in the subsection 4.3.1, the cubic terms of WL scalar can be obtain as

$$
\begin{aligned}
\mathcal{L}_{\varphi\varphi\varphi} = &+\sum_{n_2,j}\frac{g\sqrt{\alpha(n_2+1)}}{\sqrt{2}i}\widetilde{\varphi}^{2*}_{n_2+1,j}\widetilde{\varphi}^2_{n_2,j}\varphi^{1*} - \sum_{n_3,j}\frac{g\sqrt{\alpha(n_3+1)}}{\sqrt{2}i}\widetilde{\varphi}^{3*}_{n_3,j}\widetilde{\varphi}^3_{n_3+1,j}\varphi^{1*}, \\
&-\sum_{n_2,j}\frac{g\sqrt{\alpha(n_2+1)}}{\sqrt{2}i}\widetilde{\varphi}^{2*}_{n_2,j}\widetilde{\varphi}^2_{n_2+1,j}\varphi^1 + \sum_{n_3,j}\frac{g\sqrt{\alpha(n_3+1)}}{\sqrt{2}i}\widetilde{\varphi}^{3*}_{n_3+1,j}\widetilde{\varphi}^3_{n_3,j}\varphi^1.
\end{aligned} \tag{4.52}
$$

4.3.3 Ghost field

Finally, we compute the cubic terms for the ghost and WL scalar fields, which include a single φ^1. Expanding $[\varphi, c^*]^a \bar{\mathcal{D}} c^a$ by KK modes, we have

$$\begin{aligned}
[\varphi, c^*]^a \bar{\mathcal{D}} c^a &= i\varepsilon^{abc} \varphi^a c^{b*} \bar{\mathcal{D}} c^c \supset c^{*a}(-i\varepsilon_{a1b}) \bar{\mathcal{D}} c^b \varphi^1 \\
&= \widetilde{c}^{*a} U^{-1} (-i\varepsilon_{a1b}) U \bar{\mathcal{D}}_{diag} \widetilde{c}^b \varphi^1 \\
&= -\frac{\sqrt{\alpha}}{i} \widetilde{c}^{2*} a_2 \widetilde{c}^2 \varphi^1 + \frac{\sqrt{\alpha}}{i} \widetilde{c}^{3*} a_3 \widetilde{c}^3 \varphi^1.
\end{aligned} \tag{4.53}$$

Using the relation (4.27) and the orthonormality condition for mode functions, we find

$$\begin{aligned}
\mathcal{L}_{cc\varphi} = &+ \sum_{n_2, j} \frac{g\xi\sqrt{\alpha(n_2+1)}}{\sqrt{2}i} \widetilde{c}^{2*}_{n_2+1,j} \widetilde{c}^2_{n_2,j} \varphi^{1*} - \sum_{n_3, j} \frac{g\xi\sqrt{\alpha(n_3+1)}}{\sqrt{2}i} \widetilde{c}^{3*}_{n_3,j} \widetilde{c}^3_{n_3+1,j} \varphi^{1*} \\
&+ \sum_{n_2, j} \frac{g\xi\sqrt{\alpha(n_2+1)}}{\sqrt{2}i} \widetilde{c}^{2*}_{n_2,j} \widetilde{c}^2_{n_2+1,j} \varphi^{1} - \sum_{n_3, j} \frac{g\xi\sqrt{\alpha(n_3+1)}}{\sqrt{2}i} \widetilde{c}^{3*}_{n_3+1,j} \widetilde{c}^3_{n_3,j} \varphi^{1}.
\end{aligned} \tag{4.54}$$

4.4 Cancellation of one-loop corrections to WL scalar mass

In this section, we calculate the quantum corrections to WL scalar mass at one-loop for the zero mode of φ^1 and show that they are exactly canceled. In this section, we omit the symbol of tilde for mass eigenstate (for example, we write $\widetilde{A}^a_\mu$ as A^a_μ for simplicity).

4.4.1 Gauge boson loop

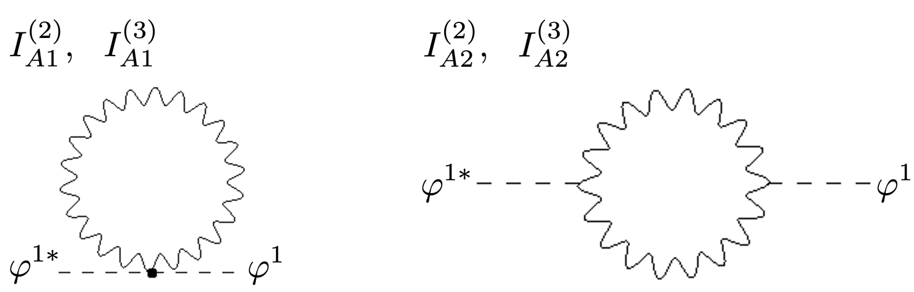

Figure 4.1: Gauge boson loop corrections $I^{(2,3)}_{A1}$ and $I^{(2,3)}_{A2}$.

As shown in figure 4.1, there are two diagrams from the gauge boson loop contributions. Superscript (2), (3) means the contributions from A^2_μ, A^3_μ loops, respectively.

Denoting $I_{A1}^{(2,3)}$ and $I_{A2}^{(2,3)}$ as the contribution from the four-point interaction and the three-point interactions respectively, these are obtained as

$$I_{A1}^{(2)} = -2ig^2|N| \sum_{n=0}^{\infty} \int \frac{d^4p}{(2\pi)^4} \left(\frac{3}{p^2+\alpha n} + \frac{\xi}{p^2+\alpha n\xi} \right), \tag{4.55}$$

$$I_{A1}^{(3)} = -2ig^2|N| \sum_{n=0}^{\infty} \int \frac{d^4p}{(2\pi)^4} \left(\frac{3}{p^2+\alpha(n+1)} + \frac{\xi}{p^2+\alpha(n+1)\xi} \right), \tag{4.56}$$

$$I_{A2}^{(2)} = 2ig^2|N| \sum_{n=0}^{\infty} \int \frac{d^4p}{(2\pi)^4} \left(\frac{3\alpha(n+1)}{(p^2+\alpha n)(p^2+\alpha(n+1))} + \frac{\alpha(n+1)\xi^2}{(p^2+\alpha n\xi)(p^2+\alpha(n+1)\xi)} \right), \tag{4.57}$$

$$\begin{aligned} I_{A2}^{(3)} = 2ig^2|N| \sum_{n=0}^{\infty} \int \frac{d^4p}{(2\pi)^4} \bigg(& \frac{3\alpha(n+1)}{(p^2+\alpha(n+1))(p^2+\alpha(n+2))} \\ & + \frac{\alpha(n+1)\xi^2}{(p^2+\alpha(n+1)\xi)(p^2+\alpha(n+2)\xi)} \bigg), \end{aligned} \tag{4.58}$$

where Wick rotation is applied in momentum integrals and the symmetry factor is involved. To obtain $I_{A2}^{(2,3)}$, we use a partial fraction decomposition

$$\frac{(1-\xi)p^2}{(p^2+\alpha n)(p^2+\alpha n\xi)} = \frac{1}{p^2+\alpha n} - \frac{\xi}{p^2+\alpha n\xi}. \tag{4.59}$$

We now consider the sum of $I_{A1}^{(2)}$ and $I_{A2}^{(2)}$ or $I_{A1}^{(3)}$ and $I_{A2}^{(3)}$:

$$\begin{aligned} I_{A1}^{(2)} + I_{A2}^{(2)} =& -6ig^2|N| \sum_{n=0}^{\infty} \int \frac{d^4p}{(2\pi)^4} \left(\frac{1}{p^2+\alpha n} - \frac{\alpha(n+1)}{(p^2+\alpha n)(p^2+\alpha(n+1))} \right) \\ & - 2ig^2|N| \sum_{n=0}^{\infty} \int \frac{d^4p}{(2\pi)^4} \left(\frac{\xi}{p^2+\alpha n\xi} - \frac{\alpha(n+1)\xi^2}{(p^2+\alpha n\xi)(p^2+\alpha(n+1)\xi)} \right), \end{aligned} \tag{4.60}$$

$$\begin{aligned} I_{A1}^{(3)} + I_{A2}^{(3)} =& -6ig^2|N| \sum_{n=0}^{\infty} \int \frac{d^4p}{(2\pi)^4} \left(\frac{1}{p^2+\alpha(n+1)} - \frac{\alpha(n+1)}{(p^2+\alpha(n+1))(p^2+\alpha(n+2))} \right) \\ & - 2ig^2|N| \sum_{n=0}^{\infty} \int \frac{d^4p}{(2\pi)^4} \left(\frac{\xi}{p^2+\alpha(n+1)\xi} - \frac{\alpha(n+1)\xi^2}{(p^2+\alpha(n+1)\xi)(p^2+\alpha(n+2)\xi)} \right). \end{aligned} \tag{4.61}$$

The integrand of the first line in $I_{A1}^{(2)} + I_{A2}^{(2)}$ can be deformed as

$$\begin{aligned} \frac{1}{p^2+\alpha n} - \frac{\alpha(n+1)}{(p^2+\alpha n)(p^2+\alpha(n+1))} &= \frac{1}{p^2+\alpha n} - (n+1)\left(\frac{1}{p^2+\alpha n} - \frac{1}{p^2+\alpha(n+1)} \right) \\ &= -\frac{n}{p^2+\alpha n} + \frac{n+1}{p^2+\alpha(n+1)}, \end{aligned} \tag{4.62}$$

and we thus find a crucial result

$$\begin{aligned}&\sum_{n=0}^{\infty}\int\frac{d^4p}{(2\pi)^4}\left(\frac{1}{p^2+\alpha n}-\frac{\alpha(n+1)}{(p^2+\alpha n)(p^2+\alpha(n+1))}\right)\\ =&\sum_{n=0}^{\infty}\int\frac{d^4p}{(2\pi)^4}\left(-\frac{n}{p^2+\alpha n}+\frac{n+1}{p^2+\alpha(n+1)}\right)=0\end{aligned}\tag{4.63}$$

by the shift $n\to n+1$ in the first term. The same result holds for the integrand of the first line in $I^{(3)}_{A1}+I^{(3)}_{A2}$. As for the integrand of the second line in $I^{(2)}_{A1}+I^{(2)}_{A2}$, $I^{(3)}_{A1}+I^{(3)}_{A2}$, the same structure can be easily found after the change of variable $p^2=\xi q^2$. Thus, we conclude

$$I^{(2)}_{A1}+I^{(2)}_{A2}=0,\quad I^{(3)}_{A1}+I^{(3)}_{A2}=0,\tag{4.64}$$

which implies that the quantum corrections from the gauge boson loop are canceled. We emphasize that this cancellation holds for an arbitrary ξ.

4.4.2 WL scalar loop

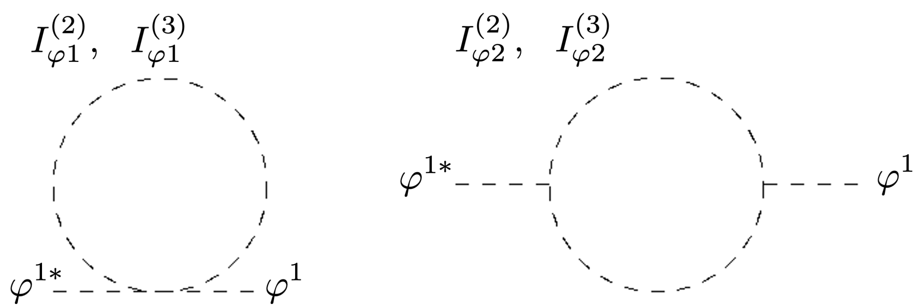

Figure 4.2: WL scalar loop corrections $I^{(2,3)}_{\varphi1}$ and $I^{(2,3)}_{\varphi2}$.

As shown in figure 4.2, there are also two diagrams from the WL scalar field loop contributions. Superscript (2), (3) means the contributions from φ^2, φ^3 loops, respectively. Denoting $I^{(2,3)}_{\varphi1}$ and $I^{(2,3)}_{\varphi2}$ as the contribution from the four-point interaction and the three-point interactions respectively, we have

$$I_{\varphi 1}^{(2)} = -ig^2|N|\sum_{n=0}^{\infty}\int \frac{d^4p}{(2\pi)^4}\frac{1}{p^2+\frac{\alpha}{2}((1+\xi)n+1)}, \tag{4.65}$$

$$I_{\varphi 1}^{(3)} = -ig^2|N|\sum_{n=0}^{\infty}\int \frac{d^4p}{(2\pi)^4}\frac{1}{p^2+\frac{\alpha}{2}((1+\xi)n+\xi)}, \tag{4.66}$$

$$I_{\varphi 2}^{(2)} = \frac{ig^2|N|}{2}\sum_{n=0}^{\infty}\int \frac{d^4p}{(2\pi)^4}\frac{\alpha(n+1)}{(p^2+\frac{\alpha}{2}((1+\xi)n+1))(p^2+\frac{\alpha}{2}((1+\xi)(n+1)+1))}, \tag{4.67}$$

$$I_{\varphi 2}^{(3)} = \frac{ig^2|N|}{2}\sum_{n=0}^{\infty}\int \frac{d^4p}{(2\pi)^4}\frac{\alpha(n+1)}{(p^2+\frac{\alpha}{2}((1+\xi)n+\xi))(p^2+\frac{\alpha}{2}((1+\xi)(n+1)+\xi))}. \tag{4.68}$$

4.4.3 Ghost field loop

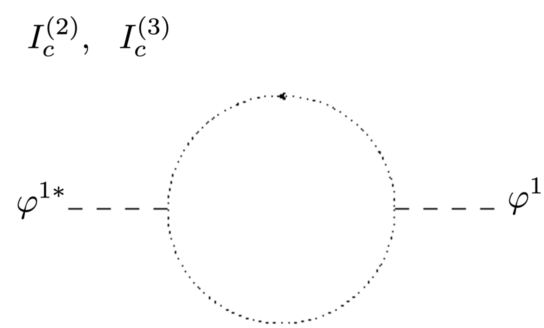

Figure 4.3: Ghost field loop correction $I_c^{(2,3)}$

As for the ghost loop contributions, we have only to consider a diagram shown in figure 4.3. Superscript (2), (3) means the contributions from c^2, c^3 loops, respectively. Denoting $I_c^{(2,3)}$ as the contribution from the interaction including ghost fields, $I_c^{(2,3)}$ are

$$I_c^{(2)} = \frac{ig^2|N|\xi^2}{2}\sum_{n=0}^{\infty}\int \frac{d^4p}{(2\pi)^4}\frac{\alpha(n+1)}{(p^2+\alpha(n+\frac{1}{2}))(p^2+\alpha(n+\frac{3}{2}))}, \tag{4.69}$$

$$I_c^{(3)} = \frac{ig^2|N|\xi^2}{2}\sum_{n=0}^{\infty}\int \frac{d^4p}{(2\pi)^4}\frac{\alpha(n+1)}{(p^2+\alpha(n+\frac{1}{2}))(p^2+\alpha(n+\frac{3}{2}))}, \tag{4.70}$$

where a change of variable $p^2 \to \xi p^2$ is performed in momentum integral. Notice that we need to consider an overall sign (-1) for the ghost loop.

4.4.4 Cancellation between WL scalar loop and ghost loop contributions

As you have seen in subsection 4.4.1, one-loop corrections to the zero mode WL scalar mass are canceled between two diagrams of gauge boson loop. In this subsection, we show the cancellation between the corrections from the WL scalar field and the ghost field loops.

First, let us consider the case $\xi = 0$ (Landau gauge). In this case, the contributions from the ghost field (4.69) and (4.70) trivially vanish since they are proportional to ξ^2: $I_c^{(2)} = I_c^{(3)} = 0$. These results in the $\xi = 0$ case can be understood that ghost fields have no interaction with WL scalar fields (see eq.(4.54)). Thus, we have only to calculate the remaining contributions from the WL scalar field loop from eq.(4.65) to (4.68) in the $\xi = 0$ case. The summation of WL scalar field contribution can be found

$$I_{\varphi 1}^{(2)} + I_{\varphi 2}^{(2)} = -ig^2|N| \sum_{n=0}^{\infty} \int \frac{d^4p}{(2\pi)^4} \left(\frac{1}{p^2 + \frac{\alpha}{2}(n+1)} - \frac{\frac{\alpha}{2}(n+1)}{(p^2 + \frac{\alpha}{2}(n+1))(p^2 + \frac{\alpha}{2}(n+2))} \right), \tag{4.71}$$

$$I_{\varphi 1}^{(3)} + I_{\varphi 2}^{(3)} = -ig^2|N| \sum_{n=0}^{\infty} \int \frac{d^4p}{(2\pi)^4} \left(\frac{1}{p^2 + \frac{\alpha}{2}n} - \frac{\frac{\alpha}{2}(n+1)}{(p^2 + \frac{\alpha}{2}n)(p^2 + \frac{\alpha}{2}(n+1))} \right). \tag{4.72}$$

Using the results (4.62) and (4.63), we can easily find that these contributions are canceled:

$$I_{\varphi 1}^{(2)} + I_{\varphi 2}^{(2)} = 0, \quad I_{\varphi 1}^{(3)} + I_{\varphi 2}^{(3)} = 0. \tag{4.73}$$

Next, we consider a more non-trivial case $\xi = 1$ (Feynman gauge), in which we expect non-trivial cancellations between the corrections from the WL scalar field and ghost field loops. The summation of the WL scalar and ghost field contributions can be found

$$\begin{aligned} I_{\varphi 1}^{(2)} + I_{\varphi 2}^{(2)} + I_c^{(2)} =& -ig^2|N| \sum_{n=0}^{\infty} \int \frac{d^4p}{(2\pi)^4} \left(\frac{1}{p^2 + \alpha(n+\frac{1}{2})} \right. \\ & \left. -\frac{1}{2} \frac{\alpha(n+1)}{(p^2 + \alpha\left(n+\frac{1}{2}\right))(p^2 + \alpha\left(n+\frac{3}{2}\right))} - \frac{1}{2} \frac{\alpha(n+1)}{(p^2 + \alpha\left(n+\frac{1}{2}\right))(p^2 + \alpha(n+\frac{3}{2}))} \right) \\ =& -ig^2|N| \sum_{n=0}^{\infty} \int \frac{d^4p}{(2\pi)^4} \left(\frac{1}{p^2 + \alpha(n+\frac{1}{2})} - \frac{\alpha(n+1)}{(p^2 + \alpha\left(n+\frac{1}{2}\right))(p^2 + \alpha(n+\frac{3}{2}))} \right), \end{aligned} \tag{4.74}$$

$$
\begin{aligned}
I^{(3)}_{\varphi 1}+I^{(3)}_{\varphi 2}+I^{(3)}_{c} =& - ig^2|N|\sum_{n=0}^{\infty}\int\frac{d^4p}{(2\pi)^4}\left(\frac{1}{p^2+\alpha(n+\frac{1}{2})}\right.\\
&\left.-\frac{1}{2}\frac{\alpha(n+1)}{(p^2+\alpha(n+\frac{1}{2}))(p^2+\alpha(n+\frac{3}{2}))}-\frac{1}{2}\frac{\alpha(n+1)}{(p^2+\alpha(n+\frac{1}{2}))(p^2+\alpha(n+\frac{3}{2}))}\right)\\
=& - ig^2|N|\sum_{n=0}^{\infty}\int\frac{d^4p}{(2\pi)^4}\left(\frac{1}{p^2+\alpha(n+\frac{1}{2})}-\frac{\alpha(n+1)}{(p^2+\alpha(n+\frac{1}{2}))(p^2+\alpha(n+\frac{3}{2}))}\right).
\end{aligned}
\tag{4.75}
$$

Using the results (4.62) and (4.63) again, we conclude that these contributions are also canceled:

$$
I^{(2)}_{\varphi 1}+I^{(2)}_{\varphi 2}+I^{(2)}_{c}=0, \quad I^{(3)}_{\varphi 1}+I^{(3)}_{\varphi 2}+I^{(3)}_{c}=0. \tag{4.76}
$$

It would be also interesting that the cancellation between the WL scalar and the ghost loop contributions is shown in an arbitrary gauge parameter ξ as in the case of the gauge field loop contributions.

4.5 Fermion

In the above sections, we have shown that the quantum corrections to WL scalar mass from the gauge, the WL scalar and the ghost field loops are canceled at one-loop level. In this section, we will see the quantum corrections from fermion loop. For simplicity, we introduce a constant magnetic flux in the direction of SU(2) Cartan part as

$$
\langle A^3_5\rangle = -\frac{1}{2}fx_6, \quad \langle A^3_6\rangle = \frac{1}{2}fx_5, \quad \langle A^{1,2}_5\rangle = \langle A^{1,2}_6\rangle = 0, \tag{4.77}
$$

and we calculate the quantum corrections to WL scalar φ^3 not φ^1. Even though the direction introducing the magnetic flux is different from eq.(4.4), the cancellations of the quantum corrections to WL scalar φ^1 or φ^3 in Yang-Mills theory are also satisfied. Thus, we will investigate the quantum corrections from fermion loop [1].

[1] The calculations in this section are unpublished results

4.5.1 SU(2) Weyl fermion

We consider a six-dimensional SU(2) Weyl fermion $\mathbf{\Psi}_d$ interacting with SU(2) gauge fields. The Lagrangian is given by

$$\begin{aligned}\mathcal{L}_f &= i\bar{\mathbf{\Psi}}_d\Gamma^M D_M \mathbf{\Psi}_d \\ &= i\bar{\mathbf{\Psi}}_d\Gamma^\mu D_\mu \mathbf{\Psi}_d + i\bar{\mathbf{\Psi}}_d\Gamma^5 D_5 \mathbf{\Psi}_d + i\bar{\mathbf{\Psi}}_d\Gamma^6 D_6 \mathbf{\Psi}_d,\end{aligned} \tag{4.78}$$

where the covariant derivatives are $D_M = \partial_M - igA_M^a T^a$ and T^a are SU(2) generators. $\mathbf{\Psi}_d$ is constructed by

$$\mathbf{\Psi}_d = \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix}, \quad \Psi_1 = \begin{pmatrix} \psi_{1L} \\ \psi_{1R} \end{pmatrix}, \quad \Psi_2 = \begin{pmatrix} \psi_{2L} \\ \psi_{2R} \end{pmatrix}, \tag{4.79}$$

where $\psi_{1L,1R}$ and $\psi_{2L,2R}$ are given by

$$\psi_{1L,2L} = \begin{pmatrix} \psi_{1,2} \\ 0 \end{pmatrix}, \quad \psi_{1R,2R} = \begin{pmatrix} 0 \\ \bar{\chi}_{1,2} \end{pmatrix} \tag{4.80}$$

as eq.(3.14). Note that $\Psi_{1,2}$ satisfies $\Gamma^7\Psi_{1,2} = -\Psi_{1,2}$.

To derive the mass-squared operators, we need to find Dirac equations for $\psi_{1L,1R}$ and $\psi_{2L,2R}$. The results are

$$\gamma^\mu\partial_\mu\psi_{1L} + i(\bar{\partial} + gfz)\psi_{1R} = 0, \tag{4.81}$$

$$\gamma^\mu\partial_\mu\psi_{1R} - i(\partial - gf\bar{z})\psi_{1L} = 0, \tag{4.82}$$

$$\gamma^\mu\partial_\mu\psi_{2L} + i(\bar{\partial} - gfz)\psi_{2R} = 0, \tag{4.83}$$

$$\gamma^\mu\partial_\mu\psi_{2R} - i(\partial + gf\bar{z})\psi_{2L} = 0, \tag{4.84}$$

where we ignore the interaction terms. Acting $\gamma^\mu\partial_\mu$ on eqs.(4.81) to (4.84), Klein-Gordon equations

$$\Box\psi_{1L} - (\bar{\partial} + gfz)(\partial - gf\bar{z})\psi_{1L} = 0, \tag{4.85}$$

$$\Box\bar{\psi}_{1R} - (\bar{\partial} - gfz)(\partial + gf\bar{z})\bar{\psi}_{1R} = 0, \tag{4.86}$$

$$\Box\psi_{2L} - (\bar{\partial} - gfz)(\partial + gf\bar{z})\psi_{2L} = 0, \tag{4.87}$$

$$\Box\bar{\psi}_{2R} - (\bar{\partial} + gfz)(\partial - gf\bar{z})\bar{\psi}_{2R} = 0 \tag{4.88}$$

can be obtained. From these Klein-Gordon equations, we define creation and annihilation operators as

$$a_{1L} = \frac{i}{\sqrt{\alpha}}(\bar{\partial} + gfz), \quad a_{1L}^{\dagger} = \frac{i}{\sqrt{\alpha}}(\partial - gf\bar{z}), \tag{4.89}$$

$$a_{1R} = \frac{i}{\sqrt{\alpha}}(\partial + gf\bar{z}), \quad a_{1R}^{\dagger} = \frac{i}{\sqrt{\alpha}}(\bar{\partial} - gfz), \tag{4.90}$$

$$a_{2L} = \frac{i}{\sqrt{\alpha}}(\partial + gf\bar{z}), \quad a_{2L}^{\dagger} = \frac{i}{\sqrt{\alpha}}(\bar{\partial} - gfz), \tag{4.91}$$

$$a_{2R} = \frac{i}{\sqrt{\alpha}}(\bar{\partial} + gfz), \quad a_{2R}^{\dagger} = \frac{i}{\sqrt{\alpha}}(\partial - gf\bar{z}). \tag{4.92}$$

Note that $a_{1L} = a_{2R}$, $a_{1L}^{\dagger} = a_{2R}^{\dagger}$, $a_{1R} = a_{2L}$ and $a_{1R}^{\dagger} = a_{2L}^{\dagger}$ are satisfied. The commutation relations are computed as

$$[a_{iL,iR}, a_{iL,iR}^{\dagger}] = 1, \quad [a_{iL,iR}, a_{iL,iR}] = 0, \quad [a_{iL,iR}, a_{jL,jR}] = 0, \quad [a_{iL,iR}, a_{jR,jL}^{\dagger}] = 1, \tag{4.93}$$

where $i, j = 1, 2,\ i \neq j$, and the corresponding subscripts (iL, iR, etc.) are ordered. Using these creation and annihilation operators, we can read the mass-squared operators as

$$\mathcal{M}_{1L}^2 = \alpha a_{1L} a_{1L}^{\dagger}, \quad \mathcal{M}_{2L}^2 = \alpha a_{2L}^{\dagger} a_{2L}, \tag{4.94}$$

$$\mathcal{M}_{1R}^2 = \alpha a_{1R}^{\dagger} a_{1R}, \quad \mathcal{M}_{2R}^2 = \alpha a_{2R} a_{2R}^{\dagger}. \tag{4.95}$$

The mode functions on ground state satisfy $a_{iL}\xi_{0,j}^{(i)} = 0,\ a_{iR}\bar{\xi}_{0,j}^{(i)} = 0$ and higher mode functions are represented as

$$\xi_{n,j}^{(i)} = \frac{i^n}{\sqrt{n!}}(a_{iL}^{\dagger})^n \xi_{0,j}^{(i)}, \quad \bar{\xi}_{n,j}^{(i)} = \frac{i^n}{\sqrt{n!}}(a_{iR}^{\dagger})^n \bar{\xi}_{0,j}^{(i)}, \tag{4.96}$$

and the orthonormality condition is satisfied. The relation between the higher mode functions and creation (annihilation) operators has

$$a_{iL}\xi_{n,j}^{(i)} = i\sqrt{n}\xi_{n-1,j}^{(i)}, \quad a_{iL}^{\dagger}\xi_{n,j}^{(i)} = -i\sqrt{n+1}\xi_{n+1,j}^{(i)}, \tag{4.97}$$

$$a_{iR}\bar{\xi}_{n,j}^{(i)} = i\sqrt{n}\bar{\xi}_{n-1,j}^{(i)}, \quad a_{iR}^{\dagger}\bar{\xi}_{n,j}^{(i)} = -i\sqrt{n+1}\bar{\xi}_{n+1,j}^{(i)}. \tag{4.98}$$

Thus, KK expansions for $\psi_{iL}, \psi_{iR} (i = 1, 2)$ can be obtained by

$$\psi_{iL} = \sum_{n,j} \psi_{iL,n,j}\xi_{n,j}^{(i)}, \quad \bar{\psi}_{iR} = \sum_{n,j} \bar{\psi}_{iR,n,j}\bar{\xi}_{n,j}^{(i)}. \tag{4.99}$$

4.5.2 Effective Lagrangian

We represent the four-dimensional effective Lagrangian in terms of $\psi_{iL,iR}$. Noting that we are not interested in the interaction of fermion and non-Abelian gauge fields and we ignore these interactions. We extract necessary terms from the Lagrangian (4.78) for our purpose as

$$\begin{aligned}\mathcal{L}_f \supset\ & i\bar{\psi}_{1L}\gamma^\mu\partial_\mu\psi_{1L} + i\bar{\psi}_{1R}\gamma^\mu\partial_\mu\psi_{1R} + i\bar{\psi}_{2L}\gamma^\mu\partial_\mu\psi_{2L} + i\bar{\psi}_{2R}\gamma^\mu\partial_\mu\psi_{2R} \\ & - \frac{\sqrt{\alpha}}{i}\bar{\psi}_{1L}a_{1L}\psi_{1R} + \frac{\sqrt{\alpha}}{i}\bar{\psi}_{1R}a^\dagger_{1L}\psi_{1L} - \frac{\sqrt{\alpha}}{i}\bar{\psi}_{2L}a^\dagger_{2L}\psi_{2R} + \frac{\sqrt{\alpha}}{i}\bar{\psi}_{2R}a_{2L}\psi_{2L} \\ & - \sqrt{2}g\varphi^{3*}(\bar{\psi}_{1L}\psi_{1R} - \bar{\psi}_{2L}\psi_{2R}) - \sqrt{2}g\varphi^3(\bar{\psi}_{1R}\psi_{1L} - \bar{\psi}_{2R}\psi_{2L}). \end{aligned} \tag{4.100}$$

Using the relation eq.(4.97), eq.(4.98) and the orthonormality condition, the four-dimensional Lagrangian for fermion can be obtained by

$$\begin{aligned}\mathcal{L}_{4f} \supset & \sum_{n,j}\Big[i\bar{\psi}_{1L,n,j}\gamma^\mu\partial_\mu\psi_{1L,n,j} + i\bar{\psi}_{1R,n,j}\gamma^\mu\partial_\mu\psi_{1R,n,j} \\ & \qquad + i\bar{\psi}_{2L,n,j}\gamma^\mu\partial_\mu\psi_{2L,n,j} + i\bar{\psi}_{2R,n,j}\gamma^\mu\partial_\mu\psi_{2R,n,j}\Big] \\ & + \sum_{n,j}\Big[-\sqrt{\alpha(n+1)}\bar{\psi}_{1L,n+1,j}\psi_{1R,n,j} + \sqrt{\alpha(n+1)}\bar{\psi}_{2L,n+1,j}\psi_{2R,n,j} + \text{h.c.}\Big] \\ & - \sum_{n,j}\Big[\sqrt{2}g\varphi^{3*}(\bar{\psi}_{1L,n,j}\psi_{1R,n,j} - \bar{\psi}_{2L,n,j}\psi_{2R,n,j}) \\ & \qquad + \sqrt{2}g\varphi^3(\bar{\psi}_{1R,n,j}\psi_{1L,n,j} - \bar{\psi}_{2R,n,j}\psi_{2L,n,j})\Big]. \end{aligned} \tag{4.101}$$

For the reason that different KK modes are mixed in the mass terms, we rewrite eq.(4.101). Using eq.(4.80), we decompose eq.(4.101) into the following form:

$$\begin{aligned}\mathcal{L}_{4f} \supset & \sum_{n,j}\Big[-i\psi_{1,n,j}\sigma^\mu\partial_\mu\bar{\psi}_{1,n,j} - i\psi_{2,n,j}\sigma^\mu\partial_\mu\bar{\psi}_{2,n,j} - i\chi_{1,n,j}\sigma^\mu\partial_\mu\bar{\chi}_{1,n,j} - i\chi_{2,n,j}\sigma^\mu\partial_\mu\bar{\chi}_{2,n,j}\Big] \\ & + \sum_{n,j}\Big[\sqrt{\alpha(n+1)}\bar{\psi}_{1,n+1,j}\bar{\chi}_{1,n,j} - \sqrt{\alpha(n+1)}\bar{\psi}_{2,n+1,j}\bar{\chi}_{2,n,j} + \text{h.c.}\Big] \\ & + \sum_{n,j}\Big[\sqrt{2}g\varphi^{3*}(\bar{\psi}_{1,n,j}\bar{\chi}_{1,n,j} - \bar{\psi}_{2,n,j}\bar{\chi}_{2,n,j}) + \sqrt{2}g\varphi^3(\chi_{1,n,j}\psi_{1,n,j} - \chi_{2,n,j}\psi_{2,n,j})\Big]. \end{aligned} \tag{4.102}$$

For Dirac fermion, we define $\Psi_{i,n,j}$ as

$$\Psi_{i,n,j} = \begin{pmatrix} \psi_{i,n+1,j} \\ \bar{\chi}_{i,n,j} \end{pmatrix}, \tag{4.103}$$

where we understand $\chi_{i,-1,j} = 0$ for $n = -1$. Thus, we rewrite eq.(4.102) in terms of $\Psi_{i,n,j}$,

$$\begin{aligned}\mathcal{L}_{4f} \supset & \sum_{n=-1}^{\infty} \sum_{j=0}^{|N|-1} \Big[i\overline{\Psi}_{1,n,j}\gamma^{\mu}\partial_{\mu}\Psi_{1,n,j} + i\overline{\Psi}_{2,n,j}\gamma^{\mu}\partial_{\mu}\Psi_{2,n,j} \Big] \\ & + \sum_{n=-1}^{\infty} \sum_{j=0}^{|N|-1} \Big[-\sqrt{\alpha(n+1)}\ \overline{\Psi}_{1,n,j}\Psi_{1,n,j} + \sqrt{\alpha(n+1)}\ \overline{\Psi}_{2,n,j}\Psi_{2,n,j} \Big] \\ & + \sum_{n=-1}^{\infty} \sum_{j=0}^{|N|-1} \Big[\sqrt{2} g \varphi^{3*} (-\overline{\Psi}_{1,n,j} P_R \Psi_{1,n+1,j} + \overline{\Psi}_{2,n,j} P_R \Psi_{2,n+1,j}) \\ & \qquad + \sqrt{2} g \varphi^{3} (-\overline{\Psi}_{1,n+1,j} P_L \Psi_{1,n,j} + \overline{\Psi}_{2,n+1,j} P_L \Psi_{2,n,j}) \Big], \end{aligned} \tag{4.104}$$

where $P_L = (1-\gamma^5)/2$, $P_R = (1+\gamma^5)/2$ are the projection operators.

4.5.3 Fermion loop

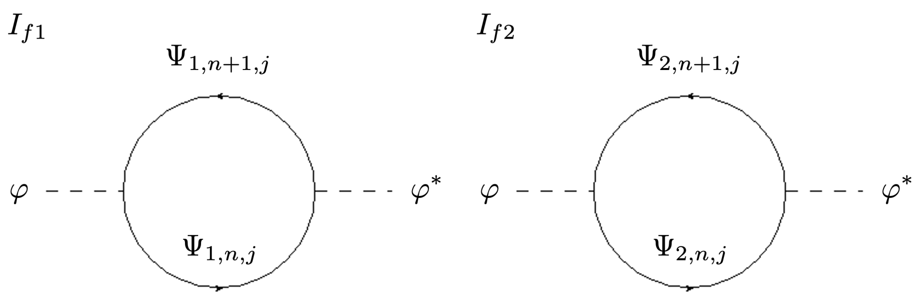

Figure 4.4: Fermion loop correction I_{f1} and I_{f2}.

As shown in figure 4.4, there are two diagrams from the fermion loop contributions. Denoting I_{f1} and I_{f2} as the contribution from $\Psi_{1,n,j}$ and $\Psi_{2,n,j}$ respectively, we obtain

$$I_{f1} = I_{f2} = +4ig^2|N| \sum_n \int \frac{d^4k}{(2\pi)^4} \frac{k^2}{(k^2+\alpha n)(k^2+\alpha(n+1))}. \tag{4.105}$$

Eq.(4.105) is the same form for eq.(3.61), and these contributions vanish by the shift $n \to n+1$. Thus, we conclude

$$I_{f1} = I_{f2} = 0. \tag{4.106}$$

4.6 Higher dimensional operator

Since the higher dimensional gauge theory is non-renormalizable, the higher dimensional operators which are consistent with symmetry of the theory should be considered. The main purpose of this section is to show that the quantum corrections to the masses of WL scalar fields φ^1, φ^{1*} at one-loop are canceled even if we take into account the contributions from the higher dimensional operators. Before going to the calculation in detail, we classify the higher dimensional operators based on a dimensional analysis.

In general, we can add the gauge invariant higher dimensional operators to Lagrangian (4.1).

$$\mathcal{L}_6 = -\frac{1}{4}F^a_{MN}F^{aMN} + \frac{1}{\Lambda^2}\mathcal{O}_1(D,F) + \frac{1}{\Lambda^4}\mathcal{O}_2(D,F) + \frac{1}{\Lambda^6}\mathcal{O}_3(D,F) + \cdots , \tag{4.107}$$

where $\mathcal{O}_n(D,F)$ is a set of gauge invariant operators with covariant derivatives and field strengths. Λ is a cutoff scale of the theory and n is a degree of $1/\Lambda^2$. For $\mathcal{O}_n(D,F)$, we can determine the form of operators allowed in $\mathcal{O}_n(D,F)$ by considering mass dimension in four dimensions of $\mathcal{O}_n(D,F)$. In the case of $n=1$ (the first order in $1/\Lambda^2$), the following three operators are allowed.

$$\mathcal{O}_1(D,F) = D^4F + D^2F^2 + F^3. \tag{4.108}$$

Similarly, in the case of $n=2$ (the second order in $1/\Lambda^2$), the following four operators are allowed.

$$\mathcal{O}_2(D,F) = D^6F + D^4F^2 + D^2F^3 + F^4. \tag{4.109}$$

More explicitly, the operators $\mathcal{O}_1(D,F)$ and $\mathcal{O}_2(D,F)$ are written by[2]

$$\begin{aligned}\mathcal{O}_1(D,F) &= \mathrm{Tr}[D_LD^LD_MD_NF^{MN}] + 2\mathrm{Tr}[D_LF_{MN}D^LF^{MN}] \\ &\quad + \epsilon^{M_1N_1M_2N_2M_3N_3}\mathrm{Tr}[F_{M_1N_1}F_{M_2N_2}F_{M_3N_3}], \end{aligned}\tag{4.110}$$

$$\begin{aligned}\mathcal{O}_2(D,F) &= \mathrm{Tr}[D_KD^KD_LD^LD_MD_NF^{MN}] + \mathrm{Tr}[D_kD_LF_{MN}D^KD^LF^{MN}] \\ &\quad + \mathrm{Tr}[\epsilon^{M_1N_1M_2N_2M_3N_3}(D_LF_{M_1N_2})(D^LF_{M_2N_2})F_{M_3N_3}] + \mathrm{Tr}[F_{MN}F^{MN}F_{AB}F^{AB}],\end{aligned}\tag{4.111}$$

[2]For convenience of the calculation, a factor "2" is included in the second term of $\mathcal{O}_1(D,F)$ to cancel a factor 1/2 coming from the normalization condition of the generators.

where $\epsilon^{M_1N_1M_2N_2M_3N_3}$ is a totally anti-symmetric tensor.

In this thesis, we mainly focus on the operators (4.110), which are the leading terms of the higher dimensional operators. Since only the second term in eq.(4.110) will be found to be non-vanishing, we derive the cubic terms with a single φ^1 or φ^{1*} and the quartic terms involving two φ^1 and φ^{1*} from it, which are necessary for calculations of one-loop corrections to WL scalar mass. In the following calculations, we fix the parameter $\xi = 1$.

4.6.1 $\mathrm{Tr}[D_L D^L D_M D_N F^{MN}]$

This operator vanishes because of the traceless condition for SU(2) generators.

$$\mathrm{Tr}[D_L D^L D_M D_N F^{MN}] = (D_L D^L D_M D_N F^{MN})^a \mathrm{Tr}[t^a] = 0. \tag{4.112}$$

Thus, we need not to calculate the first term in eq.(4.110).

4.6.2 $\epsilon^{M_1N_1M_2N_2M_3N_3}\mathrm{Tr}[F_{M_1N_1}F_{M_2N_2}F_{M_3N_3}]$

The third term in eq.(4.110) also vanishes because of properties of totally anti-symmetric tensor $\epsilon^{M_1N_1M_2N_2M_3N_3}$ and the trace of product of three generators. We first note that the trace of $t^a t^b t^c$ is written as

$$\mathrm{Tr}\left[t^a t^b t^c\right] = \frac{1}{4} i\epsilon^{abc}. \tag{4.113}$$

Using this result, we can find the third term in eq.(4.110) to take the following form.

$$\begin{aligned}\epsilon^{M_1N_1M_2N_2M_3N_3}\mathrm{Tr}[F_{M_1N_1}F_{M_2N_2}F_{M_3N_3}] &= \frac{i}{4}\epsilon^{abc}\epsilon^{M_1N_1M_2N_2M_3N_3}F^a_{M_1N_1}F^b_{M_2N_2}F^c_{M_3N_3} \\ &= -\frac{i}{4}\epsilon^{abc}\epsilon^{M_1N_1M_2N_2M_3N_3}F^a_{M_1N_1}F^b_{M_2N_2}F^c_{M_3N_3},\end{aligned} \tag{4.114}$$

where we interchanged the indices $a \leftrightarrow b$ and $M_1, N_1 \leftrightarrow M_2, N_2$ in the second equality, and use the properties of two anti-symmetric tensors ϵ^{abc} and $\epsilon^{M_1N_1M_2N_2M_3N_3}$. Then we conclude

$$\epsilon^{M_1N_1M_2N_2M_3N_3}\mathrm{Tr}[F_{M_1N_1}F_{M_2N_2}F_{M_3N_3}] = 0. \tag{4.115}$$

4.6.3 $2\mathrm{Tr}[D_L F_{MN} D^L F^{MN}]$

This operator can be decomposed into the fields with four-dimensional and extra two-dimensional indices as follows.

$$\begin{aligned}2\mathrm{Tr}[D_L F_{MN} D^L F^{MN}] &= D_L F^a_{MN} D^L F^{aMN} \\ &= D_\rho F^a_{\mu\nu} D^\rho F^{a\mu\nu} + 2D_\rho F^a_{\mu m} D^\rho F^{a\mu m} + 2D_\rho F^a_{56} D^\rho F^{a56} \\ &\quad + D_l F^a_{\mu\nu} D^l F^{a\mu\nu} + 2D_l F^a_{\mu m} D^l F^{a\mu m} + 2D_l F^a_{56} D^l F^{a56}.\end{aligned} \tag{4.116}$$

Since the first term has no terms with φ^1, φ^{1*}, it is irrelevant to our calculations. We then decompose the remaining terms in eq.(4.116). Detail computations of the remaining terms in eq.(4.116) are described in appendix B. In this subsection, we show the final result,

$$\begin{aligned}\mathcal{L}_{\varphi\varphi AA} &= 8g^2\varphi^{1*}\varphi^1 \sum_{n,j} \partial_\mu \widetilde{A}^2_{\nu,n,j} \partial^\mu \widetilde{A}^{2\nu}_{n,j} + 8g^2\varphi^{1*}\varphi^1 \sum_{n,j} \partial_\mu \widetilde{A}^3_{\nu,n,j} \partial^\mu \widetilde{A}^{3\nu}_{n,j} \\ &\quad + 16g^2\varphi^{1*}\varphi^1 \sum_{n,j} \alpha n \widetilde{A}^2_{\mu,n,j} \widetilde{A}^{2\mu}_{n,j} + 16g^2\varphi^{1*}\varphi^1 \sum_{n,j} \alpha(n+1) \widetilde{A}^3_{\mu,n,j} \widetilde{A}^{3\mu}_{n,j},\end{aligned} \tag{4.117}$$

$$\begin{aligned}\mathcal{L}_{\varphi\varphi\varphi\varphi} &= 8g^2\varphi^{1*}\varphi^1 \sum_{n,j} \partial_\mu \widetilde{\varphi}^{2*}_{n,j} \partial^\mu \widetilde{\varphi}^2_{n,j} + 8g^2\varphi^{1*}\varphi^1 \sum_{n,j} \partial_\mu \widetilde{\varphi}^{3*}_{n,j} \partial^\mu \widetilde{\varphi}^3_{n,j} \\ &\quad + 16g^2\varphi^{1*}\varphi^1 \sum_{n,j} \alpha n \widetilde{\varphi}^2_{n,j} \widetilde{\varphi}^{2*}_{n,j} + 16g^2\varphi^{1*}\varphi^1 \sum_{n,j} \alpha(n+1) \widetilde{\varphi}^3_{n,j} \widetilde{\varphi}^{3*}_{n,j},\end{aligned} \tag{4.118}$$

$$\begin{aligned}\mathcal{L}_{\varphi AA1} &= +4\sqrt{2}ig \sum_{n,j} \sqrt{\alpha\,(n+1)}\partial_\mu \widetilde{A}^2_{\nu,n,j} \partial^\mu \widetilde{A}^{2\nu}_{n+1,j} \varphi^{1*} \\ &\quad - 4\sqrt{2}ig \sum_{n,j} \sqrt{\alpha(n+1)}\partial_\mu \widetilde{A}^3_{\nu,n+1,j} \partial^\mu \widetilde{A}^{3\nu}_{n,j} \varphi^{1*} \\ &\quad - 4\sqrt{2}ig \sum_{n,j} \sqrt{\alpha(n+1)}\partial_\mu \widetilde{A}^2_{\nu,n+1,j} \partial^\mu \widetilde{A}^{2\nu}_{n,j} \varphi^1 \\ &\quad + 4\sqrt{2}ig \sum_{n,j} \sqrt{\alpha(n+1)}\partial_\mu \widetilde{A}^3_{\nu,n,j} \partial^\mu \widetilde{A}^{3\nu}_{n+1,j} \varphi^1\end{aligned} \tag{4.119}$$

$$\begin{aligned}\mathcal{L}_{\varphi AA2} = &+4\sqrt{2}ig\sum_{n,j}\alpha\left(n+\frac{1}{2}\right)\sqrt{\alpha(n+1)}\widetilde{A}^{2}_{\mu,n+1,j}\widetilde{A}^{2\mu}_{n,j}\varphi^{1*}\\ &-4\sqrt{2}ig\sum_{n,j}\alpha\left(n+\frac{3}{2}\right)\sqrt{\alpha(n+1)}\widetilde{A}^{3}_{\mu,n,j}\widetilde{A}^{3\mu}_{n+1,j}\varphi^{1*}\\ &-4\sqrt{2}ig\sum_{n,j}\alpha\left(n+\frac{1}{2}\right)\sqrt{\alpha(n+1)}\widetilde{A}^{2}_{\mu,n+1,j}\widetilde{A}^{2\mu}_{n,j}\varphi^{1}\\ &+4\sqrt{2}ig\sum_{n,j}\alpha\left(n+\frac{3}{2}\right)\sqrt{\alpha(n+1)}\widetilde{A}^{3}_{\mu,n,j}\widetilde{A}^{3\mu}_{n+1,j}\varphi^{1},\end{aligned}\tag{4.120}$$

$$\begin{aligned}\mathcal{L}_{\varphi\varphi\varphi1} = &+4\sqrt{2}ig\sum_{n,j}\sqrt{\alpha(n+1)}\partial_\mu\widetilde{\varphi}^{2*}_{n+1,j}\partial^\mu\widetilde{\varphi}^{2}_{n,j}\varphi^{1*}\\ &-4\sqrt{2}ig\sum_{n,j}\sqrt{\alpha(n+1)}\partial_\mu\widetilde{\varphi}^{3*}_{n,j}\partial^\mu\widetilde{\varphi}^{3}_{n+1,j}\varphi^{1*}\\ &-4\sqrt{2}ig\sum_{n,j}\sqrt{\alpha(n+1)}\partial_\mu\widetilde{\varphi}^{2}_{n+1,j}\partial^\mu\widetilde{\varphi}^{2*}_{n,j}\varphi^{1}\\ &+4\sqrt{2}ig\sum_{n,j}\sqrt{\alpha\,(n+1)}\partial_\mu\widetilde{\varphi}^{3}_{n,j}\partial^\mu\widetilde{\varphi}^{3*}_{n+1,j}\varphi^{1}\end{aligned}\tag{4.121}$$

$$\begin{aligned}\mathcal{L}_{\varphi\varphi\varphi2} = &+4\sqrt{2}ig\sum_{n_2,j}\alpha\left(n-\frac{1}{4}\right)\sqrt{\alpha(n+1)}\widetilde{\varphi}^{2*}_{n_2+1,j}\widetilde{\varphi}^{2}_{n_2,j}\varphi^{1*}\\ &-4\sqrt{2}ig\sum_{n_3,j}\alpha\left(n+\frac{9}{4}\right)\sqrt{\alpha(n+1)}\widetilde{\varphi}^{3*}_{n_3,j}\widetilde{\varphi}^{3}_{n_3+1,j}\varphi^{1*}\\ &-4\sqrt{2}ig\sum_{n_2,j}\alpha\left(n-\frac{1}{4}\right)\sqrt{\alpha(n+1)}\widetilde{\varphi}^{2*}_{n_2,j}\widetilde{\varphi}^{2}_{n_2+1,j}\varphi^{1}\\ &+4\sqrt{2}ig\sum_{n_3,j}\alpha\left(n+\frac{9}{4}\right)\sqrt{\alpha(n+1)}\widetilde{\varphi}^{3*}_{n_3+1,j}\widetilde{\varphi}^{3}_{n_3,j}\varphi^{1},\end{aligned}\tag{4.122}$$

where we rewrite the original fields to the fields in the mass eigenstate $\widetilde{A}^a_\mu, \widetilde{\varphi}^a$. Thus, the four-dimensional interaction Lagrangian is summarized as

$$\mathcal{L}_{4,int} = \frac{1}{\Lambda^2}(\mathcal{L}_{\varphi\varphi AA} + \mathcal{L}_{\varphi\varphi\varphi\varphi} + \mathcal{L}_{\varphi AA1} + \mathcal{L}_{\varphi AA2} + \mathcal{L}_{\varphi\varphi\varphi1} + \mathcal{L}_{\varphi\varphi\varphi2}).\tag{4.123}$$

4.7 Quantum corrections to WL scalar mass from higher dimensional operators

We will explicitly show below that one-loop corrections to the WL scalar masses is indeed canceled even if the lowest term of the higher dimensional operators is present.

The statement is straightforward, but the cancellation of quantum corrections to WL scalar mass is somewhat nontrivial since the cancellation is realized among the terms with different orders of $1/\Lambda^2$.

4.7.1 One-loop Corrections from the Quartic Interactions

From the interactions in (4.117), there are four types of one-loop corrections to the WL scalar masses from the gauge boson loop contributions as the left diagram in figure 4.1, which are expressed as

$$I_{A3}^{(2)} = \frac{32ig^2}{\Lambda^2}|N|\sum_{n=0}^{\infty}\int \frac{d^4p}{(2\pi)^4}\frac{p^2}{p^2+\alpha n}, \tag{4.124}$$

$$I_{A3}^{(3)} = \frac{32ig^2}{\Lambda^2}|N|\sum_{n=0}^{\infty}\int \frac{d^4p}{(2\pi)^4}\frac{p^2}{p^2+\alpha(n+1)}, \tag{4.125}$$

$$I_{A4}^{(2)} = \frac{64ig^2}{\Lambda^2}|N|\sum_{n=0}^{\infty}\int \frac{d^4p}{(2\pi)^4}\frac{\alpha n}{p^2+\alpha n}, \tag{4.126}$$

$$I_{A4}^{(3)} = \frac{64ig^2}{\Lambda^2}|N|\sum_{n=0}^{\infty}\int \frac{d^4p}{(2\pi)^4}\frac{\alpha(n+1)}{p^2+\alpha(n+1)}. \tag{4.127}$$

The superscripts (2), (3) imply the contributions from $\widetilde{A}_\mu^2, \widetilde{A}_\mu^3$ loops respectively. $A3$ and $A4$ represent corrections from the interactions in the first line and the second line of eq.(4.117), respectively. Performing the dimensional regularization [3] for the four-dimensional momentum integral, we find

$$I_{A3} \equiv I_{A3}^{(2)} + I_{A3}^{(3)} = -i\frac{4g^2\alpha^2|N|}{\pi^2\Lambda^2}\left(\frac{4\pi}{\alpha}\right)^{\epsilon}\Gamma(\epsilon-1)\zeta[\epsilon-2,0], \tag{4.128}$$

$$I_{A4} \equiv I_{A4}^{(2)} + I_{A4}^{(3)} = +i\frac{8g^2\alpha^2|N|}{\pi^2\Lambda^2}\left(\frac{4\pi}{\alpha}\right)^{\epsilon}\Gamma(\epsilon-1)\zeta[\epsilon-2,0], \tag{4.129}$$

where $\zeta[s,a]$ is Hurwitz zeta function which is defined by eq.(D.1) in appendix D and ϵ is defined in the ordinary dimensional regularization as $d = 4-2\epsilon$. We will understand how to compute from eq.(4.124) to eq.(4.127) by using the dimensional regularization in next chapter. Summing up (4.128) and (4.129), we obtain the total gauge boson loop

[3] The reason why we employ the dimensional regularization is to keep the gauge symmetry. In our discussion, the gauge symmetry is important to forbid a mass term of the WL scalar field at tree level. Therefore, we should keep the gauge symmetry in the process of computation.

contributions to one-loop correction due to the quartic interactions.

$$I_{\varphi\varphi AA} \equiv I_{A3} + I_{A4} = +i\frac{4g^2\alpha^2|N|}{\pi^2\Lambda^2}\left(\frac{4\pi}{\alpha}\right)^{\epsilon}\Gamma(\epsilon-1)\zeta[\epsilon-2,0]. \tag{4.130}$$

Next, we consider the corrections from the WL scalar quartic interactions (4.118). There are also four types of one-loop corrections to the WL scalar masses from the WL scalar loop contributions as the left diagram in figure 4.2, which are expressed as

$$I^{(2)}_{\varphi 3} = \frac{8ig^2}{\Lambda^2}|N|\sum_{n=0}^{\infty}\int\frac{d^4p}{(2\pi)^4}\frac{p^2}{p^2+\alpha\left(n+\frac{1}{2}\right)}, \tag{4.131}$$

$$I^{(3)}_{\varphi 3} = \frac{8ig^2}{\Lambda^2}|N|\sum_{n=0}^{\infty}\int\frac{d^4p}{(2\pi)^4}\frac{p^2}{p^2+\alpha\left(n+\frac{1}{2}\right)}, \tag{4.132}$$

$$I^{(2)}_{\varphi 4} = \frac{16ig^2}{\Lambda^2}|N|\sum_{n=0}^{\infty}\int\frac{d^4p}{(2\pi)^4}\frac{\alpha n}{p^2+\alpha\left(n+\frac{1}{2}\right)}, \tag{4.133}$$

$$I^{(3)}_{\varphi 4} = \frac{16ig^2}{\Lambda^2}|N|\sum_{n=0}^{\infty}\int\frac{d^4p}{(2\pi)^4}\frac{\alpha(n+1)}{p^2+\alpha\left(n+\frac{1}{2}\right)}, \tag{4.134}$$

where the superscripts (2), (3) mean the contributions from $\widetilde{\varphi}^2, \widetilde{\varphi}^3$ loops respectively. $\varphi 3$ and $\varphi 4$ represent corrections from the interactions in the first line and the second line of eq.(4.118), respectively. Calculating these corrections similarly to the above gauge boson loop (also understanding how to compute from eq.(4.131) to eq.(4.134) in next chapter), one has

$$I_{\varphi 3} \equiv I^{(2)}_{\varphi 3} + I^{(3)}_{\varphi 3} = -i\frac{g^2\alpha^2|N|}{\pi^2\Lambda^2}\left(\frac{4\pi}{\alpha}\right)^{\epsilon}\Gamma(\epsilon-1)\zeta[\epsilon-2,1/2], \tag{4.135}$$

$$I_{\varphi 4} \equiv I^{(2)}_{\varphi 4} + I^{(3)}_{\varphi 4} = +i\frac{2g^2\alpha^2|N|}{\pi^2\Lambda^2}\left(\frac{4\pi}{\alpha}\right)^{\epsilon}\Gamma(\epsilon-1)\zeta[\epsilon-2,1/2]. \tag{4.136}$$

Summing up (4.135) and (4.136), we obtain the total WL scalar loop contributions to one-loop correction due to the WL scalar quartic interactions.

$$I_{\varphi\varphi\varphi\varphi} \equiv I_{\varphi 3} + I_{\varphi 4} = +i\frac{g^2\alpha^2|N|}{\pi^2\Lambda^2}\left(\frac{4\pi}{\alpha}\right)^{\epsilon}\Gamma(\epsilon-1)\zeta[\epsilon-2,1/2]. \tag{4.137}$$

4.7.2 One-loop Corrections from the Cubic Interactions

In the case of the corrections due to the cubic interactions, we note that one-loop corrections generate by using both the cubic interactions eqs.(4.48), (4.52) in $\mathcal{O}(\Lambda^0)$

and eqs.(4.119), (4.120), (4.121), (4.122) in $\mathcal{O}(1/\Lambda^2)$. This is a nontrivial point in calculating the corrections in the presence of the higher dimensional operators. From the interactions (4.48), (4.119) and (4.120), there are four types of one-loop corrections to the WL scalar masses from the gauge boson loop contributions as the right diagram in figure 4.1. These contributions are expressed as

$$I_{A5}^{(2)} = -\frac{32ig^2}{\Lambda^2}|N|\sum_{n=0}^{\infty}\int\frac{d^4p}{(2\pi)^4}\frac{\alpha(n+1)p^2}{(p^2+\alpha n)(p^2+\alpha(n+1))}, \tag{4.138}$$

$$I_{A5}^{(3)} = -\frac{32ig^2}{\Lambda^2}|N|\sum_{n=0}^{\infty}\int\frac{d^4p}{(2\pi)^4}\frac{\alpha(n+1)p^2}{(p^2+\alpha(n+1))(p^2+\alpha(n+2))}, \tag{4.139}$$

$$I_{A6}^{(2)} = -\frac{32ig^2}{\Lambda^2}|N|\sum_{n=0}^{\infty}\int\frac{d^4p}{(2\pi)^4}\frac{\alpha(n+1)\alpha\left(n+\frac{1}{2}\right)}{(p^2+\alpha n)(p^2+\alpha(n+1))}, \tag{4.140}$$

$$I_{A6}^{(3)} = -\frac{32ig^2}{\Lambda^2}|N|\sum_{n=0}^{\infty}\int\frac{d^4p}{(2\pi)^4}\frac{\alpha(n+1)\alpha\left(n+\frac{3}{2}\right)}{(p^2+\alpha(n+1))(p^2+\alpha(n+2))}, \tag{4.141}$$

where $A5$ or $A6$ represent the contributions from the interactions (4.48) and (4.119) or the interaction (4.48) and (4.120), respectively. Calculating these corrections by dimensional regularization, we find

$$I_{A5} \equiv I_{A5}^{(2)} + I_{A5}^{(3)} = +i\frac{4g^2\alpha^2|N|}{\pi^2\Lambda^2}\left(\frac{4\pi}{\alpha}\right)^{\epsilon}\Gamma(\epsilon-1)\zeta[\epsilon-2,0], \tag{4.142}$$

$$I_{A6} \equiv I_{A6}^{(2)} + I_{A6}^{(3)} = -i\frac{8g^2\alpha^2|N|}{\pi^2\Lambda^2}\left(\frac{4\pi}{\alpha}\right)^{\epsilon}\Gamma(\epsilon-1)\zeta[\epsilon-2,0]. \tag{4.143}$$

Summing up these results of eq.(4.142) and eq.(4.143), we obtain the total one-loop corrections to the WL scalar masses from the gauge boson loop contributions.

$$I_{\varphi AA} \equiv I_{A5} + I_{A6} = -i\frac{4g^2\alpha^2|N|}{\pi^2\Lambda^2}\left(\frac{4\pi}{\alpha}\right)^{\epsilon}\Gamma(\epsilon-1)\zeta[\epsilon-2,0]. \tag{4.144}$$

Next, we consider the corrections from eq.(4.52), eq.(4.121) and eq.(4.122), which also give four types of one-loop corrections from the WL scalar loop contributions as the right diagram in figure 4.2. These contributions are represented as

$$I_{\varphi 5}^{(2)} = -\frac{8ig^2}{\Lambda^2}|N| \sum_{n=0}^{\infty} \int \frac{d^4p}{(2\pi)^4} \frac{\alpha(n+1)p^2}{\left(p^2 + \alpha\left(n+\frac{1}{2}\right)\right)\left(p^2 + \alpha\left(n+\frac{3}{2}\right)\right)}, \tag{4.145}$$

$$I_{\varphi 5}^{(3)} = -\frac{8ig^2}{\Lambda^2}|N| \sum_{n=0}^{\infty} \int \frac{d^4p}{(2\pi)^4} \frac{\alpha(n+1)p^2}{\left(p^2 + \alpha\left(n+\frac{1}{2}\right)\right)\left(p^2 + \alpha\left(n+\frac{3}{2}\right)\right)}, \tag{4.146}$$

$$I_{\varphi 6}^{(2)} = -\frac{8ig^2}{\Lambda^2}|N| \sum_{n=0}^{\infty} \int \frac{d^4p}{(2\pi)^4} \frac{\alpha(n+1)\alpha\left(n-\frac{1}{4}\right)}{\left(p^2 + \alpha\left(n+\frac{1}{2}\right)\right)\left(p^2 + \alpha\left(n+\frac{3}{2}\right)\right)}, \tag{4.147}$$

$$I_{\varphi 6}^{(3)} = -\frac{8ig^2}{\Lambda^2}|N| \sum_{n=0}^{\infty} \int \frac{d^4p}{(2\pi)^4} \frac{\alpha(n+1)\alpha\left(n+\frac{9}{4}\right)}{\left(p^2 + \alpha\left(n+\frac{1}{2}\right)\right)\left(p^2 + \alpha\left(n+\frac{3}{2}\right)\right)}, \tag{4.148}$$

where $\varphi 5$ or $\varphi 6$ represent the contributions from the interactions (4.52) and (4.121) or the interactions (4.52) and (4.122), respectively. By similar calculations of eq.(4.142) and eq.(4.143), we find

$$I_{\varphi 5} \equiv I_{\varphi 5}^{(2)} + I_{\varphi 5}^{(3)} = +i\frac{g^2\alpha^2|N|}{\pi^2\Lambda^2}\left(\frac{4\pi}{\alpha}\right)^{\epsilon}\Gamma(\epsilon-1)\zeta[\epsilon-2, 1/2], \tag{4.149}$$

$$I_{\varphi 6} \equiv I_{\varphi 6}^{(2)} + I_{\varphi 6}^{(3)} = -i\frac{2g^2\alpha^2|N|}{\pi^2\Lambda^2}\left(\frac{4\pi}{\alpha}\right)^{\epsilon}\Gamma(\epsilon-1)\zeta[\epsilon-2, 1/2]. \tag{4.150}$$

Summing up eq.(4.149) and eq.(4.150), we obtain the total one-loop corrections to the WL scalar masses from the WL scalar loop contributions.

$$I_{\varphi\varphi\varphi} \equiv I_{\varphi 5} + I_{\varphi 6} = -i\frac{g^2\alpha^2|N|}{\pi^2\Lambda^2}\left(\frac{4\pi}{\alpha}\right)^{\epsilon}\Gamma(\epsilon-1)\zeta[\epsilon-2, 1/2]. \tag{4.151}$$

4.7.3 Cancellation of One-loop Corrections to Scalar Mass at $\mathcal{O}(1/\Lambda^2)$

Summing up all of the results (4.130), (4.137), (4.144) and (4.151), we can verify that one-loop corrections to the WL scalar masses are indeed canceled at the leading order of $\mathcal{O}(1/\Lambda^2)$.

$$I_{\varphi\varphi AA} + I_{\varphi AA} = 0, \tag{4.152}$$

$$I_{\varphi\varphi\varphi\varphi} + I_{\varphi\varphi\varphi} = 0. \tag{4.153}$$

As can be seen from the above results, the gauge loop contributions and the WL scalar loop contributions are independently canceled. In particular, the WL scalar loop contributions can be canceled without the ghost loop contributions, which is different from the case of Yang-Mills theory [27].

4.7.4 Comments on the Corrections from the Higher Dimensional Operators More Than $\mathcal{O}(1/\Lambda^4)$

We discuss the corrections from the higher dimensional operators more than $\mathcal{O}(1/\Lambda^4)$. If we use two kinds of cubic interactions (4.119), (4.120), (4.121) and (4.122), we obtain some one-loop corrections to the WL scalar mass at the second order of $1/\Lambda^2$, that is $1/\Lambda^4$. However these corrections are not canceled because we must take into account the contributions from the operators of $\mathcal{O}(1/\Lambda^0)$ and $\mathcal{O}(1/\Lambda^4)$. As an example for $\mathcal{O}(1/\Lambda^4)$, we have seen that operators (4.111) have an order $\mathcal{O}(1/\Lambda^4)$ in the section 4.6. Of these operators, the first term vanishes because of the traceless condition for SU(2) generators as was shown in the section 4.6.1 and the third term also vanishes because of the properties of totally anti-symmetric tensor and the trace of generators as was shown in the section 4.6.2. Thus, we need to consider the second and the fourth terms in eq.(4.111):

$$\mathcal{O}_2(D,F) = \mathrm{Tr}[D_k D_L F_{MN} D^K D^L F^{MN}] + \mathrm{Tr}[F_{MN}F^{MN}F_{AB}F^{AB}]. \tag{4.154}$$

Although it is relatively easy to calculate the second term in eq.(4.154) (see Appendix C), the first term in eq.(4.154) is found to have huge number of interaction terms which are relevant to the one-loop corrections to the WL scalar masses. At higher order than $\mathcal{O}(1/\Lambda^4)$, we need to consider carefully the variety of combinations among the operators which are different order of $1/\Lambda^2$ and it becomes more complicated. Such an analysis is very interesting, however it is beyond the scope of this thesis.

4.8 WL scalar as a Nambu-Goldstone boson

As in the section 3.4, the zero mode of WL scalar can be regarded as a NG boson of translational invariance in extra spaces, which is the physical reason that one-loop corrections to the WL scalar mass vanish. The transformations of translation in extra spaces are given by

$$\delta_T A_5^a = (\epsilon_5\partial_5 + \epsilon_6\partial_6)\,\widetilde{A}_5^a - \frac{f}{2}\epsilon_6\delta^{a1}, \tag{4.155}$$

$$\delta_T A_6^a = (\epsilon_5\partial_5 + \epsilon_6\partial_6)\,\widetilde{A}_6^a + \frac{f}{2}\epsilon_5\delta^{a1}, \tag{4.156}$$

where $\epsilon_{5,6}$ are constant parameters of translation in torus. These transformations can be rewritten in complex coordinate as

$$\begin{aligned}\delta_T \phi^a &= \frac{1}{\sqrt{2}}(\delta_T A_6 + i\delta_T A_5) \\ &= (\epsilon\partial + \bar{\epsilon}\bar{\partial})\varphi^a + \frac{f}{\sqrt{2}}\bar{\epsilon}\delta^{a1},\end{aligned} \tag{4.157}$$

where $\epsilon \equiv (\epsilon_5 + i\epsilon_6)/2$. The first term in eq.(4.157) vanishes because we deal with the zero mode of WL scalar φ^1. Thus,

$$\delta_T \phi^1 = \frac{f}{\sqrt{2}}\bar{\epsilon} \tag{4.158}$$

is obtained and eq.(4.158) is simply reduced to a constant shift symmetry. Eq.(4.158) means that the zero mode of WL scalar φ^1 becomes a NG boson under the translation in torus. Therefore, only the derivative terms of the zero mode of WL scalar are allowed in the Lagrangian and it is a natural result that one-loop corrections to the zero mode of WL scalar mass vanish. It is very interesting to note that the cancellations in the explicit calculations above have been shown by relying on the shift $n \to n+1$, which is a remnant of the shift symmetry discussed in previous subsections.

Chapter 5

Nonvanishing finite WL scalar mass

In previous chapters, we have shown that the quantum correction to WL scalar mass vanishes. In this chapter, we study possibilities to realize a nonvanishing finite WL scalar mass in flux compactification by analyzing the generalized loop integrals in the quantum correction to WL scalar mass at one-loop [30]. After finding the conditions for the loop integrals and mode sums in one-loop corrections to WL scalar mass to be finite, we guess the four-point and three-point interaction terms satisfying this conditions.

5.1 Summary for Kaluza-Klein mass spectrum

In previous chapters, we have discussed the KK mass spectrums. For scalar field, the KK mass is obtained by

$$m^2_{\text{scalar}} = \alpha\left(n + \frac{1}{2}\right) \tag{5.1}$$

as eq.(2.18) in subsection 2.2.

For fermion field, the KK mass is given by

$$m^2_{\text{fermion}} = \begin{cases} \alpha n \\ \alpha(n+1) \end{cases} \tag{5.2}$$

as $\mathcal{M}_-^2 = \alpha a_-^\dagger a_-$ and $\mathcal{M}_+^2 = \alpha(a_+^\dagger a_+ + 1)$ in the subsection 3.3.2, or the mass terms of eq.(4.104) in the subsection 4.5.2.

For non-Abelian gauge field (for example, consider SU(2) gauge field), the KK mass is expressed by

$$m_{YM}^2 = \begin{pmatrix} \beta(l^2+m^2) & 0 & 0 \\ 0 & \alpha n_2 & 0 \\ 0 & 0 & \alpha(n_3+1) \end{pmatrix} \tag{5.3}$$

as eq.(4.28) in the subsection 4.2.1.

5.2 The structure of loop integral: general

In this section, we first systematically analyze the divergence structure of the quantum corrections to WL scalar mass. In general, there are two types of Feynman diagrams in figure 4.1 or 4.2. From these diagrams and the results of the above subsection, the general form of loop integral in the quantum correction can be given by

$$\begin{aligned} &\sum_{n=0}^{\infty}\int \frac{d^4k}{(2\pi)^4}\frac{k^{2a}f(n)}{(k^2+\alpha\,(n+x))^b} \\ &= \frac{1}{\alpha^{b-a}}\left(\frac{4\pi}{\alpha}\right)^{\epsilon-2}\frac{\Gamma\,(a+2-\epsilon)\,\Gamma\,(\epsilon+b-a-2)}{\Gamma(b)\Gamma(2-\epsilon)}\sum_{n=0}^{\infty}\frac{f(n)}{(n+x)^{\epsilon+b-a-2}}, \end{aligned} \tag{5.4}$$

where the dimensional regularization was employed for loop integral in the second line. $\Gamma(z)$ is a gamma function, $\zeta[s,a]$ is Hurwitz zeta function which is summarized in appendix D, and $d = 4-2\epsilon$ dimensions. x is the part of KK mass characterized by the field running in the loop. $x = 1/2$ corresponds to the KK mass of scalar field (5.1). $x = 1$ or 0 mainly corresponds to the KK mass of fermion field (5.2) or the KK mass of SU(2) gauge field (5.3), respectively. a denotes the number of derivatives acting on the single field and b corresponds to the number of the propagator. Note that both a and b are non-negative numbers. Since we are interested in the quantum correction as figure 4.1 or 4.2, we focus on $b = 1$ or $b = 2$, that is four-point interaction or three-point interaction, respectively. $f(n)$ is a coefficient generated by an interaction term depending on KK mode n.

If we take the complicated form of $f(n)$, it is difficult to express the quantum correction by using Hurwitz zeta function and the discussion on the finiteness of loop integral becomes hard. Therefore, we simply take the form $f(n) = (\alpha(n+q))^c$(q is a real number and c is a non-negative number) in this thesis:

$$I(x;a,b;c,q) \equiv \frac{1}{\alpha^{b-a-c}} \left(\frac{4\pi}{\alpha}\right)^{\epsilon-2} \frac{\Gamma(a+2-\epsilon)\Gamma(\epsilon+b-a-2)}{\Gamma(b)\Gamma(2-\epsilon)} \sum_{n=0}^{\infty} \frac{(n+q)^c}{(n+x)^{\epsilon+b-a-2}}. \tag{5.5}$$

5.3 The structure of loop integral: part 1

First, we investigate the divergence structure for the quantum correction (5.5) with $c = 0$:

$$\begin{aligned} I(x;a,b) &\equiv I(x;a,b;0,q) \\ &= \frac{1}{\alpha^{b-a}} \left(\frac{4\pi}{\alpha}\right)^{\epsilon-2} \frac{\Gamma(a+2-\epsilon)\Gamma(\epsilon+b-a-2)}{\Gamma(b)\Gamma(2-\epsilon)} \zeta[\epsilon+b-a-2,x]. \end{aligned} \tag{5.6}$$

In order to realize nonvanishing finite WL scalar mass, the loop integral and mode sum for one-loop correction to WL scalar mass (5.6) must be finite. To clarify this point, we investigate

$$J(x;a,b) \equiv \frac{\Gamma(a+2-\epsilon)\Gamma(\epsilon+b-a-2)}{\Gamma(2-\epsilon)} \zeta[\epsilon+b-a-2,x] \tag{5.7}$$

in eq.(5.6). As was mentioned in the above discussion, we deal with the case $b = 1$ or $b = 2$. In the case of $b = 1$, the Gamma function part of eq.(5.7) is expressed by

$$\begin{aligned} \frac{\Gamma(a+2-\epsilon)\Gamma(\epsilon-a-1)}{\Gamma(2-\epsilon)} &= (a+2-\epsilon-1)(a+2-\epsilon-2)\cdots(2-\epsilon)\Gamma(\epsilon-a-1) \\ &= (-1)^a\Gamma(\epsilon-1). \end{aligned} \tag{5.8}$$

Thus, $J(x;a,1)$ becomes

$$J(x;a,1) = (-1)^a\Gamma(\epsilon-1)\zeta[\epsilon-a-1,x]. \tag{5.9}$$

In the case of $b = 2$, the same part of eq.(5.7) is expressed by

$$\begin{aligned} \frac{\Gamma(a+2-\epsilon)\Gamma(\epsilon-a)}{\Gamma(2-\epsilon)} &= (a+2-\epsilon-1)(a+2-\epsilon-2)\cdots(2-\epsilon)\Gamma(\epsilon-a) \\ &= (-1)^a(\epsilon-a-1)\Gamma(\epsilon-1). \end{aligned} \tag{5.10}$$

Thus, $J(x;a,2)$ becomes

$$J(x;a,2) = (-1)^a(\epsilon-a-1)\Gamma(\epsilon-1)\zeta[\epsilon-a,x]. \tag{5.11}$$

Here, Gamma function and Hurwitz zeta function can be expanded in ϵ

$$\Gamma(\epsilon-1)=\frac{\Gamma(\epsilon)}{\epsilon-1}=-\left(\frac{1}{\epsilon}-\gamma_E+1+\mathcal{O}(\epsilon)\right), \tag{5.12}$$

$$\zeta[\epsilon-p,x]=\zeta[-p,x]+\zeta^{(1,0)}[-p,x]\epsilon+\mathcal{O}(\epsilon^2), \tag{5.13}$$

where $\gamma_E = 0.5772\cdots$ is the Euler-Mascheroni constant, p is an arbitrary positive integer, and $\zeta^{(1,0)}[s,a]$ means $\zeta^{(1,0)}[s,a]=\partial\zeta[s,a]/\partial s$. Calculating $\Gamma(\epsilon-1)\zeta[\epsilon-p,x]$, divergent part will be remained because of $1/\epsilon$ in eq.(5.12) and $\zeta[-p,x]$ in eq.(5.13). However, using eq.(D.5) in appendix D, $\Gamma(\epsilon-1)\zeta[\epsilon-p,x]$ becomes finite if we take p being even. Thus, we summarize the condition for $\Gamma(\epsilon-1)\zeta[\epsilon-p,x]$ being finite as

$$\Gamma(\epsilon-1)\zeta[\epsilon-p,x]=\text{finite},\quad \text{if } p=even. \tag{5.14}$$

Applying this result to eqs.(5.9) and (5.11), $J(x;a,1)$ takes finite value at odd a, $J(x;a,2)$ does at even a.

5.4 Classification of interaction terms: part 1

From the condition (5.14), we can classify the interaction terms giving finite one-loop correction to WL scalar mass. In this section, we consider interaction terms which has no derivatives acting on φ or φ^* because we consider one-loop corrections to WL scalar mass.

5.4.1 Four-point interaction

Four-point interaction term generates a correction to WL scalar mass of the left one in figure 4.1 or 4.2. Since the diagram has a propagator, the diagram corresponds to $J(x;a,1)$ (a: odd), from which we can guess the four-point interaction terms as follows,

- scalar field loop

$$J(1/2;a,1)\to\varphi^*\varphi\partial_{\mu_1}\cdots\partial_{\mu_a}\Phi^*\partial^{\mu_1}\cdots\partial^{\mu_a}\Phi, \tag{5.15}$$

- fermion field loop

$$J(1;a,1)\to\varphi^*\varphi\bar{\psi}(\partial\!\!\!/)^{2a-1}\psi, \tag{5.16}$$

- SU(2) gauge field loop

$$J(0; a, 1) \to \varphi^* \varphi \partial_{\mu_1} \cdots \partial_{\mu_a} A^a_\nu \partial^{\mu_1} \cdots \partial^{\mu_a} A^{a\nu}. \tag{5.17}$$

We did not consider a four-point interaction with such as $\varphi^* \varphi \partial_{\mu_1} \cdots \partial_{\mu_a} \bar{\psi} \partial^{\mu_1} \cdots \partial^{\mu_a} \psi$ since the fermion mass $m_{fermion} = \sqrt{\alpha(n+1)}$ appears from a numerator in the fermion propagator and then the form of Hurwitz zeta function is complicated. On the other hand, $(\not{k})^{2a-1}$ is obtained by (5.16). Computing quantum correction, the trace of $\not{k}$ from a numerator in the propagator of fermion multiplied by $(\not{k})^{2a-1}$ is given by k^{2a}. These terms contribute to quantum correction to WL scalar mass in the case odd a.

5.4.2 Three-point inteaction

Three-point interaction term generates a correction of the right one in figure 3.4, 4.1 or 4.2. The diagram has two propagators and corresponds to $J(x; a, 2)$ (a: even), from which we can guess the three-point interaction terms as follows,

- scalar field loop

$$J(1/2; 0, 2) \to \varphi^* \Phi^* \Phi + \varphi \Phi^* \Phi, \tag{5.18}$$

$$J(1/2; a, 2) \to \varphi^* \partial_{\mu_1} \cdots \partial_{\mu_{a/2}} \Phi^* \partial^{\mu_1} \cdots \partial^{\mu_{a/2}} \Phi + \varphi \partial_{\mu_1} \cdots \partial_{\mu_{a/2}} \Phi^* \partial^{\mu_1} \cdots \partial^{\mu_{a/2}} \Phi, \tag{5.19}$$

- fermion field loop

$$J(1; a, 2) \to \varphi^* \bar{\psi} (\not{\partial})^{a-1} \psi + \varphi \bar{\psi} (\not{\partial})^{a-1} \psi, \tag{5.20}$$

- SU(2) gauge field loop

$$J(0; a, 2) \to \varphi^* \partial_{\mu_1} \cdots \partial_{\mu_{a/2}} A^a_\nu \partial^{\mu_1} \cdots \partial^{\mu_{a/2}} A^{a\nu} + \varphi \partial_{\mu_1} \cdots \partial_{\mu_{a/2}} A^a_\nu \partial^{\mu_1} \cdots \partial^{\mu_{a/2}} A^{a\nu}, \tag{5.21}$$

where $J(1/2; 0, 2)$ is allowed because of $\zeta[0, 1/2] = 0$. We are particularly interested in the interaction term in (5.18), therefore we will discuss later.

5.5 The structure of loop integral: part 2

Next, we consider the divergence structure for the quantum correction (5.5) with $c \neq 0$ [1]. As a first step, we set $c = 1$ in eq.(5.5):

$$\begin{aligned} I(x;a,b;1,q) &= \frac{1}{\alpha^{b-a-1}}\left(\frac{4\pi}{\alpha}\right)^{\epsilon-2}\frac{\Gamma(a+2-\epsilon)\,\Gamma(\epsilon+b-a-2)}{\Gamma(b)\Gamma(2-\epsilon)}\sum_{n=0}^{\infty}\frac{n+q}{(n+x)^{\epsilon+b-a-2}} \\ &= \frac{1}{\alpha^{b-a-1}}\left(\frac{4\pi}{\alpha}\right)^{\epsilon-2}\frac{\Gamma(a+2-\epsilon)\,\Gamma(\epsilon+b-a-2)}{\Gamma(b)\Gamma(2-\epsilon)} \\ &\qquad\times\Big(\zeta[\epsilon+b-a-3,x]+(q-x)\zeta[\epsilon+b-a-2,x]\Big). \end{aligned} \tag{5.22}$$

If $q \neq x$, the divergence will inevitably appears from either $\zeta[\epsilon+b-a-3,x]$ or $\zeta[\epsilon+b-a-2,x]$. To avoid the divergence and see whether the quantum correction is finite, we need to choose $q = x$ (equivalent to the choice $f(n) =$ KK mass in $c = 1$ case). Thus, the form of $f(n)$ is fixed by $f(n) = (\alpha(n+x))^c$ in $c \neq 0$ case in order to be finite for $I(x;a,b,c)$. In the $q = x$ case, eq.(5.5) has

$$\begin{aligned} I(x;a,b,c) &\equiv I(x;a,b;c,q=x) \\ &= \frac{1}{\alpha^{b-a-c}}\left(\frac{4\pi}{\alpha}\right)^{\epsilon-2}\frac{\Gamma(a+2-\epsilon)\,\Gamma(\epsilon+b-a-2)}{\Gamma(b)\Gamma(2-\epsilon)}\zeta[\epsilon+b-a-c-2,x]. \end{aligned} \tag{5.23}$$

Note that $I(x;a,b,0)$ corresponds to $I(x;a,b)$ (5.6). To investigate the finiteness of $I(x;a,b,c)$, we see

$$K(x;a,b,c) \equiv \frac{\Gamma(a+2-\epsilon)\,\Gamma(\epsilon+b-a-2)}{\Gamma(2-\epsilon)}\zeta[\epsilon+b-a-c-2,x], \tag{5.24}$$

in eq.(5.23). Substituting $b = 1$ or $b = 2$ in eq.(5.24) and using eq.(5.8) or eq.(5.10) respectively, we obtain

$$K(x;a,1,c) = (-1)^a\Gamma(\epsilon-1)\zeta[\epsilon-a-c-1,x], \tag{5.25}$$

$$K(x;a,2,c) = (-1)^a(\epsilon-a-1)\Gamma(\epsilon-1)\zeta[\epsilon-a-c,x]. \tag{5.26}$$

Applying the result (5.14) to (5.25) and (5.26), $K(x;a,1,c)$ takes finite value at odd $a+c$, $K(x;a,2,c)$ does at even $a+c$.

[1] These results in this section are extended results in [30].

5.6 Classification of interaction terms: part 2

We consider the case of four-point interaction terms ($b = 1$, $a + c$: odd) and guess their form providing finite quantum corrections to WL scalar mass,

- scalar field loop

$$K(1/2; a, 1, c) \to \varphi^* \varphi \partial_{\mu_1} \cdots \partial_{\mu_a} \Phi^* \left(a^\dagger a + \frac{1}{2} \right)^c \partial^{\mu_1} \cdots \partial^{\mu_a} \Phi, \tag{5.27}$$

- fermion field loop

$$K(1; a, 1, c) \to \varphi^* \varphi \bar{\psi} (\partial\!\!\!/)^{2a-1} (a^\dagger a + 1)^c \psi \tag{5.28}$$

- SU(2) gauge field loop

$$K(0; a, 1, c) \to \varphi^* \varphi \partial_{\mu_1} \cdots \partial_{\mu_a} A^a_\nu (a^\dagger a)^c \partial^{\mu_1} \cdots \partial^{\mu_a} A^{a\nu}. \tag{5.29}$$

The case of three-point interaction term is hard to guess because the three-point interaction term cannot be expressed in terms of a mass-squared operator. Thus, we do not consider the three-point interaction terms in this section.

5.7 The structure of loop integral: part 3

Due to the presence of annihilation and creation operators, there are interactions between the field with different KK mode indices. In this case, we consider the following divergence structure of the quantum corrections to WL scalar mass [2]:

$$\sum_{n=0}^{\infty} \int \frac{d^4k}{(2\pi)^4} \frac{k^{2a} f(n, x)}{(k^2 + \alpha(n + x))(k^2 + \alpha(n + y))}, \tag{5.30}$$

where $f(n, x)$ is a coefficient generated by an interaction term depending on KK mode n and x, y are the parts of the KK mass characterized by the field running in the loop and satisfy $x \neq y$ and $y - x \in \mathbb{Z}$. Focusing on the denominator in the above integrand, we use a partial fraction decomposition:

$$\frac{1}{(k^2 + \alpha(n + x))(k^2 + \alpha(n + y))} = \frac{1}{\alpha(y - x)} \left(\frac{1}{k^2 + \alpha(n + x)} - \frac{1}{k^2 + \alpha(n + y)} \right). \tag{5.31}$$

[2] These calculations in this section are unpublished results.

Thus,

$$\sum_{n=0}^{\infty}\int\frac{d^4k}{(2\pi)^4}\frac{k^{2a}f(n,x)}{(k^2+\alpha(n+x))(k^2+\alpha(n+y))}$$
$$=\frac{1}{\alpha(y-x)}\left(\sum_{n=0}^{\infty}\int\frac{d^4k}{(2\pi)^4}\frac{k^{2a}f(n,x)}{k^2+\alpha(n+x)}-\sum_{n=0}^{\infty}\int\frac{d^4k}{(2\pi)^4}\frac{k^{2a}f(n,x)}{k^2+\alpha(n+y)}\right) \tag{5.32}$$

is obtained. It would be interesting that eq.(5.32) implies that the quantum correction from three-point interactions between the fields with different KK mode indices is decomposed into the ones from four-point interaction. If we assume $f(n,x)=(\alpha(n+x))^c$ (c is a non-negative number), the divergence structure of the quantum corrections to WL scalar mass is expressed as

$$I_c(x,y;a,c)\equiv\sum_{n=0}^{\infty}\int\frac{d^4k}{(2\pi)^4}\frac{k^{2a}(\alpha(n+x))^c}{(k^2+\alpha(n+x))(k^2+\alpha(n+y))}$$
$$=\frac{1}{\alpha(y-x)}\left(I(x;a,1,c)-\sum_{n=0}^{\infty}\int\frac{d^4k}{(2\pi)^4}\frac{k^{2a}(\alpha(n+x))^c}{k^2+\alpha(n+y)}\right), \tag{5.33}$$

where we use eq.(5.23). Denoting the second term in eq.(5.33) as X, X is computed as

$$X\equiv\sum_{n=0}^{\infty}\int\frac{d^4k}{(2\pi)^4}\frac{k^{2a}(\alpha(n+x))^c}{k^2+\alpha(n+y)}$$
$$=\frac{1}{\alpha^{1-a}}\left(\frac{4\pi}{\alpha}\right)^{\epsilon-2}\frac{\Gamma(a+2-\epsilon)\,\Gamma(\epsilon-a-1)}{\Gamma(2-\epsilon)}\sum_{n=0}^{\infty}\frac{\alpha^c\{(n+y)+(x-y)\}^c}{(n+y)^{\epsilon-a-1}}$$
$$=\frac{(-1)^a}{\alpha^{1-a-c}}\left(\frac{4\pi}{\alpha}\right)^{\epsilon-2}\Gamma(\epsilon-1)\sum_{n=0}^{\infty}\frac{1}{(n+y)^{\epsilon-a-1}}\sum_{k=0}^{c}{}_cC_k(n+y)^k(x-y)^{c-k}$$
$$=\sum_{k=0}^{c}{}_cC_k(x-y)^{c-k}\alpha^{c-k}\left[\frac{(-1)^a}{\alpha^{1-a-k}}\left(\frac{4\pi}{\alpha}\right)^{\epsilon-2}\Gamma(\epsilon-1)\zeta[\epsilon-a-k-1,y]\right]$$
$$=\sum_{k=0}^{c}{}_cC_k(x-y)^{c-k}\alpha^{c-k}I(y;a,1,k), \tag{5.34}$$

where eq.(5.8) is used in the third equality and eq.(5.23), eq.(5.25) with $b=1$ and $c=k$ are used in the last equality. Note that we define ${}_0C_k=0$ if $c=0$ is taken. Therefore, $I_c(x,y;a,c)$ has

$$I_c(x,y;a,c)=\frac{1}{\alpha(y-x)}\left[I(x;a,1,c)-\sum_{k=0}^{c}{}_cC_k(x-y)^{c-k}\alpha^{c-k}I(y;a,1,k)\right]. \tag{5.35}$$

We try to rewrite $I(y;a,1,k)$ in $I_c(x,y;a,c)$. By using eq.(D.4), $I(y;a,1,k)$ is calculated by

$$
\begin{aligned}
I(y;a,1,k) &= \frac{(-1)^a}{\alpha^{1-a-k}}\left(\frac{4\pi}{\alpha}\right)^{\epsilon-2}\Gamma(\epsilon-1)\zeta[\epsilon-a-k-1,x+(y-x)] \\
&= I(x;a,1,k) - \frac{(-1)^a}{\alpha^{1-a-k}}\left(\frac{4\pi}{\alpha}\right)^{\epsilon-2}\Gamma(\epsilon-1)\sum_{m=0}^{y-x-1}\frac{1}{(m+x)^{\epsilon-a-k-1}}. \qquad (5.36)
\end{aligned}
$$

Thus, $I_c(x,y;a,c)$ is rewritten as

$$
\begin{aligned}
I_c(x,y;a,c) = &\frac{1}{\alpha(y-x)}\left(I(x;a,1,c) - \sum_{k=0}^{c}{}_cC_k(x-y)^{c-k}\alpha^{c-k}I(x;a,1,k)\right) \\
&+ \frac{1}{\alpha(y-x)}\sum_{k=0}^{c}{}_cC_k(x-y)^{c-k}\frac{(-1)^a}{\alpha^{1-a-c}}\left(\frac{4\pi}{\alpha}\right)^{\epsilon-2}\Gamma(\epsilon-1)\sum_{m=0}^{y-x-1}\frac{1}{(m+x)^{\epsilon-a-k-1}} \\
= &-\frac{1}{\alpha(y-x)}\sum_{k=0}^{c-1}{}_cC_k(x-y)^{c-k}\alpha^{c-k}I(x;a,b,k) \\
&+ \frac{1}{\alpha(y-x)}\frac{(-1)^a}{\alpha^{1-a-c}}\left(\frac{4\pi}{\alpha}\right)^{\epsilon-2}\Gamma(\epsilon-1)\sum_{m=0}^{y-x-1}\frac{(m+2x-y)^c}{(m+x)^{\epsilon-a-1}} \qquad (5.37)
\end{aligned}
$$

In order for $I_c(x,y;a,c)$ to be finite, the second term in eq.(5.37) needs to be vanished because it involves the divergent term. In general, it is difficult to vanish the second term. If we however impose $y-x-1=0$, $I_c(x,y;a,c)$ is reduced as

$$
\begin{aligned}
I_c(x,x+1;a,c) = &-\frac{1}{\alpha}\sum_{k=0}^{c-1}{}_cC_k(-1)^{c-k}\alpha^{c-k}I(x;a,1,k) \\
&+ \frac{1}{\alpha}\frac{(-1)^a}{\alpha^{1-a}}\left(\frac{4\pi}{\alpha}\right)^{\epsilon-2}\Gamma(\epsilon-1)x^{a+1-\epsilon}(\alpha(x-1))^c. \qquad (5.38)
\end{aligned}
$$

To vanish the second term in eq.(5.38), we choose $x=0$ which corresponds to the KK mass of fermion field or $x=1$ except for $c=0$ which corresponds to the KK mass of gauge field, respectively.

On the other hand, the second term in eq.(5.38) will remain if we choose $x=1/2$, which corresponds to the KK mass of scalar field. In the $x=1/2$ case, we need to deal with a polynomial in the KK mass of scalar field. We define below the polynomial version of the divergence structure:

$$
\begin{aligned}
I_{\rm poly}(x;a)[\lambda_c,\cdots,\lambda_0] \equiv\ &\lambda_c I_c(x,x+1;a,c) + \lambda_{c-1}I_{c-1}(x,x+1;a,c-1) + \cdots \\
&+ \cdots + \lambda_1 I_1(x,x+1;a,1) + \lambda_0 I_0(x,x+1;a,0), \qquad (5.39)
\end{aligned}
$$

where we fix $b = 1$ and $y = x + 1$ and $\lambda_i (i = 1, \cdots c)$ are real numbers. We note that $I_{\rm poly}(x; a)[\lambda_c, \cdots, \lambda_0]$ is expressed by

$$I_{\rm poly}(x;a)[\lambda_c,\cdots,\lambda_0] = \sum_{n=0}^{\infty}\int \frac{d^4k}{(2\pi)^4}\frac{k^{2a} f_{\rm poly}(n,x)}{(k^2+\alpha(n+x))(k^2+\alpha(n+x+1))}, \tag{5.40}$$

$$f_{\rm poly}(n,x) = \lambda_c(\alpha(n+x))^c + \lambda_{c-1}(\alpha(n+x))^{c-1} + \cdots + \lambda_0(\alpha(n+x))^0. \tag{5.41}$$

As an illustration, we consider the following $f_{\rm poly}(n, x)$:

$$\begin{aligned} f_{\rm poly}(n,x) &= (\alpha(n+x))^2 + \alpha(r+s)(\alpha(n+x)) + \alpha^2 rs \\ &= \alpha((n+x)+r)\alpha((n+x)+s), \end{aligned} \tag{5.42}$$

where r, s are real numbers. From the above example, we read $\lambda_2 = 1,\ \lambda_1 = \alpha(r + s),\ \lambda_0 = \alpha^2 rs$. By using $\lambda_{0,1,2}$, eq.(5.38) and eq.(5.39), $I_{\rm poly}(x; a)[1, \alpha(r + s), \alpha^2 rs]$ involves

$$\begin{aligned} &I_{\rm poly}(x;a)[1,\alpha(r+s),\alpha^2 rs] \\ &\supset -\frac{(-1)^a}{\alpha^{1-a}}\left(\frac{4\pi}{\alpha}\right)^{\epsilon-2}\Gamma(\epsilon-1)x^{a+1-\epsilon}\alpha^2\{x-(1-r)\}\{x-(1-s)\}. \end{aligned} \tag{5.43}$$

Taking $x = 1/2$, eq.(5.43) vanishes if we choose r or s for a half. That is, $f_{\rm poly}(n, x)$ is represented by

$$f_{poly}(n,x) = \alpha(n+1)\alpha\left(\left(n+\frac{1}{2}\right)+s\right). \tag{5.44}$$

The factor $\alpha(n + 1)$ (or $(\sqrt{\alpha(n + 1)})^2$) implies that creation operators raise KK mode by one as eq.(3.40) or eq.(4.27). This implication is consistent with the condition of $y - x - 1 = 0$.

5.8 Summary of the structure of loop integral

We summarize the structure of loop integral in the previous sections. In general, the structure of loop integrals has

$$\begin{aligned} I(x;a,b;c,q) &\equiv \sum_{n=0}^{\infty}\int\frac{d^4k}{(2\pi)^4}\frac{k^{2a}(\alpha(n+q))^c}{(k^2+\alpha\,(n+x))^b} \\ &= \frac{1}{\alpha^{b-a-c}}\left(\frac{4\pi}{\alpha}\right)^{\epsilon-2}\frac{\Gamma\,(a+2-\epsilon)\,\Gamma\,(\epsilon+b-a-2)}{\Gamma(b)\Gamma(2-\epsilon)}\sum_{n=0}^{\infty}\frac{(n+q)^c}{(n+x)^{\epsilon+b-a-2}}, \end{aligned} \tag{5.45}$$

where a, b, c are non-negative numbers. According to the finiteness of loop integral, we mainly deal with the following loop integral

$$I(x;a,b,c) \equiv I(x;a,b;c,q=x) = \frac{1}{\alpha^{b-a-c}} \left(\frac{4\pi}{\alpha}\right)^{\epsilon-2} K(x;a,b,c), \tag{5.46}$$

$$I(x;a,b) \equiv I(x;a,b;0,q) = \frac{1}{\alpha^{b-a}} \left(\frac{4\pi}{\alpha}\right)^{\epsilon-2} J(x;a,b), \tag{5.47}$$

where $K(x;a,b,c)$ and $J(x;a,b)$ with $b=1$ or $b=2$ are expressed as

$$K(x;a,1,c) = (-1)^a \Gamma(\epsilon-1)\zeta[\epsilon-a-c-1,x], \tag{5.48}$$

$$K(x;a,2,c) = (-1)^a (\epsilon-a-1)\Gamma(\epsilon-1)\zeta[\epsilon-a-c,x], \tag{5.49}$$

$$J(x;a,1) = (-1)^a \Gamma(\epsilon-1)\zeta[\epsilon-a-1,x], \tag{5.50}$$

$$J(x;a,2) = (-1)^a (\epsilon-a-1)\Gamma(\epsilon-1)\zeta[\epsilon-a,x]. \tag{5.51}$$

Note that $K(x;a,b,0)$ is reduced to $J(x;a,b)$.

If there are interactions between the field with different KK mode indices, we consider the following divergence structure of the quantum corrections to WL scalar mass as in section 5.7. The final results are given by

$$I_c(x,y;a,c) \equiv \sum_{n=0}^{\infty} \int \frac{d^4k}{(2\pi)^4} \frac{k^{2a}(\alpha(n+x))^c}{(k^2+\alpha(n+x))(k^2+\alpha(n+y))} \tag{5.52}$$

$$= -\frac{1}{\alpha(y-x)} \sum_{k=0}^{c-1} {}_cC_k (x-y)^{c-k} \alpha^{c-k} I(x;a,b,k) + \frac{1}{\alpha(y-x)} \frac{(-1)^a}{\alpha^{1-a-c}} \left(\frac{4\pi}{\alpha}\right)^{\epsilon-2} \Gamma(\epsilon-1) \sum_{m=0}^{y-x-1} \frac{(m+2x-y)^c}{(m+x)^{\epsilon-a-1}}. \tag{5.53}$$

In order for $I_c(x,y;a,c)$ to be finite, the following form is used:

$$I_c(x,x+1;a,c) = -\frac{1}{\alpha} \sum_{k=0}^{c-1} {}_cC_k (-1)^{c-k} \alpha^{c-k} I(x;a,1,k) + \frac{1}{\alpha} \frac{(-1)^a}{\alpha^{1-a}} \left(\frac{4\pi}{\alpha}\right)^{\epsilon-2} \Gamma(\epsilon-1) x^{a+1} (\alpha(x-1))^c. \tag{5.54}$$

Note that we define ${}_0C_k = 0$ if $c=0$ is taken.

Moreover, we define the polynomial version of the divergence structure:

$$
\begin{aligned}
I_{\text{poly}}(x;a)[\lambda_c,\cdots,\lambda_0] &\equiv \sum_{n=0}^{\infty}\int \frac{d^4k}{(2\pi)^4}\frac{k^{2a}f_{\text{poly}}(n,x)}{(k^2+\alpha(n+x))(k^2+\alpha(n+x+1))} \\
&= \lambda_c I_c(x,x+1;a,c)+\lambda_{c-1}I_{c-1}(x,x+1;a,c-1)+\cdots \\
&\quad +\cdots+\lambda_1 I_1(x,x+1;a,1)+\lambda_0 I_0(x,x+1;a,0), \qquad (5.55)
\end{aligned}
$$

$$
f_{\text{poly}}(n,x) = \lambda_c(\alpha(n+x))^c+\lambda_{c-1}(\alpha(n+x))^{c-1}+\cdots+\lambda_0(\alpha(n+x))^0. \qquad (5.56)
$$

5.9 Examples

By using above formula, we can compute the quantum corrections to WL scalar mass in previous chapters. In this section, we apply the above formula to the quantum corrections in previous chapters.

5.9.1 Scalar type

The quantum corrections with from the KK scalar field contributions have been seen in subsection 3.3.1, 4.4.2 with $\xi = 1$ or 4.4.3. By using eq.(5.46) and eq.(5.55), we first compute I_{b4pt} and I_{3bpt} in subsection 3.3.1:

$$
I_{b4pt} = -2ig^2|N|I(1/2;0,1,0), \qquad (5.57)
$$

$$
\begin{aligned}
I_{b3pt} &= +2ig^2|N|I_{\text{poly}}(1/2;0)[1,\alpha/2] \\
&= 2ig^2|N|\left(I_1(1/2,3/2;0,1)+\frac{\alpha}{2}I_0(1/2,3/2;0,0)\right) \\
&= 2ig^2|N|\left(I(1/2;0,1,0)-\frac{1}{4\alpha}\left(\frac{4\pi}{\alpha}\right)^{\epsilon-2}\Gamma(\epsilon-1)+\frac{1}{4\alpha}\left(\frac{4\pi}{\alpha}\right)^{\epsilon-2}\Gamma(\epsilon-1)\right) \\
&= 2ig^2|N|I(1/2;0,1,0). \qquad (5.58)
\end{aligned}
$$

For $I_1(1/2,3/2;0,1)$ and $I_0(1/2,3/2;0,0)$, we used eq.(5.54). Therefore, I_{b4pt} and I_{b3pt} are canceled.

Next, we consider eq.(4.135) and eq.(4.136). Eq.(4.135) is the sum of eq.(4.131) and eq.(4.132):

$$I_{\varphi 3} = \frac{16ig^2}{\Lambda^2}|N|\sum_{n=0}^{\infty}\int \frac{d^4p}{(2\pi)^4}\frac{p^2}{p^2+\alpha\left(n+\frac{1}{2}\right)} = \frac{16ig^2}{\Lambda^2}|N|I(1/2;1,1)$$
$$= -i\frac{g^2\alpha^2|N|}{\pi^2\Lambda^2}\left(\frac{4\pi}{\alpha}\right)^{\epsilon}\Gamma(\epsilon-1)\zeta[\epsilon-2,1/2], \tag{5.59}$$

where we used eq.(5.47) (or eq.(5.46) with $c = 0$). Also, eq.(4.136) is the sum of eq.(4.133) and eq.(4.134):

$$I_{\varphi 4} = \frac{16ig^2}{\Lambda^2}|N|\sum_{n=0}^{\infty}\int \frac{d^4p}{(2\pi)^4}\frac{\alpha n+\alpha(n+1)}{p^2+\alpha\left(n+\frac{1}{2}\right)} = \frac{16ig^2}{\Lambda^2}|N|\times 2I(1/2;0,1,1)$$
$$= i\frac{2g^2\alpha^2|N|}{\pi^2\Lambda^2}\left(\frac{4\pi}{\alpha}\right)^{\epsilon}\Gamma(\epsilon-1)\zeta[\epsilon-2,1/2], \tag{5.60}$$

where we used eq.(5.46). These results are consistent with eq.(4.135) and eq.(4.136).

Finally, we consider eq.(4.149) and eq.(4.150). Eq.(4.149) is the sum of eq.(4.145) and eq.(4.146). Using eq.(5.47), eq.(5.54) and eq.(5.55), $I_{\varphi 5}$ is reproduced by

$$I_{\varphi 5} = -\frac{16ig^2}{\Lambda^2}|N|\sum_{n=0}^{\infty}\int \frac{d^4p}{(2\pi)^4}\frac{\alpha(n+1)p^2}{\left(p^2+\alpha\left(n+\frac{1}{2}\right)\right)\left(p^2+\alpha\left(n+\frac{3}{2}\right)\right)}$$
$$= -\frac{16ig^2}{\Lambda^2}|N|I_{\text{poly}}(1/2;1)[1,\alpha/2]$$
$$= -\frac{16ig^2}{\Lambda^2}|N|\left(I_1(1/2,3/2;1,1)+\frac{\alpha}{2}I_0(1/2,3/2;1,0)\right)$$
$$= -\frac{16ig^2}{\Lambda^2}|N|\left(I(1/2;1,1)+\frac{1}{8}\left(\frac{4\pi}{\alpha}\right)^{\epsilon-2}\Gamma(\epsilon-1)-\frac{1}{8}\left(\frac{4\pi}{\alpha}\right)^{\epsilon-2}\Gamma(\epsilon-1)\right)$$
$$= +i\frac{g^2\alpha^2|N|}{\pi^2\Lambda^2}\left(\frac{4\pi}{\alpha}\right)^{\epsilon}\Gamma(\epsilon-1)\zeta[\epsilon-2,1/2]. \tag{5.61}$$

Also, by using eq.(5.47), eq.(5.54) and eq.(5.55), eq.(4.150), which is the sum of eq.(4.147) and eq.(4.148), is computed by

$$I_{\varphi 6} = -\frac{8ig^2}{\Lambda^2}|N|\sum_{n=0}^{\infty}\int \frac{d^4p}{(2\pi)^4}\frac{\alpha(n+1)\alpha\,(2n+2)}{\left(p^2+\alpha\left(n+\frac{1}{2}\right)\right)\left(p^2+\alpha\left(n+\frac{3}{2}\right)\right)}$$
$$= -\frac{16ig^2}{\Lambda^2}|N|I_{\text{poly}}(1/2;0)[1,\alpha,\alpha^2/4]$$
$$= -\frac{16ig^2}{\Lambda^2}|N|\left(I_2(1/2,3/2;0,2)+\alpha I_1(1/2,3/2;0,1)+\frac{\alpha^2}{4}I_0(1/2,3/2;0,0)\right). \tag{5.62}$$

Using eq.(5.54), $I_2(1/2,3/2;0,2)$, $I_1(1/2,3/2;0,1)$ and $I_0(1/2,3/2;0,0)$ are expressed as

$$I_2(1/2,3/2;0,2) = -\frac{1}{\alpha}\Big(\alpha^2 I(1/2;0,1,0) - 2\alpha I(1/2;0,1,1)\Big) + \frac{1}{8}\left(\frac{4\pi}{\alpha}\right)^{\epsilon-1}\Gamma(\epsilon-1), \tag{5.63}$$

$$I_1(1/2,3/2;0,1) = I(1/2;0,1,0) - \frac{1}{4\alpha}\left(\frac{4\pi}{\alpha}\right)^{\epsilon-1}\Gamma(\epsilon-1), \tag{5.64}$$

$$I_0(1/2,3/2;0,1) = \frac{1}{2\alpha^2}\left(\frac{4\pi}{\alpha}\right)^{\epsilon-1}\Gamma(\epsilon-1). \tag{5.65}$$

Therefore, using eq.(5.46), $I_{\varphi 6}$ is reproduced by

$$\begin{aligned} I_{\varphi 6} &= -\frac{16ig^2}{\Lambda^2}|N| \times 2I(1/2;0,1,1) \\ &= -i\frac{2g^2\alpha^2|N|}{\pi^2\Lambda^2}\left(\frac{4\pi}{\alpha}\right)^{\epsilon}\Gamma(\epsilon-1)\zeta[\epsilon-2,1/2]. \end{aligned} \tag{5.66}$$

5.9.2 fermion/gauge type

The quantum corrections from the KK fermion or the KK gauge field contributions have been seen in subsections 3.3.2 or 4.4.1. First, we again calculate eq.(3.4) in the subsection 3.3.2, which is the contribution from fermion loop.

$$I_f = +4ig^2|N|I_0(0,1;1,0) = 0, \tag{5.67}$$

where we used eq.(5.54). Note that $I_0(0,1;1,0)$ vanishes because of ${}_0C_k = 0$ and $x = 0$. Thus, we conclude that the contribution from fermion loop vanishes.

Next, we see $I^{(2)}_{A1}$ and $I^{(2)}_{A2}$ in the subsection 4.4.1 as an example. For simplicity, we take $\xi = 1$ in this subsection.

$$I^{(2)}_{A1} = -8ig^2|N|I(0;0,1), \tag{5.68}$$

$$\begin{aligned} I^{(2)}_{A2} &= 8ig^2|N|I_{\text{poly}}(0;0)[1,\alpha] = 8ig^2|N|\Big(I_1(0,1;0,1) + \alpha I_0(0,1;0,0)\Big) \\ &= 8ig^2|N|I(0;0,1). \end{aligned} \tag{5.69}$$

Note that $I_1(0,1;0,1)$ is reduced to $I(0;0,1)$ (or $I(0;0,1,0)$) and $I_0(0,1;0,0)$ vanishes because of ${}_0C_k = 0$ and $x = 0$. Thus, $I^{(2)}_{A1}$ and $I^{(2)}_{A2}$ are canceled. As above calculations, we show that $I^{(3)}_{A1} + I^{(3)}_{A2}$ vanishes.

Next, we consider I_{A3} and I_{A4} in the subsection 4.7. We compute eqs.(4.124) and (4.125) by using eq.(5.47) and eq.(4.126) and eq.(4.127) by using eq.(5.46):

$$I_{A3}^{(2)} = \frac{32ig^2}{\Lambda^2}|N|I(0;1,1) = -i\frac{2g^2\alpha^2|N|}{\pi^2\Lambda^2}\left(\frac{4\pi}{\alpha}\right)^{\epsilon}\Gamma(\epsilon-1)\zeta[\epsilon-2,0], \tag{5.70}$$

$$I_{A3}^{(3)} = \frac{32ig^2}{\Lambda^2}|N|I(1;1,1) = -i\frac{2g^2\alpha^2|N|}{\pi^2\Lambda^2}\left(\frac{4\pi}{\alpha}\right)^{\epsilon}\Gamma(\epsilon-1)\zeta[\epsilon-2,1], \tag{5.71}$$

$$I_{A4}^{(2)} = \frac{64ig^2}{\Lambda^2}|N|I(0;0,1,1) = i\frac{4g^2\alpha^2|N|}{\pi^2\Lambda^2}\left(\frac{4\pi}{\alpha}\right)^{\epsilon}\Gamma(\epsilon-1)\zeta[\epsilon-2,0], \tag{5.72}$$

$$I_{A4}^{(3)} = \frac{64ig^2}{\Lambda^2}|N|I(1;0,1,1) = i\frac{4g^2\alpha^2|N|}{\pi^2\Lambda^2}\left(\frac{4\pi}{\alpha}\right)^{\epsilon}\Gamma(\epsilon-1)\zeta[\epsilon-2,1]. \tag{5.73}$$

Noting that $\zeta[s,1]=\zeta[s,0]$ is satisfied, $I_{A3}=I_{A3}^{(2)}+I_{A3}^{(3)}$ and $I_{A4}=I_{A4}^{(2)}+I_{A4}^{(3)}$ are derived.

Finally, we deal with I_{A5} and I_{A6} in the subsection 4.7.2. By applying eq.(5.54) and eq.(5.55) to $I_{A5}^{(2)}$ and $I_{A5}^{(3)}$, we obtain

$$\begin{aligned} I_{A5}^{(2)} &= -\frac{32ig^2}{\Lambda^2}|N|I_{\text{poly}}(0;1)[1,\alpha] = \frac{32ig^2}{\Lambda^2}|N|\Big(I_1(0,1;1,1)+\alpha I_0(0,1;1,0)\Big) \\ &= \frac{32ig^2}{\Lambda^2}|N|\Big(I(0;1,1)+\alpha\times 0\Big) = i\frac{2ig^2\alpha^2|N|}{\pi^2\Lambda^2}\left(\frac{4\pi}{\alpha}\right)^{\epsilon}\Gamma(\epsilon-1)\zeta[\epsilon-2,0] \end{aligned} \tag{5.74}$$

$$\begin{aligned} I_{A5}^{(3)} &= -\frac{32ig^2}{\Lambda^2}|N|I_1(1,2;1,1) = -\frac{32ig^2}{\Lambda^2}|N|I(1;1,1) \\ &= i\frac{2ig^2\alpha^2|N|}{\pi^2\Lambda^2}\left(\frac{4\pi}{\alpha}\right)^{\epsilon}\Gamma(\epsilon-1)\zeta[\epsilon-2,1], \end{aligned} \tag{5.75}$$

where we apply eq.(5.47) to eq.(5.74) and eq.(5.75) in the last equality. Because of $\zeta[s,1]=\zeta[s,0]$, $I_{A5}=I_{A5}^{(2)}+I_{A5}^{(3)}$ is derived. Similarly, $I_{A6}^{(2)}$ and $I_{A6}^{(3)}$ are

$$\begin{aligned} I_{A6}^{(2)} &= -\frac{32ig^2}{\Lambda^2}|N|I_{\text{poly}}(0;0)[1,3\alpha/2,\alpha^2/2] \\ &= -\frac{32ig^2}{\Lambda^2}|N|\left(I_2(0,1;0,2)+\frac{3\alpha}{2}I_1(0,1;0,1)+\frac{\alpha^2}{2}I_0(0,1;0,0)\right) \\ &= -\frac{32ig^2}{\Lambda^2}|N|\left(-\frac{1}{\alpha}\Big(\alpha^2 I(0;0,1)-2\alpha I(0;0,1,1)\Big)+\frac{3\alpha}{2}I(0;0,1)+\frac{\alpha^2}{2}\times 0\right) \\ &= -\frac{32ig^2}{\Lambda^2}|N|\left(2I(0;0,1,1)+\frac{\alpha}{2}I(0;0,1)\right) \\ &= -i\frac{2g^2\alpha^2|N|}{\pi^2\Lambda^2}\left(\frac{4\pi}{\alpha}\right)^{\epsilon}\left(2\Gamma(\epsilon-1)\zeta[\epsilon-2,0]+\frac{1}{2}\Gamma(\epsilon-1)\zeta[\epsilon-1,0]\right), \end{aligned} \tag{5.76}$$

$$
\begin{aligned}
I_{A6}^{(3)} &= -\frac{32ig^2}{\Lambda^2}|N| I_{\text{poly}}(1;0)[1,\alpha/2,0] \\
&= -\frac{32ig^2}{\Lambda^2}|N| \Big(I_2(1,2;0,2) + \frac{\alpha}{2} I_1(1,2;0,1) \Big) \\
&= -\frac{32ig^2}{\Lambda^2}|N| \Big(-\frac{1}{\alpha}\Big(\alpha^2 I(1;0,1) - 2\alpha I(1;0,1,1)\Big) + \frac{\alpha}{2} I(1;0,1) \Big) \\
&= -\frac{32ig^2}{\Lambda^2}|N| \Big(2I(1;0,1,1) - \frac{\alpha}{2} I(1;0,1) \Big) \\
&= -i\frac{2g^2\alpha^2|N|}{\pi^2\Lambda^2} \left(2\Gamma(\epsilon-1)\zeta[\epsilon-2,1] - \frac{1}{2}\Gamma(\epsilon-1)\zeta[\epsilon-1,1] \right),
\end{aligned}
\tag{5.77}
$$

where eq.(5.47) and eq.(5.46) are used in the last equality of eq.(5.76) and eq.(5.77). Although $I_{A6}^{(2)}$ or $I_{A6}^{(3)}$ have a divergent term, the sum of $I_{A6}^{(2)}$ and $I_{A6}^{(3)}$ cancels the divergent term because of $\zeta[s,1] = \zeta[s,0]$ and then eq.(4.143) can be realized.

5.10 Nonvanishing finite WL scalar mass

In the section 5.4, we have classified the interaction terms generating finite quantum correction at one-loop. In this section, we focus on eq.(5.18) since it has no derivatives and is the simplest interaction term of all interaction terms in the section 5.4.

We consider the following Lagrangian given by eqs.(3.3), (3.5) and (5.18):

$$
\mathcal{L} = -\frac{1}{4}F_{MN}F^{MN} - D_M\Phi^* D^M\Phi + \kappa(\phi^*\Phi^*\Phi + \phi\Phi^*\Phi), \tag{5.78}
$$

where κ is a dimensionless coupling constant. ϕ involves the flux background $\langle\phi\rangle$ and the fluctuation φ as eq.(3.2). Thus, Lagrangian (5.78) is rewritten as

$$
\begin{aligned}
\mathcal{L} \supset &-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - D_\mu\Phi^* D^\mu\Phi - m_{\text{scalar}}^2\Phi^*\Phi \\
&- ig\sqrt{2\alpha}\varphi^*\Phi^* a^\dagger\Phi + ig\sqrt{2\alpha}\varphi\Phi^* a\Phi - 2g^2\varphi^*\varphi\Phi^*\Phi \\
&+ \kappa(\varphi^*\Phi^*\Phi + \varphi\Phi^*\Phi) + \kappa(\langle\phi^*\rangle\Phi^*\Phi + \langle\phi\rangle\Phi^*\Phi),
\end{aligned}
\tag{5.79}
$$

where we note that the unnecessary terms are omitted. To derive a four-dimensional effective Lagrangian by KK reduction, we need to use eq.(2.22) or eq.(2.23). Integrating over T^2, the four-dimensional effective Lagrangian is obtained by

$$\begin{aligned}\mathcal{L}_{4D} = &-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \partial^\mu\varphi^*\partial_\mu\varphi \\ &+ \sum_{n,j}\Bigg(-D_\mu\Phi^*_{n,j}D^\mu\Phi_{n,j} - \alpha\left(n+\frac{1}{2}\right)\Phi^*_{n,j}\Phi_{n,j} \\ &- ig\sqrt{2\alpha(n+1)}\varphi^*\Phi^*_{n+1,j}\Phi_{n,j} + ig\sqrt{2\alpha(n+1)}\varphi\Phi^*_{n,j}\Phi_{n+1,j} - 2g^2\varphi^*\varphi\Phi^*_{n,j}\Phi_{n,j} \\ &+ \kappa\varphi^*\Phi^*_{n,j}\Phi_{n,j} + \kappa\varphi\Phi^*_{n,j}\Phi_{n,j} + \kappa\left\langle\phi\right\rangle_I\Phi^*_{n,j}\Phi_{n,j} + \kappa\left\langle\phi^*\right\rangle_I\Phi^*_{n,j}\Phi_{n,j}\Bigg),\end{aligned} \tag{5.80}$$

where $\langle\phi\rangle_I$ and $\langle\phi^*\rangle_I$ are defined by

$$\langle\phi\rangle_I = \int_{T^2} dx^2\,\langle\phi\rangle\,\bar{\xi}_{n,j}\xi_{n',j'}, \quad \langle\phi^*\rangle_I = \int_{T^2} dx^2\,\langle\phi^*\rangle\,\bar{\xi}_{n,j}\xi_{n',j'}. \tag{5.81}$$

When $\langle\phi\rangle = f\bar{z}/\sqrt{2}$, $\langle\phi\rangle_I$ and $\langle\phi^*\rangle_I$ lead to zero because of odd function with respect to integral variables z or $\bar{z}$.

5.10.1 Diagrammatic computation

If $\kappa = 0$, we reproduce the result (3.47). On the other hand, we get a new quantum correction to WL scalar mass in the $\kappa \neq 0$ case. Computing the right diagram in figure 3.1, the result has

$$\begin{aligned}\mathcal{I} &= +i\kappa^2\sum_{n,j}\int\frac{d^4k}{(2\pi)^4}\frac{1}{\left(k^2+\alpha\left(n+\frac{1}{2}\right)\right)^2} = i\kappa^2|N|I(1/2;0,2) \\ &= \frac{i\kappa^2|N|}{\alpha^2}\left(\frac{4\pi}{\alpha}\right)^{\epsilon-2}\Gamma(\epsilon)\zeta[\epsilon,1/2] = -i\frac{\kappa^2|N|\ln 2}{32\pi^2}\left(\frac{4\pi}{\alpha}\right)^{\epsilon} + \mathcal{O}(\epsilon),\end{aligned} \tag{5.82}$$

where

$$\zeta[\epsilon,1/2] = 0 - \epsilon\frac{\ln 2}{2} \tag{5.83}$$

are used in the last equality of eq.(5.82). This correction (5.82) is finite in $\epsilon \to 0$ limit. Thus, the quantum correction to WL scalar mass at one-loop is given by

$$\delta m^2 = i\mathcal{I} = \frac{|N|\ln 2}{32\pi^2}\frac{\kappa^2}{L^2}. \tag{5.84}$$

Note that we introduced a factor of torus area L^2, which comes from the normalization factors for KK mode function. Obviously, we can also understand that $\delta m^2 = 0$ is reproduced for eq.(5.84) with $\kappa = 0$ in six-dimensional scalar QED (see subsection 3.3.1).

One of the interesting phenomenological applications is that the quantum correction δm^2 to WL scalar mass can be interpreted as Higgs mass. This idea is based on gauge-Higgs unification, namely a zero-mode of WL scalar φ is regarded as Higgs field. Even if the compactification scale is Planck scale $1/L \sim \mathcal{O}(M_{Planck})$, Higgs mass could be realized by the interaction term (5.18) generated by some dynamics at $\mathcal{O}(1)$ TeV scale. This is analogous to the mass of pion as a pseudo NG boson for chiral symmetry. The reason why the pion mass is not Planck scale is that chiral symmetry is dynamically broken at extremely lower energy scale comparing to the Planck scale, namely, QCD scale.

In this theory, the WL scalar cannot be actually identified with Higgs field in the SM since the WL scalar in this theory is not an SU(2) doublet. It would be studied that an SU(2) doublet is realized by the WL scalar field in six-dimensional SU(3) Yang-Mills theory and SU(3) gauge symmetry is broken to U(1)×U(1) or U(1) [39].

5.10.2 Effective potential analysis

We can calculate the quantum correction to WL scalar mass in terms of effective potential. In our Lagrangian (5.80), we read the KK mass spectrum of Φ to be $\alpha(n+1/2) - \kappa \langle\phi\rangle_I - \kappa \langle\phi^*\rangle_I$. Thus, the four-dimensional effective potential is given by

$$V = \sum_{n=0}^{\infty} \int \frac{d^4k}{(2\pi)^4} \ln\left(k^2 + \alpha\left(n + \frac{1}{2}\right) - \kappa\langle\phi\rangle_I - \kappa\langle\phi^*\rangle_I\right), \tag{5.85}$$

where we take into account a degree of freedom of complex scalar field Φ. To obtain the quantum correction to WL scalar mass from four-dimensional effective potential, we differentiate the effective potential with respect to $\langle\phi\rangle_I$ and $\langle\phi^*\rangle_I$. Thus, δm^2 is obtained as

$$\begin{aligned} \delta m^2 &= \left.\frac{\partial^2 V}{\partial\langle\phi\rangle_I \partial\langle\phi^*\rangle_I}\right|_{\langle\phi\rangle_I=0} \\ &= -\kappa^2 \sum_{n=0}^{\infty} \int \frac{d^4k}{(2\pi)^4} \frac{1}{\left(k^2 + \alpha\left(n+\frac{1}{2}\right)\right)^2} = i\mathcal{I}. \end{aligned} \tag{5.86}$$

This result (5.86) is consistent with eq.(5.82) or eq.(5.84).

5.10.3 WL scalar as a pseudo Nambu-Goldstone boson

We have seen that the zero-mode of WL scalar φ becomes a NG boson of translational invariance in extra spaces as in section 3.4 or 4.8 if $\kappa = 0$. This is the physical reason that the quantum correction to WL scalar mass vanishes. If $\kappa \neq 0$, $\varphi\Phi^*_{n,j}\Phi_{n,j}$ (or $\varphi^*\Phi^*_{n,j}\Phi_{n,j}$) in eq.(5.80) is expected to break the translational invariance explicitly. To confirm it, we consider the following local six-dimensional transformation [25]

$$\varphi' = \varphi - \frac{1}{\sqrt{2}}\partial\Lambda, \quad \Phi' = e^{g\Lambda}\Phi, \quad \Phi^{*\prime} = e^{-g\Lambda}\Phi^*, \tag{5.87}$$

where $\Lambda = f(\epsilon\bar{z} - \bar{\epsilon}z)$. Infinitesimal transformations of $\epsilon, \bar{\epsilon}$ are expressed as

$$\delta_\Lambda\varphi = -\frac{1}{\sqrt{2}}\partial\Lambda, \quad \delta_\Lambda\Phi = g\Lambda\Phi, \quad \delta_\Lambda\Phi^* = -g\Lambda\Phi^*. \tag{5.88}$$

Transformations of φ and Φ are the combination of translation δ_T and infinitesimal transformation δ_Λ,

$$\delta\varphi = (\delta_T + \delta_\Lambda)\varphi = \sqrt{2}f\bar{\epsilon}, \tag{5.89}$$

$$\delta\Phi = (\delta_T + \delta_\Lambda)\Phi = -i\sqrt{\alpha}(\epsilon a^\dagger + \bar{\epsilon}a)\Phi. \tag{5.90}$$

Using eq.(2.22) and eq.(3.40), we obtain

$$\delta\Phi = -i\sqrt{\alpha}\sum_{n,j}\Phi_{n,j}(\epsilon a^\dagger + \bar{\epsilon}a)\xi_{n,j} = \sum_{n,j}\delta\Phi_{n,j}\xi_{n,j}, \tag{5.91}$$

$$\delta\Phi_{n,j} = -i\sqrt{\alpha}(\epsilon\sqrt{n+1}\Phi_{n+1,j} + \bar{\epsilon}\sqrt{n}\Phi_{n-1,j}). \tag{5.92}$$

For $\delta\Phi^*_{n,j}$, it is given by complex conjugate of eq.(5.92),

$$\delta\Phi^*_{n,j} = +i\sqrt{\alpha}(\bar{\epsilon}\sqrt{n+1}\Phi^*_{n+1,j} + \epsilon\sqrt{n}\Phi^*_{n-1,j}). \tag{5.93}$$

Let us confirm the explicit breaking of translational invariance of the interaction term $\varphi\Phi^*_{n,j}\Phi_{n,j}$. First, a transformation of $\Phi^*_{n,j}\Phi_{n,j}$ is

$$\begin{aligned}\delta\left(\sum_{n,j}\Phi^*_{n,j}\Phi_{n,j}\right) &= i\sqrt{\alpha}\sum_{n,j}\Big(\bar{\epsilon}\sqrt{n+1}\Phi^*_{n+1,j}\Phi_{n,j} + \epsilon\sqrt{n}\Phi^*_{n-1,j}\Phi_{n,j} \\ &\qquad - \epsilon\sqrt{n+1}\Phi^*_{n,j}\Phi_{n+1,j} - \bar{\epsilon}\sqrt{n}\Phi^*_{n,j}\Phi_{n-1,j}\Big) \\ &= 0,\end{aligned} \tag{5.94}$$

by the shift $n \to n+1$. Thus, the mass term of $\Phi_{n,j}$ is invariant. For $\varphi\Phi^*_{n,j}\Phi_{n,j}$, a transformation is

$$\delta\left(\sum_{n,j}\varphi\Phi^*_{n,j}\Phi_{n,j}\right) = (\delta\varphi)\sum_{n,j}\Phi^*_{n,j}\Phi_{n,j} + \varphi\delta\left(\sum_{n,j}\Phi^*_{n,j}\Phi_{n,j}\right)$$
$$= \sqrt{2}f\bar{\epsilon}\sum_{n,j}\Phi^*_{n,j}\Phi_{n,j} \neq 0. \tag{5.95}$$

This result means the explicit breaking of translational invariance in extra spaces. For $\kappa \neq 0$, the zero-mode of WL scalar is identified with a pseudo NG boson of translational invariance in extra spaces.

One might claim that the interaction terms (5.18) are not gauge invariant since φ or φ^* transforms under the gauge symmetry as eq.(5.87). In order to overcome such a claim, φ or φ^* should be expressed by a gauge invariant non-local Wilson line operator and the interaction terms (5.18) should be regarded as one of the terms of expanding the Wilson line operators in small φ or φ^*. Noting that the Wilson line operators[3]

$$U_5 = \exp\left[ig\oint A_5 dx^5\right], \quad U_6 = \exp\left[ig\oint A_6 dx^6\right] \tag{5.96}$$

can be written in terms of φ, φ^* and $z, \bar{z}$ as

$$U_5 = \exp\left[\frac{g}{\sqrt{2}}\oint(\varphi dz + \varphi d\bar{z} - \varphi^* dz - \varphi^* d\bar{z})\right], \tag{5.97}$$
$$U_6 = \exp\left[\frac{g}{\sqrt{2}}\oint(\varphi dz - \varphi d\bar{z} + \varphi^* dz - \varphi^* d\bar{z})\right], \tag{5.98}$$

we find that the cubic terms introduced in this thesis can be expressed by the non-local Wilson line operators

$$i(U_5 - U_5^\dagger)\Phi^*\Phi - i(U_6 - U_6^\dagger)\Phi^*\Phi \supset 2\sqrt{2}ig\oint \varphi d\bar{z}\Phi^*\Phi - 2\sqrt{2}ig\oint \varphi^* dz\Phi^*\Phi$$
$$= 2\sqrt{2}ig_4\varphi\Phi^*\Phi - 2\sqrt{2}ig_4\varphi^*\Phi^*\Phi \tag{5.99}$$

where g_4 is a gauge coupling constant in four dimensions. Note that the $\Phi^*\Phi$ term cannot be included in (5.99). If this term is allowed, the WL scalar mass would be divergent.

We note how the finite WL scalar mass can be expressed in terms of the Wilson line operators. If the WL scalar mass is generated in the broken phase, where the VEV

[3] In non-Abelian case, the path ordering must be taken into account, $U_{5,6} = P\exp[ig\oint A_{5,6}dx^{5,6}]$.

of the WL scalar field is non-zero, it is straightforward to express the WL scalar mass by the Wilson line operators as in the gauge-Higgs unification. As for the WL scalar field mass in the present thesis, the mass is generated in the unbroken phase and is independent of the VEV of the WL scalar field. Therefore, we cannot express the WL scalar field mass by the Wilson line operators explicitly.

Under the constant shift of $A_5 \to A_5 - f\epsilon_6/2, A_6 \to A_6 + f\epsilon_5/2$, the operators

$$U_5 - U_5^\dagger = 2i\sin\left[g\oint A_5 dx^5\right], \quad U_6 - U_6^\dagger = 2i\sin\left[g\oint A_6 dx^6\right] \tag{5.100}$$

are not obviously invariant, which means that the interaction terms (5.99) explicitly break the shift symmetry. It is not easy to clarify the origin of the interaction terms eq.(5.99), which is beyond the scope of this thesis. We expect that the origin of the interaction terms might be connected to the quantum gravity effects, nontrivial backgrounds such as a vortex, or some non-perturbative dynamics.

Chapter 6

Application to inflationary theory

In this chapter, we propose a new inflation scenario in flux compactification [31]. In this scenario, we identify a zero mode WL scalar field of extra components of the higher dimensional gauge field with an inflaton. Following the section 5.10, we give an explicit inflation model in a six-dimensional scalar QED, which is shown to be consistent with Planck 2018 data.

6.1 Setup and one-loop effective potential

Inflation is a very attractive scenario to expand the space in the early universe exponentially and to solve many problems (for example, Horizon problem and flatness problem) in the standard Big Bang cosmology. Its existence is supported by observations of cosmological parameters [32]. Although inflation has been considered to happen by a scalar field called as inflaton, there is still no compelling model of inflation. In a slow-roll scenario of the inflation, the scalar potential is required to be flat and stable under quantum corrections, which usually causes an unnatural fine-tuning of parameters of the theory unless we have some dynamics or symmetry to control the inflaton dynamics. For instance, the inflaton in natural inflation [40, 41] is identified with the pseudo Nambu-Goldstone boson of some global symmetry. In extranatural inflation [42], the inflaton is identified with the WL scalar field of the gauge field in higher dimensions without magnetic flux. In [43], the inflaton and the curvaton are identified with the WL scalar fields in a six-dimensional gauge theory.

Our setup in this chapter is the same as the section 5.10. In particular, we follow eq.(5.78) or eq.(5.80). By using $g_4 = g_6/L$ which is a four-dimensional gauge coupling constant and $\kappa_4 = \kappa_6/L$ which is a four-dimensional coupling constant, eq.(5.80) is rewritten as

$$\begin{aligned}\mathcal{L}_{4D} = &-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \partial^\mu\varphi^*\partial_\mu\varphi \\ &+\sum_{n,j}\left(-D_\mu\Phi^*_{n,j}D^\mu\Phi_{n,j} - \alpha\left(n+\frac{1}{2}\right)\Phi^*_{n,j}\Phi_{n,j}\right. \\ &- ig_4\sqrt{2\alpha(n+1)}\varphi^*\Phi^*_{n+1,j}\Phi_{n,j} + ig_4\sqrt{2\alpha(n+1)}\varphi\Phi^*_{n,j}\Phi_{n+1,j} - 2g_4^2\varphi^*\varphi\Phi^*_{n,j}\Phi_{n,j} \\ &\left.+\kappa_4\varphi^*\Phi^*_{n,j}\Phi_{n,j} + \kappa_4\varphi\Phi^*_{n,j}\Phi_{n,j} + \kappa_4\langle\phi\rangle_I\Phi^*_{n,j}\Phi_{n,j} + \kappa_4\langle\phi^*\rangle_I\Phi^*_{n,j}\Phi_{n,j}\right), \qquad (6.1)\end{aligned}$$

where $\alpha = 2gf$. Following [42], we regard WL scalar φ as an inflaton in this chapter.

One-loop effective potential depending on φ can be described as

$$V(\varphi,\varphi^*) = N\sum_{n=0}^{\infty}\int\frac{d^4k}{(2\pi)^4}\ln\left(k^2 + \alpha\left(n+\frac{1}{2}\right) + M^2(\varphi,\varphi^*)\right), \qquad (6.2)$$

where we have taken into account loop contributions from the bulk scalar field Φ. N is a number of the degeneracy, and $M^2(\varphi,\varphi^*)$ is a field-dependent mass for the bulk scalar field Φ.

As for $M^2(\varphi,\varphi^*)$, we consider two limiting cases for a free parameter $U(1)$ gauge coupling, namely $g_4 \ll 1$ and $g_4 \gg 1$. For that purpose, we read $M^2(\varphi,\varphi^*)$ from eq.(6.1) as

$$M^2(\varphi,\varphi^*) = -\kappa_4\varphi^* - \kappa_4\varphi + 2g_4^2\varphi^*\varphi. \qquad (6.3)$$

While only the first two terms in eq.(6.3) are considered in the $g_4 \ll 1$ case, the last term proportional to g_4^2 in eq.(6.3) is also considered in the $g_4 \gg 1$ case in addition to the first two terms. In the case of $g_4 \simeq \mathcal{O}(1)$, the terms linear in g_4 in eq.(6.1) should be also taken into account in $M^2(\varphi,\varphi^*)$. However, the obtained eigenvalues of $M^2(\varphi,\varphi^*)$ become complicated and makes the computation of the effective potential hard. Therefore, we do not discuss this case in this thesis.

We can express the effective potential by using Schwinger representation as

$$\begin{aligned}V &= -N\sum_{n=0}^{\infty}\int\frac{d^4k}{(2\pi)^4}\int_0^\infty\frac{dt}{t}e^{-k^2t-\alpha\left(n+\frac{1}{2}\right)t}e^{-M^2(\varphi,\varphi^*)t} \\ &= -N\frac{1}{16\pi^2}\int_0^\infty\frac{dt}{t^3}\frac{e^{-\frac{\alpha}{2}t}}{1-e^{-\alpha t}}e^{-M^2(\varphi,\varphi^*)t}. \qquad (6.4)\end{aligned}$$

To proceed a calculation of the effective potential further, we focus on an integral representation of Hurwitz zeta function [1]

$$\zeta[s,a] = \frac{1}{\Gamma(s)} \int_0^\infty dt \frac{t^{s-1}e^{-at}}{1-e^{-t}}, \quad \mathrm{Re}\ s > 1. \tag{6.5}$$

Then, the effective potential and its derivatives by φ can be expressed by

$$V = -N\frac{\alpha^2}{16\pi^2}\lim_{\epsilon\to 0}\Gamma(\epsilon-2)\zeta\left[\epsilon-2, \frac{1}{2}+\frac{1}{\alpha}M^2(\varphi,\varphi^*)\right], \tag{6.6}$$

$$V_\varphi = -N\frac{\alpha\kappa}{16\pi^2}\lim_{\epsilon\to 0}\Gamma(\epsilon-1)\zeta\left[\epsilon-1, \frac{1}{2}+\frac{1}{\alpha}M^2(\varphi,\varphi^*)\right], \tag{6.7}$$

$$V_{\varphi\varphi^*} = -N\frac{\kappa^2}{16\pi^2}\lim_{\epsilon\to 0}\Gamma(\epsilon)\zeta\left[\epsilon, \frac{1}{2}+\frac{1}{\alpha}M^2(\varphi,\varphi^*)\right], \tag{6.8}$$

where a parameter ϵ is introduced to regularize the integral of t. In particular, we can check that the $\epsilon \to 0$ limit indeed agrees with the results in case of $M^2(\varphi,\varphi^*) = 0$ obtained in section 5.10 by diagrammatic calculations using the dimensional regularization.

In the $g_4 \ll 1$ case, we ignore $2g_4^2\varphi^*\varphi$ in eq.(6.3) as mentioned above. For convenience, we define the dimensionless variables in a four-dimensional sense as

$$z = \frac{\varphi}{M_P}, \quad y = M_P\frac{\kappa_4}{\alpha}, \tag{6.9}$$

$M^2(\varphi,\varphi^*)/\alpha$ is then expressed by

$$\frac{1}{\alpha}M^2(\varphi,\varphi^*) = -(z+z^*)y = -2xy, \quad \mathrm{Re}\ z = x. \tag{6.10}$$

Thus, the effective potential is rewritten by

$$V = -N\frac{\alpha^2}{16\pi^2}\lim_{\epsilon\to 0}\Gamma(\epsilon-2)\zeta\left[\epsilon-2, \frac{1}{2}-2xy\right], \tag{6.11}$$

and the effective potential is shown in figure 6.1. If $L \sim M_P^{-1}$, the effective potential is close to flat as y (or κ_4) takes smaller value. Taking into account for the consistency with the original theory [24, 25], the small value of y is favored. If $y \ll 1$, κ_4 is small, which is independent of g_4. This implies that linear terms in g_4 can be neglected because we can always take $g_4 \ll \kappa_4 L$.

[1] For analysis of the effective potential using Hurwitz zeta function, an interesting study is done in [39].

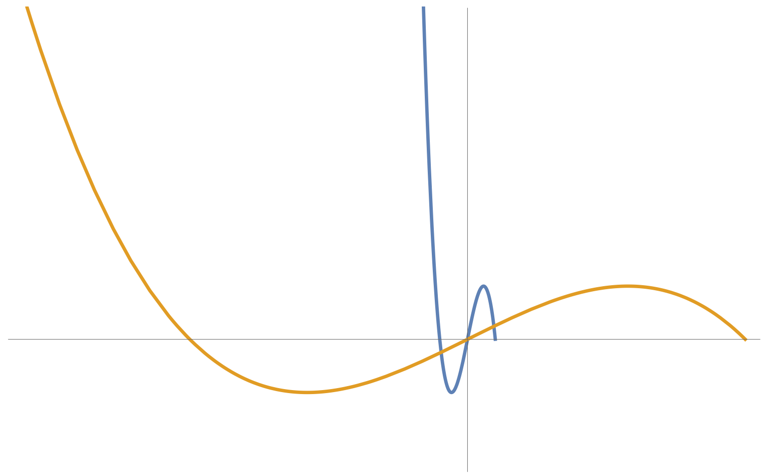

Figure 6.1: Schematic picture of the effective potential in the case of $g_4 \ll 1$. The blue and yellow lines shows $y = 1.0 \times 10^0$, $y = 1.0 \times 10^{-1}$ respectively.

In the $g_4 \gg 1$ case, $M^2(\varphi, \varphi^*)/\alpha$ is expressed by

$$\begin{aligned}\frac{1}{\alpha}M^2(\varphi,\varphi^*) &= -(z+z^*)y + 2\frac{g_4^2 M_P^2}{\alpha}|z|^2 \\ &= -2uy + 2G(u^2+v^2), \end{aligned} \tag{6.12}$$

where $z \equiv u + iv$ and $G \equiv g_4^2 M_P^2/\alpha$ are defined in the second equality. Note that G is almost an order of g_4^2 because α is independent of g_4. Setting $u = v$ for simplicity, the effective potential is given by

$$V = -N\frac{\alpha^2}{16\pi^2}\lim_{\epsilon\to 0}\Gamma(\epsilon-2)\zeta\left[\epsilon-2, \frac{1}{2} - 2uy + 4Gu^2\right], \tag{6.13}$$

which is shown in figure 6.2. This effective potential in the case of $g_4 \gg 1$ behaves as $V \propto \Gamma[\epsilon - 2]\zeta[\epsilon - 2, 4Gu^2]$. Comparing with the potential in figure 6.1, it seems difficult to apply the potential in figure 6.2 to an inflation model.

6.2 Inflationary parameters

Using the four-dimensional effective potential for the WL scalar field (6.6), we propose a cosmological inflation model in flux compactifiaction, where the WL scalar field is identified with an inflaton.

Figure 6.2: Schematic picture of the effective potential in the case of $g_4 \gg 1$. We take $y = 1$ for simplicity. The yellow and blue lines shows $G = 1.0 \times 10^2$, $G = 1.0 \times 10^3$ respectively.

Slow-roll parameters ϵ and η in our model are given by

$$\epsilon \equiv \frac{M_P^2}{2}\left(\frac{V_\varphi}{V}\right)^2 = \frac{M_P^2}{2}\left(\frac{\kappa_4}{\alpha}\lim_{\epsilon\to 0}\frac{\Gamma(\epsilon-1)}{\Gamma(\epsilon-2)}\frac{\zeta\left[\epsilon-1, \frac{1}{2}+\frac{1}{\alpha}M^2(\varphi,\varphi^*)\right]}{\zeta\left[\epsilon-2, \frac{1}{2}+\frac{1}{\alpha}M^2(\varphi,\varphi^*)\right]}\right)^2, \tag{6.14}$$

$$\eta \equiv M_P^2\frac{V_{\varphi\varphi^*}}{V} = M_P^2\left(\frac{\kappa_4^2}{\alpha^2}\lim_{\epsilon\to 0}\frac{\Gamma(\epsilon)}{\Gamma(\epsilon-2)}\frac{\zeta\left[\epsilon, \frac{1}{2}+\frac{1}{\alpha}M^2(\varphi,\varphi^*)\right]}{\zeta\left[\epsilon-2, \frac{1}{2}+\frac{1}{\alpha}M^2(\varphi,\varphi^*)\right]}\right). \tag{6.15}$$

Using eq.(D.6), we can further simplify eq.(6.14) and eq.(6.15),

$$\epsilon = \frac{y^2}{2}\left(-2\frac{\zeta\left[-1, \frac{1}{2}+\frac{1}{\alpha}M^2(\varphi,\varphi^*)\right]}{\zeta\left[-2, \frac{1}{2}+\frac{1}{\alpha}M^2(\varphi,\varphi^*)\right]}\right)^2 = \frac{9y^2}{2}\left(\frac{B_2(\frac{1}{2}+\frac{1}{\alpha}M^2(\varphi,\varphi^*))}{B_3(\frac{1}{2}+\frac{1}{\alpha}M^2(\varphi,\varphi^*))}\right)^2, \tag{6.16}$$

$$\eta = y^2\left((-1)(-2)\frac{\zeta\left[0, \frac{1}{2}+\frac{1}{\alpha}M^2(\varphi,\varphi^*)\right]}{\zeta\left[-2, \frac{1}{2}+\frac{1}{\alpha}M^2(\varphi,\varphi^*)\right]}\right) = 6y^2\frac{B_1(\frac{1}{2}+\frac{1}{\alpha}M^2(\varphi,\varphi^*))}{B_3(\frac{1}{2}+\frac{1}{\alpha}M^2(\varphi,\varphi^*))}. \tag{6.17}$$

Slow-roll conditions to realize inflation require $\epsilon \ll 1, |\eta| \ll 1$.

The number of e-folding before the end of inflation is

$$N_* = \frac{1}{M_P^2}\int_{\varphi_f}^{\varphi_*}\frac{V}{V_\varphi}d\varphi = \frac{2}{3y}\int_{\varphi_*}^{\varphi_f}\frac{B_3\left(\frac{1}{2}+\frac{1}{\alpha}M^2(\varphi,\varphi^*)\right)}{B_2\left(\frac{1}{2}+\frac{1}{\alpha}M^2(\varphi,\varphi^*)\right)}d\varphi. \tag{6.18}$$

To solve the horizon and flatness problems, the number of e-folding N_* has to be at least $50 < N_* < 60$. φ_f is the value of the end of inflation determined by $\epsilon(\varphi_f) = 1$, which violates the slow-roll conditions. φ_* is determined so that the e-folding can satisfy $50 < N_* < 60$.

The spectral index and the tensor-to-scalar ratio are given in a slow-roll approximation as

$$n_s = 1 - 6\epsilon + 2\eta, \quad r = 16\epsilon. \tag{6.19}$$

Planck 2018 data [32] gives constraints on $n_s = 0.9649 \pm 0.0042$ and $r < 0.10$.

6.3 Numerical results

In this section, our numerical results are shown.

6.3.1 $g_4 \ll 1$ case

In this case, $M^2(\varphi, \varphi^*)/\alpha$ corresponds to eq.(6.10), where the slow-roll parameters ϵ and η are provided by

$$\epsilon = \frac{9y^2}{2}\left(\frac{B_2(\frac{1}{2} - 2xy)}{B_3(\frac{1}{2} - 2xy)}\right)^2, \quad \eta = 6y^2 \frac{B_1(\frac{1}{2} - 2xy)}{B_3(\frac{1}{2} - 2xy)}. \tag{6.20}$$

To compute the e-folding N_*, we need to know the value of end of inflation $x_f = \varphi_f/M_P$, which is determined by the condition of the end of inflation $\epsilon(x_f) = 1$. The number of e-folding is

$$N_* = \frac{2}{3y}\int_{x_i}^{x_f} \frac{B_3(\frac{1}{2} - 2xy)}{B_2(\frac{1}{2} - 2xy)} dx, \tag{6.21}$$

where $x_i = \mathrm{Re}\varphi_*/M_P$. Sample of our numerical solutions x_i, x_f, N_* at some points of y are shown in Table 6.1, where the e-folding $N_* = 50, 60$ are taken. One might think that our results are not reliable since the value of the WL scalar field is quite larger than the Planck scale, which is beyond an applicability of the effective field theory. However, the gauge symmetry in our theory is not broken by quantum gravity effects and forbids any dangerous higher dimensional local operators suppressed by the Planck scale as well as the non-derivative local operators of the WL scalar field. Therefore, our results are reliable.

	y	x_i	x_f	N_*
A_{50}	1.0×10^{-2}	-30.4669	-25.3611	50.002
A_{60}	1.0×10^{-2}	-31.0285	-25.3611	60.0098
B_{50}	5.0×10^{-3}	-55.2516	-50.3573	50.0016
B_{60}	5.0×10^{-3}	-55.7738	-50.3573	60.0004
C_{50}	1.0×10^{-3}	-255.063	-250.354	50.0144
C_{60}	1.0×10^{-3}	-255.549	-250.354	60.0162
D_{50}	5.0×10^{-4}	-505.038	-500.354	50.0099
D_{60}	5.0×10^{-4}	-505.519	-500.354	60.0087
E_{50}	1.0×10^{-4}	-2505.018	-2500.35	50.0098
E_{60}	1.0×10^{-4}	-2505.495	-2500.35	60.0078

Table 6.1: Sample of our numerical solutions x_i, x_f, N_* at some points of y.

Using the numerical solutions in table 6.1, the slow-roll parameters ϵ, η, the spectral index n_s, and the scalar-to-tensor ratio r are calculated and shown in table 6.2. Comparing our results in table 6.2 with n_s and r in Planck 2018 data, our results are found to be relatively good agreement with the data. If y is taken to be a large value such as $y = 1.0 \times 10^2$, n_s and r cannot be satisfied with Planck 2018 data.

	ϵ	η	n_s	r
A_{50}	0.00683107	0.00494671	0.968907	0.109297
A_{60}	0.00582958	0.00444092	0.973904	0.0932733
B_{50}	0.00594149	0.00271376	0.969779	0.0950639
B_{60}	0.00502904	0.00245613	0.974738	0.0804646
C_{50}	0.00517205	0.000586594	0.970141	0.0827528
C_{60}	0.00432933	0.000534704	0.975093	0.0692693
D_{50}	0.00507359	0.000296245	0.970151	0.0811775
D_{60}	0.00423959	0.000270297	0.975103	0.0678334
E_{50}	0.00499408	0.0000597248	0.970155	0.0799053
E_{60}	0.00416705	0.0000545352	0.975107	0.0666728

Table 6.2: Inflation parameters ϵ, η, n_s, r obtained from our model.

Our results are shown in (n_s, r) plot of figure 6.3 from Planck 2018 data [32]. Orange circles are our results where small and large ones correspond to $N_* = 50$ and $N_* = 60$, respectively. As the parameter y is decreased, our results in (n_s, r) plot go downward. Our results are within a parameter region indicating the combining data of Planck TT, TE, EE+lowE+lensing at CL95%.

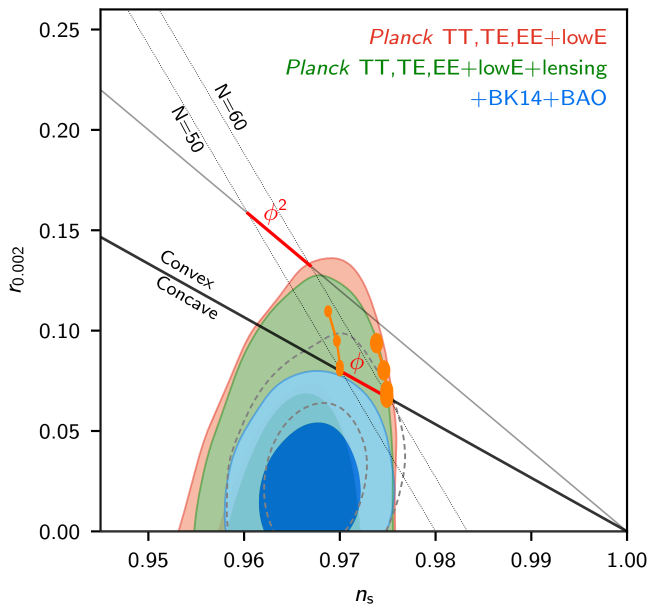

Figure 6.3: Our results (table 6.2) in the n_s-r plot from Planck 2018 data [32]. Orange circles are our results, where the small and large ones represent $N_* = 50$ and $N_* = 60$, respectively.

From the parameter y, we can estimate the value of κ_6, which provides the compactification scale and the 6D Planck scale M_{6P} in our model. $\kappa_6 L$ is determined by y

	L [GeV^{-1}]($\kappa_6 = 10^x L$)	$M_{6P} = \sqrt{M_P/L}$
$y = 1.0 \times 10^{-2}$	$3.20941 \times 10^{-x/2-10}$	$1.9497 \times 10^{x/4+14}$
$y = 5.0 \times 10^{-3}$	$2.26939 \times 10^{-x/2-10}$	$2.3186 \times 10^{x/4+14}$
$y = 1.0 \times 10^{-3}$	$1.0149 \times 10^{-x/2-10}$	$3.46711 \times 10^{x/4+14}$
$y = 5.0 \times 10^{-4}$	$7.17646 \times 10^{-x/2-11}$	$4.12311 \times 10^{x/4+14}$
$y = 1.0 \times 10^{-4}$	$3.20941 \times 10^{-x/2-11}$	$6.16549 \times 10^{x/4+14}$

Table 6.3: The value of L, M_{6P}.

as follows.

$$y = M_P \frac{\kappa_4}{\alpha} = M_P \frac{\kappa_6 / L}{4\pi N / L^2} = \frac{M_P L \kappa_6}{4\pi N}$$
$$\Leftrightarrow \kappa_6 = 4\pi N \frac{y}{M_P L} \tag{6.22}$$

where the number of degeneracy is assumed to be $N = 10$. If we assume $\kappa_6 = m_{inflaton}L$ and $m_{inflaton} = 10^x$ GeV, L is estimated. $\kappa_6 L,\ L,\ M_{6P}$ are shown in table 6.3.

Now, we discuss how small the gauge coupling g_4 is required for a successful inflation. Although the gauge coupling itself g_4 is a free parameter, the constraint from slow-roll parameter condition can be obtained through the coupling constant κ_6, which can be derived from $\epsilon \ll 1$,

$$\frac{3}{\sqrt{2}} y B_2(1/2 - 2x_i y) \ll B_3(1/2 - 2x_i y),$$

which implies,

$$y \ll \sqrt{\frac{2\sqrt{2}x_i + 1}{4(12x_i^2 + 8\sqrt{2}x_i^3)}}. \tag{6.23}$$

In the condition $\epsilon \ll 1$, $y \ll 1$ is immediately found. Therefore, we obtain $\kappa_6 \ll 10^{-19}/L$, which means $\kappa_6 \ll 1$ because the maximum value of $10^{-19}/L$ is at $1/L \sim M_P$. As mentioned in section 6.1, we can always take the free parameter g_4 less than κ_6. For a successful inflation in our model, we have only to take the free parameter gauge coupling g_4 such that $g_4 \ll \kappa_6$ and this can be always possible.

6.3.2 $g_4 \gg$1 case

In this case, $M^2(\varphi, \varphi^*)$ corresponds to (6.12). Under the simplification $u = v$, ϵ and η are expressed by

$$\epsilon = \frac{9y^2}{2}\left(\frac{B_2(\frac{1}{2} + 4Gu^2)}{B_3(\frac{1}{2} + 4Gu^2)}\right)^2, \quad \eta = 6y^2 \frac{B_1(\frac{1}{2} + 4Gu^2)}{B_3(\frac{1}{2} + 4Gu^2)}, \tag{6.24}$$

where we ignore $-2uy$ because y is small. The number of e-folding is

$$N_* = \frac{2}{3y}\int_{u_i}^{u_f} \frac{B_3(\frac{1}{2} + 4Gu^2)}{B_2(\frac{1}{2} + 4Gu^2)} du. \tag{6.25}$$

As in the $g_4 \ll 1$ case, we obtain the value of u_i and u_f for a value of G. Taking $G = 1,0 \times 10^3$ as an example, we find $u_i = -3.8403$ and $u_f = -0.7282$. Using these values, n_s and r are $n_s = 0.99569$ and $r = 0.0206896$. r is consistent with Planck 2018 constraint, but n_s is not. Thus, comparing with the potential in $g_4 \ll 1$ case, the potential in the $g_4 \gg 1$ case is not suitable for the inflation.

6.3.3 The vacuum energy during inflation

In order for our model to be consistent with inflationary setup, the vacuum energy during inflation should be smaller than 4D Planck scale and the compactification scale. We verify this requirement. As you can see from table 6.1, $x_f y$ takes $1/4$ during inflation. Thus, the vacuum energy becomes

$$\begin{aligned} V_{vac} = \langle V \rangle &= -N \frac{\alpha^2}{16\pi^2} \lim_{\epsilon \to 0} \Gamma(\epsilon - 2)\zeta[\epsilon - 2, 1/2] \\ &= -N \frac{\alpha^2}{16\pi^2} \left(\frac{1}{2} \zeta^{(1,0)}[-2, 1/2] \right) \\ &= -\frac{3N^3\zeta(3)}{32\pi^2} \frac{1}{L^4} \end{aligned} \tag{6.26}$$

where we take into account in the second equality that the VEV of inflaton field is zero during inflation, and

$$\zeta^{(1,0)}[-2, 1/2] = \frac{2\zeta(3)}{16\pi^2} \tag{6.27}$$

is used in the third equality. Setting $N = 10$, V_{vac} is estimated to be $\mathcal{O}(10) \times L^{-4} \sim \mathcal{O}(10) \times \left(\frac{M_{6P}}{M_P}\right)^4 M_{6P}^4$. In large extra dimensions, 6D Planck scale is smallar than 4D Planck scale $M_{6P} < M_P$, unless the compactification scale is the 4D Planck scale. Therefore, $1/L^4 < |V_{vac}| < M_P^4$ are satisfied as long as the compactification scale is smaller than 4D Planck scale.

Chapter 7

Conclusion

In this thesis, we have considered six-dimensional field theories with magnetic flux compactification toward the approach to the hierarchy problem. In chapter 2, we gave a basis of flux compactification. Based on quantum mechanics in magnetic field, we discussed a six-dimensional field theory with flux compactification. The key in flux compactification is that Kaluza-Klein mass is discretized such as Landau level. It is also a feature to identify covariant derivatives in extra spaces with creation and annihilation operators.

In chapter 3, we reviewed Abelian gauge theories in six dimensions without or with flux [24, 25]. We first discussed Abelian gauge theories in six dimensions without flux (scalar QED and QED), and then we obtained the finite quantum corrections to WL scalar mass at one-loop (see eq.(3.36) or eq.(3.37)). Next, we considered Abelian gauge theories in six dimensions with flux (scalar QED and QED). In these theories, the quantum corrections to WL scalar mass vanished for the sake of magnetic flux (see eq.(3.62)). At the end of this chapter, we mentioned that the physical reason of this cancellation is the shift symmetry from translation in compact spaces, which forbids the mass term of scalar field.

In chapter 4, we extended Abelian gauge theories to non-Abelian gauge theories and also calculated the quantum corrections to WL scalar mass. Concretely, we consider a six-dimensional SU(2) Yang-Mills theory with flux compactification. After deriving the four-dimensional effective Lagrangian, we computed the quantum corrections to WL scalar mass. As in the previous chapter, we showed that the quantum corrections to WL scalar mass vanish. Moreover, we added the higher dimensional operators and showed that the quantum corrections to WL scalar mass also vanish at the first order of $1/\Lambda^2$ (Λ is a cutoff scale).

In chapter 5, we studied possibilities to realize a nonvanishing finite WL scalar mass in flux compactification. We analyzed the generalized loop integrals in the quantum correction to WL scalar mass at one-loop. Then, the conditions for the loop integrals and mode sum in one-loop corrections to WL scalar mass to be finite could be obtained. From these conditions, we guessed the four-point and three-point interaction terms satisfying this conditions. Moreover, an argument was generalized to the quantum corrections from the interactions between the different KK modes. Finally, we considered the Lagrangian (5.78) and illustrated the finite quantum correction to the WL scalar mass (see eq.(5.84)).

In chapter 6, we proposed an inflation scenario in flux compactification as an application of the result in chapter 5. We identified a zero mode WL scalar field of extra components of the higher dimensional gauge field with an inflaton in this chapter. We gave the four-dimensional effective potential and calculated the inflationary parameters. The spectral index and the tensor-scalar ratio were computed in our model, and then we compared our results with Planck 2018 data (see figure 6.3).

We cannot directly apply the results in chapter 3, 4 to the hierarchy problem since the quantum corrections to WL scalar mass are canceled and Higgs mass cannot be realized at one-loop level. To avoid the feature of this exact cancellation to the hierarchy problem, we extend NG boson to pseudo NG boson as in chapter 5. By extending to pseudo NG boson, the quantum correction to WL scalar mass are generated (see eq.(5.84)) at the scale, where translational symmetry is explicitly broken at a scale much smaller than the compactification scale. Then, it has a possibility to solve the hierarchy problem. Concretely, even if the compactification scale becomes Planck scale $1/L \sim \mathcal{O}(M_{Planck})$, Higgs mass could be realized by the interaction $\varphi\Phi^*\Phi$ generated by some dynamics at $\mathcal{O}(1)$ TeV scale.

There are still some issues to be explored. First, we do not know a new bulk scalar Φ. For example, this scalar field might be the candidate for dark matter. In any case, we need to study what the bulk scalar Φ is. Second, we do not understand the origin of $\varphi\Phi^*\Phi$ (or governing dynamics) and a new coupling κ. As for the origin of $\varphi\Phi^*\Phi$, we discussed that WL scalar φ or φ^* should be expressed by a gauge invariant non-local Wilson line operator in chapter 5. However, it is not easy to clarify the origin of $\varphi\Phi^*\Phi$. We expect that the origin of $\varphi\Phi^*\Phi$ might be connected to the quantum gravity effects, nontrivial backgrounds such as a vortex, or some non-perturbative dynamics. These issues are left for our future study.

We do not also construct a realistic model with flux compactification. In particular, we have not succeeded in applying the flux compactification to gauge-Higgs unification when the WL scalar field is identified with the SM Higgs field. Recently, as an application to gauge-Higgs unification, we discuss the gauge symmetry breaking of six-dimensional theories in flux compactification with a magnetic flux background and a constant vacuum expectation for the WL scalar fields [39]. In [39], the pattern of the electroweak symmetry breaking is shown to be realized in Yang-Mills theory. The realistic model is not however constructed in that the theory does not contain fermions. These issues are also left for our future study.

If we resolve the above issues, we may consider that WL scalar is regarded as Higgs field and an inflaton (called as Higgs inflation) [44–46]. If Higgs inflation succeeds in gauge-Higgs unification with flux compactification, we can explain the origin of inflaton. Although a new coupling (or a corresponding quantity) is constrained by the reheating temperature and the inflaton energy scale (or compact scale) is guessed by scalar power spectrum amplitude [32], we will address these issues in future by considering the unification of WL scalar field, Higgs field and inflaton.

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Appendix A

Poisson Resummation

Poisson resummation is a technique of the calculation for the one-loop effective potential in higher dimensional field theory. In general, if Fourier inverse transformation is defined by

$$\hat{f}(k) = \int_{-\infty}^{\infty} e^{-ikx} f(x) dx, \tag{A.1}$$

the following relation is satisfied:

$$\sum_{n=-\infty}^{\infty} f(n) = \sum_{m=-\infty}^{\infty} \hat{f}(2\pi m), \tag{A.2}$$

where n, m are integer. Eq.(A.2) is the general Poisson resummation. We show this relation as follows.

We use a periodic function $F(x) = f(x+n)$ and consider Fourier transform of this function.

$$\begin{aligned}
\sum_{n=-\infty}^{\infty} f(x+n) &= \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{e^{ikx}}{2\pi} \hat{f}(k) dk e^{ikn} \\
&= \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} e^{2\pi ikx} \hat{f}(2\pi k) dk e^{2\pi ikn} \\
&= \int_{-\infty}^{\infty} dk e^{2\pi ikx} \hat{f}(2\pi k) \left(\sum_{n=-\infty}^{\infty} e^{2\pi ikn} \right).
\end{aligned}$$

Here, using the following formula

$$\sum_{n=-\infty}^{\infty} e^{2\pi ikn} = \sum_{m=-\infty}^{\infty} \delta(k-m), \tag{A.3}$$

we have

$$\sum_{n=-\infty}^{\infty} f(x+n) = \sum_{m=-\infty}^{\infty} e^{2\pi imx}\hat{f}(2\pi m). \tag{A.4}$$

Setting $x = 0$, we obtain eq.(A.2).

In this thesis, we deal with the following function:

$$f(m) = \exp\left[-\frac{(m+a)^2}{R^2 l}\right]. \tag{A.5}$$

By applying Fourier inverse transformation to $f(m)$, $\hat{f}(k)$ can be obtained as

$$\begin{aligned}\hat{f}(k) &= \int_{-\infty}^{\infty} e^{-ikx}\exp\left[-\frac{(x+a)^2}{R^2 l}\right]dx \\ &= \int_{-\infty}^{\infty} \exp\left[-\frac{1}{R^2 l}\{x^2 + (2a + ikR^2 l)x\} - \frac{a^2}{R^2 l}\right]dx \\ &= \int_{-\infty}^{\infty} \exp\left[-\frac{1}{R^2 l}\left\{x + \frac{2a + ikR^2 l}{2}\right\}^2 + ika - \frac{1}{4}k^2 R^2 l\right]dx \\ &= R\sqrt{\pi l}e^{ika}e^{-\frac{1}{4}k^2 R^2 l}. \end{aligned} \tag{A.6}$$

Therefore, we get

$$\sum_{n=-\infty}^{\infty} \exp\left[-\frac{(n+a)^2}{R^2 l}\right] = R\sqrt{\pi l}\sum_{m=-\infty}^{\infty} e^{2\pi ima}e^{-\pi^2 lm^2 R^2}. \tag{A.7}$$

Eq.(A.7) is the same as eq.(3.34).

Appendix B

Detail of $2\mathrm{Tr}[D_L F_{MN} D^L F^{MN}]$

The second term in eq.(4.116) is calculated as

$$
\begin{aligned}
2D_\rho F^a_{\mu m} D^\rho F^{a\mu m} \supset &+4g\varepsilon_{abc}\partial_\mu\partial_\nu\varphi^a\partial^\nu A^{b\mu}\varphi^{c*} + 4g\varepsilon_{abc}\partial_\mu\partial_\nu\bar{\varphi}^a\partial^\nu A^{b\mu}\varphi^c \\
&- 2\sqrt{2}ig\varepsilon_{abc}\partial_\mu\mathcal{D}A^a_\nu\partial^\mu A^{b\nu}\varphi^{c*} + 2\sqrt{2}ig\varepsilon_{abc}\partial_\mu\bar{\mathcal{D}}A^a_\nu\partial^\mu A^{b\nu}\varphi^c \\
&+ 4g^2\varphi^{a*}\varphi^a\partial_\mu A^b_\nu\partial^\mu A^{b\nu} - 4g^2\varphi^{a*}\varphi^b\partial_\mu A^a_\nu\partial^\mu A^{b\nu}.
\end{aligned}
\tag{B.1}
$$

The third term in eq.(4.116) is calculated as

$$
\begin{aligned}
2D_\mu F^a_{56} D^\mu F^{a56} \supset &-2\sqrt{2}g\partial_\mu(\mathcal{D}\varphi^{a*} + \bar{\mathcal{D}}\varphi^a)[\partial^\mu\varphi, \varphi^*]^a - 2\sqrt{2}g\partial_\mu(\mathcal{D}\varphi^{a*} + \bar{\mathcal{D}}\varphi^a)[\varphi, \partial^\mu\varphi^*]^a \\
&+ 2g^2[\partial_\mu\varphi, \varphi^*]^a[\partial^\mu\varphi, \varphi^*]^a + 4g^2[\partial_\mu\varphi, \varphi^*]^a[\varphi, \partial^\mu\varphi^*]^a + 2g^2[\varphi, \partial_\mu\varphi^*]^a[\varphi, \partial^\mu\varphi^*]^a \\
&- 2\varepsilon_{abc}\partial_\mu(\mathcal{D}\varphi^{a*} + \bar{\mathcal{D}}\varphi^a)(\mathcal{D}\varphi^{b*} + \bar{\mathcal{D}}\varphi^b)A^{c\mu} \\
&+ g^2 A^a_\mu A^{a\mu}(\mathcal{D}\varphi^{b*} + \bar{\mathcal{D}}\varphi^b)(\mathcal{D}\varphi^{b*} + \bar{\mathcal{D}}\varphi^b) \\
&- g^2 A^a_\mu A^{b\mu}(\mathcal{D}\varphi^{a*} + \bar{\mathcal{D}}\varphi^a)(\mathcal{D}\varphi^{b*} + \bar{\mathcal{D}}\varphi^b).
\end{aligned}
\tag{B.2}
$$

The fourth term in eq.(4.116) is calculated as

$$
\begin{aligned}
D_l F^a_{\mu\nu} D^l F^{a\mu\nu} \supset &\, 2\sqrt{2}ig\varepsilon_{abc}\varphi^a\Big(\partial_\mu\bar{\mathcal{D}}A^b_\nu\partial^\mu A^{c\nu} - \partial_\mu\bar{\mathcal{D}}A^b_\nu\partial^\nu A^{c\mu}\Big) \\
&- 2\sqrt{2}ig\varepsilon_{abc}\varphi^{a*}\Big(\partial_\mu\mathcal{D}A^b_\nu\partial^\mu A^{c\nu} - \partial_\mu\mathcal{D}A^b_\nu\partial^\nu A^{c\mu}\Big) \\
&+ 4g^2\varphi^{a*}\varphi^a(\partial_\mu A^a_\nu\partial^\mu A^{a\nu} - \partial_\mu A^a_\nu\partial^\nu A^{a\mu}) \\
&- 4g^2\varphi^{a*}\varphi^b(\partial_\mu A^a_\nu\partial^\mu A^{b\nu} - \partial_\mu A^a_\nu\partial^\nu A^{b\mu}).
\end{aligned}
\tag{B.3}
$$

The fifth term in eq.(4.116) is calculated as

$$\begin{aligned}
2D_l F^a_{\mu m} D^l F^{a\mu m} \supset & -2\sqrt{2}g\bar{\mathcal{D}}(\partial_\mu \varphi^a)[\varphi, \partial^\mu \varphi^*]^a + 2\sqrt{2}g\mathcal{D}(\partial_\mu \varphi^{a*})[\varphi^*, \partial^\mu \varphi]^a \\
& - 4g^2[\varphi^*, \partial^\mu \varphi]^a[\varphi, \partial^\mu \varphi^*]^a - 4ig\bar{\mathcal{D}}(\partial_\mu \varphi^a)[\varphi, \bar{\mathcal{D}}A^\mu]^a + 4ig\mathcal{D}\bar{\mathcal{D}}A^a_\mu[\varphi^*, \partial^\mu \varphi]^a \\
& - 4ig\bar{\mathcal{D}}(\partial_\mu \varphi^a)\mathcal{D}([A^\mu, \varphi^*]^a) + 4ig\bar{\mathcal{D}}\mathcal{D}A^a_\mu[\varphi, \partial^\mu \varphi^*]^a - 4ig[\varphi^*, \mathcal{D}A_\mu]^a\mathcal{D}(\partial^\mu \varphi^{a*}) \\
& - 4ig\bar{\mathcal{D}}([A_\mu, \varphi]^a)\mathcal{D}(\partial^\mu \varphi^{a*}) - 2\sqrt{2}g\bar{\mathcal{D}}\mathcal{D}A^a_\mu[\varphi, \bar{\mathcal{D}}A^\mu]^a + 2\sqrt{2}g[\varphi^*, \mathcal{D}A_\mu]^a\mathcal{D}\bar{\mathcal{D}}A^{a\mu} \\
& - 4g^2[\varphi^*, \mathcal{D}A_\mu]^a[\varphi, \bar{\mathcal{D}}A^\mu]^a - 2\sqrt{2}g\bar{\mathcal{D}}\mathcal{D}A^a_\mu\mathcal{D}([A^\mu, \varphi^*]^a) + 4g^2\bar{\mathcal{D}}\mathcal{D}A^a_\mu[\varphi, [A^\mu, \varphi^*]]^a \\
& - 4g^2[\varphi^*, \mathcal{D}A_\mu]^a\mathcal{D}([A^\mu, \varphi^*]^a) + 2\sqrt{2}g\bar{\mathcal{D}}([A_\mu, \varphi]^a)\mathcal{D}\bar{\mathcal{D}}A^{a\mu} \\
& - 4g^2\bar{\mathcal{D}}([A_\mu, \varphi]^a)[\varphi, \bar{\mathcal{D}}A^\mu]^a + 4g^2[\varphi^*, [A_\mu, \varphi]]^a\mathcal{D}\bar{\mathcal{D}}A^{a\mu} \\
& - 4g^2\bar{\mathcal{D}}([A_\mu, \varphi]^a)\mathcal{D}([A^\mu, \varphi^*]^a).
\end{aligned} \tag{B.4}$$

The sixth term in eq.(4.116) is calculated as

$$\begin{aligned}
2D_l F^a_{56} D^l F^{a56} \supset & -\sqrt{2}g\bar{\mathcal{D}}\mathcal{D}\varphi^{a*}\mathcal{D}([\varphi, \varphi^*]^a) - \sqrt{2}g\bar{\mathcal{D}}^2\varphi^a\mathcal{D}([\varphi, \varphi^*]^a) \\
& - \sqrt{2}g\bar{\mathcal{D}}([\varphi, \varphi^*]^a)\mathcal{D}^2\varphi^{a*} - \sqrt{2}g\bar{\mathcal{D}}([\varphi, \varphi^*]^a)\mathcal{D}\bar{\mathcal{D}}\varphi^a + 2g^2\bar{\mathcal{D}}([\varphi, \varphi^*]^a)\mathcal{D}([\varphi, \varphi^*]^a) \\
& - \sqrt{2}g\bar{\mathcal{D}}\mathcal{D}\varphi^{a*}[\varphi, \mathcal{D}\varphi^*]^a - \sqrt{2}g\bar{\mathcal{D}}\mathcal{D}\varphi^{a*}[\varphi, \bar{\mathcal{D}}\varphi]^a + 2g^2\bar{\mathcal{D}}\mathcal{D}\varphi^{a*}[\varphi, [\varphi, \varphi^*]]^a \\
& - \sqrt{2}g\bar{\mathcal{D}}^2\varphi^a[\varphi, \mathcal{D}\varphi^*]^a - \sqrt{2}g\bar{\mathcal{D}}^2\varphi^a[\varphi, \bar{\mathcal{D}}\varphi]^a + 2g^2\bar{\mathcal{D}}([\varphi, \varphi^*])[\varphi, \mathcal{D}\varphi^*]^a \\
& + \sqrt{2}g\mathcal{D}^2\varphi^{a*}[\varphi^*, \mathcal{D}\varphi^*]^a + \sqrt{2}g\mathcal{D}^2\varphi^{a*}[\varphi^*, \bar{\mathcal{D}}\varphi]^a + \sqrt{2}g\mathcal{D}\bar{\mathcal{D}}\varphi^a[\varphi^*, \mathcal{D}\varphi^*]^a \\
& + \sqrt{2}g\mathcal{D}\bar{\mathcal{D}}\varphi^a[\varphi^*, \bar{\mathcal{D}}\varphi]^a - 2g^2\mathcal{D}\bar{\mathcal{D}}\varphi^a[\varphi^*, [\varphi, \varphi^*]]^a - 2g^2\mathcal{D}([\varphi, \varphi^*])[\varphi^*, \bar{\mathcal{D}}\varphi]^a \\
& - 2g^2[\varphi^*, \mathcal{D}\varphi^*]^a[\varphi, \bar{\mathcal{D}}\varphi]^a - 2g^2[\varphi^*, \bar{\mathcal{D}}\varphi]^a[\varphi, \mathcal{D}\varphi^*]^a \\
& + 2\sqrt{2}g^2 f\bar{\mathcal{D}}([\varphi, \varphi^*]^a)[\varphi, \delta]^a - 2\sqrt{2}g^2 f\mathcal{D}([\varphi, \varphi^*]^a)[\varphi^*, \delta]^a \\
& - 2\sqrt{2}g^2 f[\varphi^*, \mathcal{D}\varphi^*]^a[\varphi, \delta]^a - 2\sqrt{2}g^2 f[\varphi^*, \bar{\mathcal{D}}\varphi]^a[\varphi, \delta]^a \\
& - 2\sqrt{2}g^2 f[\varphi, \mathcal{D}\varphi^*]^a[\varphi^*, \delta]^a - 2\sqrt{2}g^2 f[\varphi, \bar{\mathcal{D}}\varphi]^a[\varphi^*, \delta]^a.
\end{aligned} \tag{B.5}$$

δ is a Kronecker's delta which appears when F^a_{56} is expanded around the VEV $\langle A_{5,6}\rangle$ as eq.(4.13). In these decompositions, we have extracted only the cubic terms with a single φ^1 or φ^{1*} and quartic terms with φ^1 and φ^{1*}, which give contributions to one-loop corrections to the the WL scalar mass. After rewriting the original fields to the fields in the mass eigenstate $\widetilde{A}^a_\mu$, $\widetilde{\varphi}^a$, we expand the terms except for the first term in eq.(4.116) in terms of KK modes. Using the orthonormality condition for mode functions, we obtain four-dimensional interaction terms eqs.(4.117)-(4.122).

Appendix C

Tr$[F^4]$

We consider the operator $\mathrm{Tr}[F^4]$ at the second order of $\mathcal{O}(1/\Lambda^2)$. This operator can be expressed as

$$\mathrm{Tr}[F_{MN}F^{MN}F_{AB}F^{AB}] = F^a_{MN}F^{bMN}F^c_{AB}F^{dAB}\mathrm{Tr}[t^at^bt^ct^d]. \tag{C.1}$$

Using $[t^a, t^b] = i\epsilon^{abc}t^c$ and $\{t^a, t^b\} = \delta^{ab}1_{2\times 2}/2$, $\mathrm{Tr}[t^at^bt^ct^d]$ becomes

$$\begin{aligned}
\mathrm{Tr}[t^at^bt^ct^d] &= \frac{1}{4}\mathrm{Tr}\Big[([t^a, t^b] + \{t^a, t^b\})([t^c, t^d] + \{t^c, t^d\})\Big] \\
&= \frac{1}{4}\mathrm{Tr}\left[\left(i\epsilon^{abl}t^l + \frac{1}{2}\delta^{ab}1_{2\times 2}\right)\left(i\epsilon^{cdm}t^m + \frac{1}{2}\delta^{cd}1_{2\times 2}\right)\right] \\
&= \frac{1}{8}(\delta^{ab}\delta^{cd} - \delta^{ac}\delta^{bd} + \delta^{ad}\delta^{bd}).
\end{aligned}$$

Thus, $\mathrm{Tr}[F^4]$ is described as

$$\begin{aligned}
\mathrm{Tr}[F_{MN}F^{MN}F_{AB}F^{AB}] &= \frac{1}{8}(\delta^{ab}\delta^{cd} - \delta^{ac}\delta^{bd} + \delta^{ad}\delta^{bd})F^a_{MN}F^{bMN}F^c_{AB}F^{dAB} \\
&= \frac{1}{8}F^a_{MN}F^{aMN}F^b_{AB}F^{bAB} \\
&= 2\left(-\frac{1}{4}F^a_{MN}F^{aMN}\right)^2.
\end{aligned} \tag{C.2}$$

Eq.(C.2) means that the interaction between WL scalar and gauge fields is reproduced from the square of eq.(4.1) or eq.(4.6). Since we are interested in the three-point intxeraction terms involving φ or φ^* and the four-point interaction terms involving φ and φ^*, we extract these three-point and four-point interaction terms from eq.(C.2).

The result is

$$
\begin{aligned}
&\mathrm{Tr}[F_{MN}F^{MN}F_{AB}F^{AB}] \\
&\supset -2f^2 \Bigg(-\frac{g}{\sqrt{2}} \Big\{ -\mathcal{D}A^a_\mu[A^\mu,\varphi^*]^a + \bar{\mathcal{D}}A^{a\mu}[A_\mu,\varphi]^a \Big\} + g^2[A_\mu,\varphi]^a[A^\mu,\varphi^*]^a \\
&\qquad\qquad + \frac{g}{\sqrt{2}} \Big\{ \mathcal{D}\varphi^{a*}[\varphi,\varphi^*]^a + \bar{\mathcal{D}}\varphi^a[\varphi,\varphi^*]^a \Big\} - \frac{1}{2}g^2[\varphi,\varphi^*]^a[\varphi,\varphi^*]^a \Bigg) .
\end{aligned} \tag{C.3}
$$

Eq.(C.3) is the same structure of the last term of the third line, the fourth and fifth line in eq.(4.44) except for the coefficients. We have computed the right hand side of eq.(C.3) in the section 4.3. Thus, we can calculate the quantum corrections to WL scalar mass from eq.(C.3). The structures of loop integrals in the quantum corrections are the same as the subsection 4.4.1 and 4.4.2 with $\xi = 1$. These loop integrals vanish as in the subsection 4.4.1. The quantum corrections are also generated by using the cubic interactions (C.3), (4.48) and (4.52) in $\mathcal{O}(1/\Lambda^4)$. Since cubic interactions (C.3), (4.48) and (4.52) are the same structure except for the coefficients, the quantum corrections by using these interactions also vanish.

Appendix D

Hurwiz zeta function

In this appendix, we summarize the property of Hurwitz zeta function [47]. Hurwitz zeta function is defined as

$$\zeta[s,a] = \sum_{n=0}^{\infty} \frac{1}{(n+a)^s}. \tag{D.1}$$

It is known that Hurwitz zeta function is related to Riemann zeta function by the following identical equations

$$\zeta[s,1] = \zeta(s), \tag{D.2}$$

$$\zeta[s,1/2] = (2^s - 1)\zeta(s). \tag{D.3}$$

Hurwitz zeta function also satisfies the following formula

$$\zeta[s,a] = \zeta[s,a+m] + \sum_{n=0}^{m-1} \frac{1}{(n+a)^s}. \tag{D.4}$$

Note that $\zeta[s,1] = \zeta[s,0]$ is satisfied. Since Riemann zeta function has a property $\zeta(-2n) = 0$ (n is a natural number), Hurwitz zeta function also satisfies

$$\zeta[-2n,1] = 0, \quad \zeta[-2n,1/2] = 0. \tag{D.5}$$

In particular, $\zeta[s,1/2]$ satisfies $\zeta[0,1/2] = 0$ in $s = 0$ case. Furthermore, Hurwitz zeta function can be expressed by Bernoulli polynomials $B_n(x)$ as follows

$$\zeta[-n,x] = -\frac{B_{n+1}(x)}{n+1}. \tag{D.6}$$

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